

# Geodetic airborne laser altimetry of Breidamerkurjökull and Skeidarárjökull, Iceland, and Jakobshavns Isbræ, West Greenland

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**ABSTRACT.** Two geodetic airborne laser altimeter (ALA) systems coupled to Global Positioning System receivers acquired submeter-resolution topographic profiles of the lower parts of Breidamerkurjökull and Skeidarárjökull, Iceland, in May 1989 and September 1991 (Skeidarárjökull) and of Jakobshavns Isbræ, Greenland, in April 1992. Maximum measured crevasse depths on Breidamerkurjökull, Skeidarárjökull and Jakobshavns Isbræ were 20.7, 36.1 and 50.2 m, respectively. Crevasse spacings were 43 m (45 crevasses km<sup>-1</sup>) for Breidamerkurjökull, 46–51 m (25 crevasses km<sup>-1</sup>) for Skeidarárjökull and 20–40 m (lower part) or 50–80 m (upper part) of Jakobshavns Isbræ (27 crevasses km<sup>-1</sup>). Surface slopes were ~2.4° for the lower 11 km of Breidamerkurjökull, ~0.8° for the lower 10 km of Skeidarárjökull and 1.55° for the lower 28 km of Jakobshavns Isbræ (with a range of 0.55° for the final 17 km to ~6.3° for a steep central part several km in length). Average longitudinal strain-rate values, estimated by assuming a bulk ice temperature of 0°C and a density of 880 kg m<sup>-3</sup>, ranged from 0.12 a<sup>-1</sup> for Breidamerkurjökull, to 0.63 a<sup>-1</sup> for Skeidarárjökull; values for Jakobshavns Isbræ fell between 1.3 and 1.7 a<sup>-1</sup>. Remote sensing of glacier microtopography by ALA offers a potential new tool for determining crevasse morphology, spatial density and spacing, meter-scale local slopes, long-wavelength gradients and derived strain rates.

## INTRODUCTION

In mathematical models of glacier dynamics, subglacial topography, ice thickness, crevasse depth and surface slope are important boundary conditions (Nye, 1957; Lliboutry, 1965; Paterson, 1981; Hutter, 1983). Typical surface-feature spacing wavelengths, including those associated with crevasses and other surface undulations, can be determined using spectral-analysis techniques, if topographic data with sufficient resolution are available. This question of resolution is a long-standing issue in quantitative geology, but it is especially relevant to glaciers because of the difficulty of measuring many of the common surface features on these landforms (e.g. crevasses) (Price, 1982; Williams and others, 1991). Remote-sensing methods for directly determining glacier-surface topography at a resolution suitable for defining or constraining fundamental glaciological variables, such as longitudinal strain rates are therefore justified (Thomas and others, 1991). Therefore, in this study, our goal is objectively and systematically to measure the depths, widths, spacings, and spatial densities of glacier-surface deformation features (e.g. crevasses), and from these measurements gain additional insights into glacier-flow dynamics and the mechanics of deformation.

On 28 May 1989, during a series of airborne laser altimeter (ALA)/Global Positioning System (GPS) surveys of various landforms in southern Iceland (Garvin and Williams, 1992a, b), ALA/GPS profiles were acquired of the lower parts of two outlet glaciers. Breidamerkurjökull and Skeidarárjökull are outlet glaciers of Vatnajökull, a large icecap (8300 km<sup>2</sup>) in southeastern Iceland with 43 named outlet glaciers (Williams, 1986a, 1987). Breidamerkurjökull was selected for an ALA/GPS survey because it had previously been the site of an intensive aerial photogrammetric and field study of its terminus (10 m contour interval) and proglacial deposits by the Department of Geography at the University of Glasgow (Price, 1982; Welch, 1967; Howarth and Welch, 1969) and an analysis of its recession since 1894 by Sigbjarnarson (1970). Breidamerkurjökull originates in the southeastern part of Vatnajökull, and drains about 14% of the total area of the icecap (Sigbjarnarson, 1970). On 23 September 1991, after Skeidarárjökull had nearly completed a major surge, another ALA/GPS survey was carried out. On 22 April 1992, as part of a NASA laser/radar altimetry mission over the Greenland ice sheet by R.H. Thomas and colleagues, an ALA/GPS survey was completed of the lower 30 km of Jakobshavns Isbræ, the fastest-flowing

glacier known (Williams, 1986b; Echelmeyer and others, 1991).

## METHODS

Remote sensing of topography has most typically involved satellite radar-altimeter sensors such as Seasat, GEOS-3, Geosat and most recently ERS-1; see Bufton (1989) for general details. Such orbital altimeter systems provide kilometer-scale horizontal sampling of sea-surface and ice-sheet topography, with a statistical vertical precision of greater than 1 m. However, such radar systems are unable to measure the spatially varying topography of glaciers at length scales of meters to tens of meters. Stereophotogrammetric analysis of low-altitude aerial photographs was, until recently, the only means for determining sub-10 m scale topographic characteristics of rugged, complex surfaces, such as those commonly encountered on highly crevassed regions of glaciers or steeply sloping glaciers. New airborne and spaceborne synthetic aperture radar interferometric techniques offer great promise for mapping in two dimensions the 10–100 m scale topographic properties of glacier surfaces at vertical resolutions approaching 1 m (Zebker and Goldstein, 1986; Gabriel and others, 1989).

Bufton (1989), Bufton and others (1991) and Gardner (1992) describe the current state-of-the-art in airborne and spaceborne laser-altimeter instruments, with attention to those sensors developed by the National Aeronautics and Space Administration (NASA). We exploited recent technological innovations in a NASA laser-altimeter system known as the Airborne Oceanographic Lidar (AOL) (Hoge and others, 1984) and the Airborne Terrain Laser Altimeter System (ATLAS), both operating in the terrain profiling mode, to acquire geodetically precise topographic profiles of the two outlet glaciers in southeastern Iceland and one in West Greenland. The incorporation of Global Positioning System (GPS) aircraft-navigation instrumentation into the AOL system (Krabill and Martin, 1987) permitted us to quantify aircraft vertical position to approximately 30–50 cm over length scales of tens to hundreds of km. The combination of a high-sample-rate aircraft laser altimeter system, such as the AOL with kinematic GPS tracking of aircraft position enabled us, for the first time, from an aircraft, to measure surface topography of a glacier with a vertical precision of approximately 30 cm root-mean-square (r.m.s.). Our GPS-tracked AOL system topographic profile down Breidamerkurjökull in May 1989 was the initial demonstration of a remote-sensing method we propose to call “geodetic airborne laser altimetry” or GALA. A high signal-to-noise GALA topographic profile extending from a position on Breidamerkurjökull at 64.2° N, 16.3° W, down-glacier to the terminus at 64.0° N, 16.2° W was acquired with horizontal sampling of ~0.5 m and with vertical control of approximately 30 cm r.m.s. The total profile length of 16.6 km from Breidamerkurjökull to the ocean required ~3 min to acquire and generated about 36 000 independent elevation samples across the glacier surface. The data were acquired at an approximately constant altitude of ~400 m above the surface of the glacier. Further details

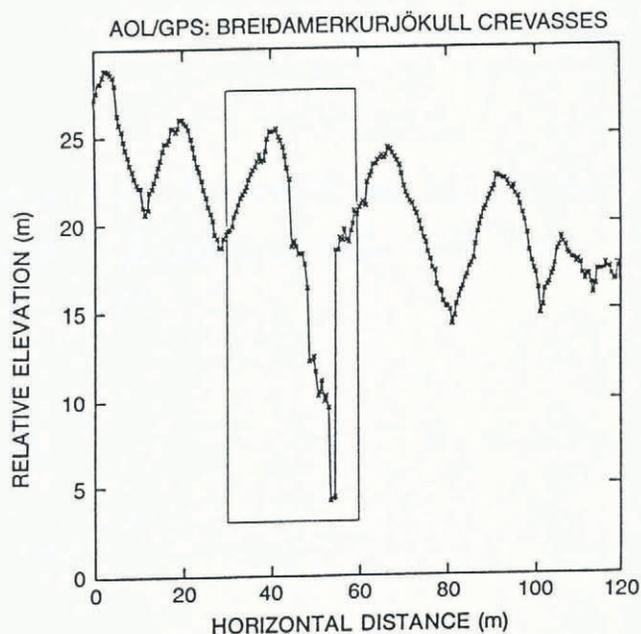


Fig. 1. Airborne laser altimeter (AOL) profile of a deep crevasse (20.7 m) near the terminus of Breidamerkurjökull, Iceland, on 28 May 1989.

of the design and application of GALA instrumentation can be found in Krabill and Martin (1987), Bufton and others (1991), Hoge and others (1984). The rest of this paper demonstrates how GALA topographic data can be used to quantify morphologic characteristics of any glacier surface, with examples from two glaciers in Iceland and one in Greenland.

## GALA DATA

A NASA P-3 research aircraft equipped with a GALA system (for example, AOL with Motorola “Eagle” GPS receivers) was deployed to: (1) southeastern Iceland on 28 May 1989 to acquire topographic profiles of the lower part of Breidamerkurjökull (Fig. 1) and Skeidarárjökull; (2) southeastern Iceland on 23 September 1991 to acquire topographic profiles of the lower part of the post-surge Skeidarárjökull (Figs 2a, b and 3); and (3) Greenland on 22 April 1992 to acquire topographic profiles of the lower part of the fast-flowing Jakobshavn Isbræ (Figs 4 and 5a, b, c). The capability of directly measuring the extreme topography of crevasses (Figs 1 and 5c) by means of GALA techniques enables quantitative comparisons of derived physical parameters such as strain rate ( $\dot{\epsilon}_x$ ) across the entire length of a complex glacier. The application of standard signal analysis methods (Turcotte, 1992) for rapid determination of wavelengths (for example, typical spacing) of surface features such as crevasses is possible with GALA datasets.

## CREVASSE DEPTH

Quantification of the microtopography and horizontal spacing of major transverse crevasses provides significant

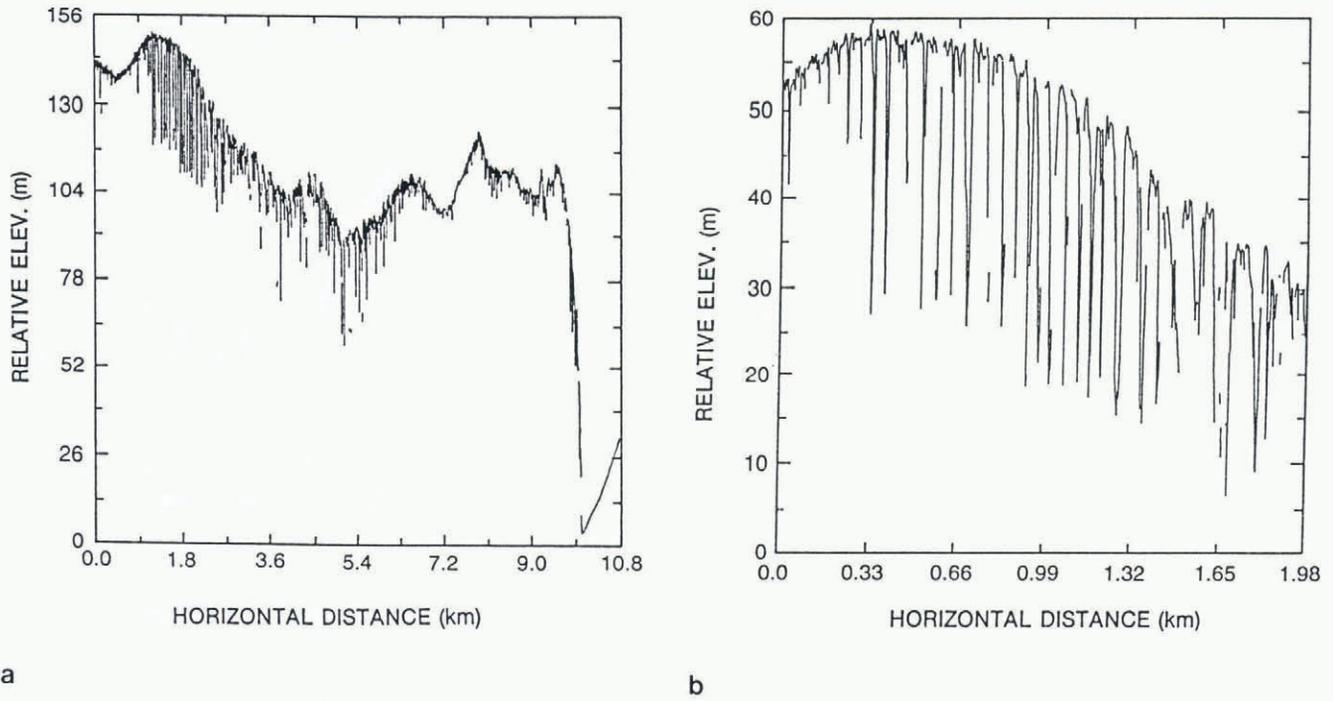


Fig. 2. Airborne laser altimeter (ATLAS) profiles of crevasses on Skeidarárjökull, Iceland, on 23 September 1991, following a surge: a, about 10 km up-glacier from the terminus (right side of profile) and b, a region 8 km up-glacier from the terminus (see Figure 3 for oblique aerial photograph).



Fig. 3. Oblique 35 mm aerial photograph looking east at a highly crevassed region of Skeidarárjökull, Iceland, following a surge. Photograph acquired by J. B. Garvin simultaneously with the ALA profile shown in Fig. 2b.

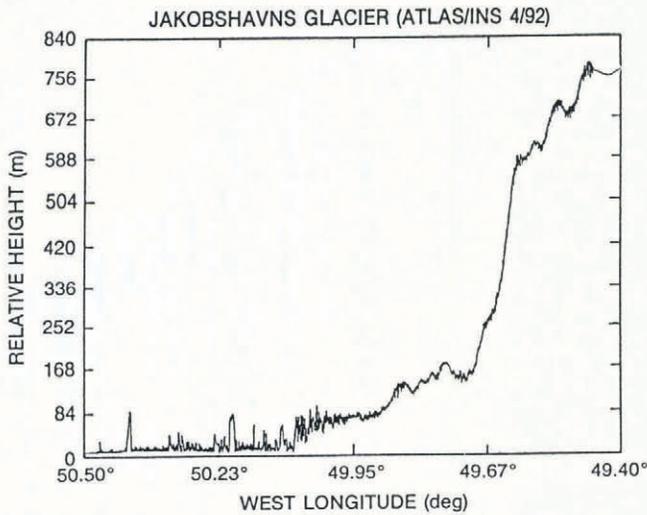


Fig. 4. Airborne laser altimeter (ATLAS) profile of the lower 17 km of Jakobshavns Isbræ, Greenland, on 22 April 1992. Iceberg-filled fiord to the left of the terminus. Note the concentration of crevasses and surface undulations up-glacier from the terminus.

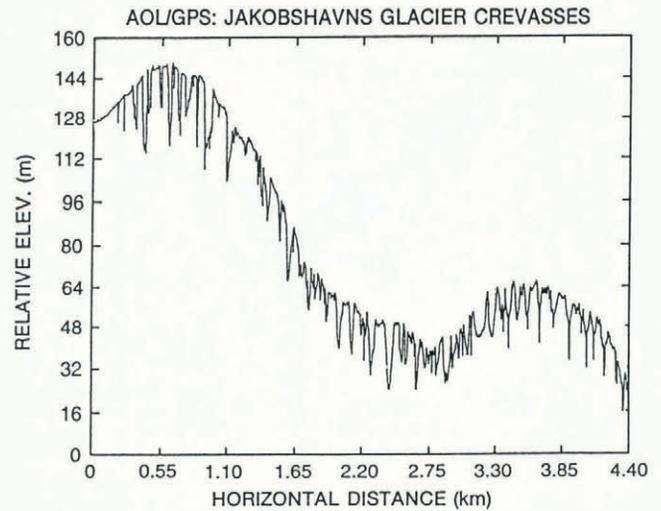
new information about glacier surfaces. The ALA successfully measured crevasse depths on the lower part of the three glaciers. Only Breidamerkurjökull seems to have been affected by significant snow accumulation in its crevasses; typical crevasse depths ranged from 8.6 to 13.5 m. The maximum crevasse depth was 20.7 m near the terminus (Fig. 1). Skeidarárjökull had crevasse depths of from 18 to 22 m; the maximum depth measured was 36.1 m. Jakobshavns Isbræ had typical crevasses in a depth range of 21 to 25 m; the maximum crevasse depth was 50.2 m (Fig. 5c). At wavelengths between 10 and 100 m, highly crevassed glacier surfaces display large topographic variances (up to 22 m on Breidamerkurjökull) with local slopes in excess of 41°. This observation suggests that advanced synthetic aperture radar systems, including those employing interferometric techniques to measure topography at spatial scales from 10–100 m, will have difficulties separating the effects of 10–30 cm roughness from 10–100 m scale topographic variance associated with deep crevasses (Gabriel and others, 1989).

**CREVASSE SPACING**

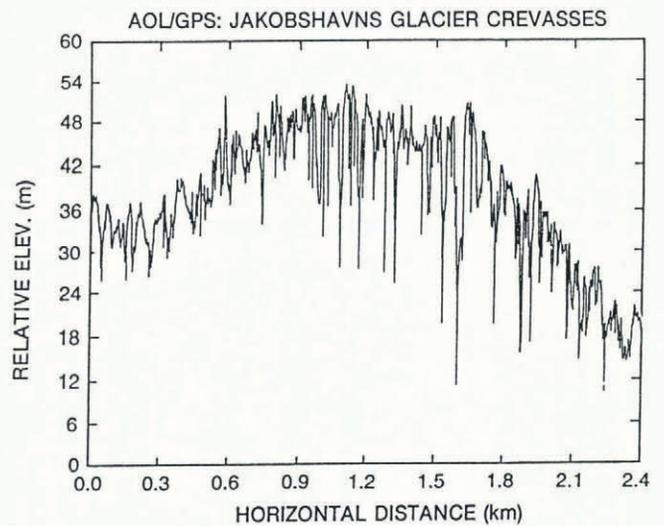
Spectral analysis of periodic topography has been employed for more than 30 years as a standard technique for terrain characterization (Turcotte, 1992; Garvin and Williams, 1992a). We employed such techniques to estimate the power-spectral density (PSD) function of a surface in the form of an equation that relates the amplitude (or power) of the topography *A* to a given wavelength ( $\lambda$ ):

$$A = k\lambda^\beta,$$

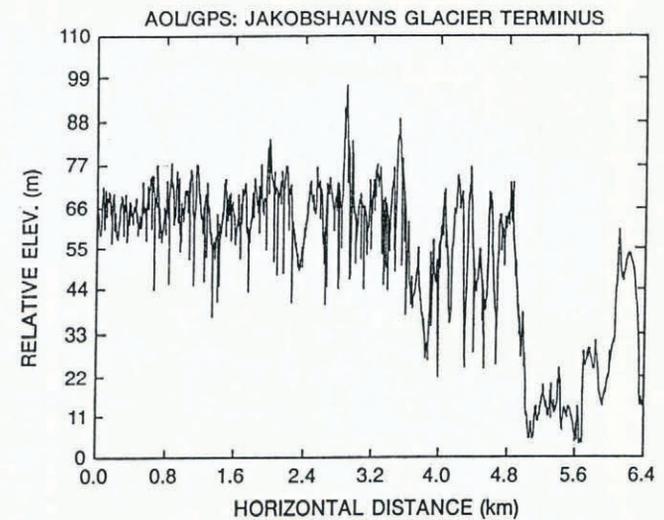
where *k* and  $\beta$  are constants derived from a least-squares regression of the data relating *A* to  $\lambda$  for a topographic profile. We can then investigate whether there are



a



b



c

Fig. 5. Enlarged part of airborne laser altimeter (AOL) profile shown in Figure 4: a, crevasses on surface undulations in the upper part of the profile; b, crevasses on a surface undulation in the middle part of the profile; and c, the 70 m high terminus and lower part of the profile.

significant “dominant wavelengths” associated with a given surface sub-unit by subtracting the best-fitting power law from the original PSD function. From this process, we can identify statistically significant residual wavelengths and attempt to relate them to periodic crevasse spacing. A PSD analysis of the lower, snow-free part of Breidamerkurjökull gave a typical crevasse spacing of 43 m; Skeidarárjökull had crevasses typically spaced from 46 to 51 m. Jakobshavns Isbræ had a typical crevasse spacing of 20–41 m in its lower part and 50–80 m in its upper part.

### CREVASSE DENSITY

The spatial density of crevasses on the three glaciers was estimated from the GALA data. For the lower few kilometers of Breidamerkurjökull, the spatial density of crevasses along the center line of the glacier was  $\sim 45$  crevasses  $\text{km}^{-1}$  (Table 1), in contrast with 25 crevasses  $\text{km}^{-1}$  for the final 10 km (e.g. 10 km up-glacier from the terminus) of Skeidarárjökull. The crevasse density statistics for typical crevassed sections of Jakobshavns Isbræ varied from  $\sim 19$  crevasses  $\text{km}^{-1}$  (as measured from the 3.27 m sampling resolution of the ATLAS dataset) to  $\sim 27$  crevasses  $\text{km}^{-1}$  (as estimated from the 0.54 m sampling resolution of the AOL data). This demonstrates that a six-fold increase in horizontal sampling frequency (for example, from 3.27 to 0.54 m) increases the detectability of apparent crevasses by up to 46%.

### SURFACE SLOPE

Surface slopes for Breidamerkurjökull were  $\sim 2.44^\circ$  for the lower 13 km and  $\sim 0.8^\circ$  for the lower 10 km of Skeidarárjökull. Local slopes, depending on the baseline selected, can be quite variable. Jakobshavns Isbræ has a  $1.55^\circ$  slope over its lower 28 km; a slope of  $\sim 6.3^\circ$  over its central part,

and a slope of  $0.55^\circ$  (Fig. 4) over the final 17 km, including the floating terminus.

### DERIVED STRAIN RATE

In his summary on the physics of glaciers, Paterson (1981) presented a simple theory to account for the maximum depth of crevasses originally formulated by Nye (1957). Nye (1957) first developed a simple relationship between maximum crevasse depth  $d$  and longitudinal strain rate  $\dot{\epsilon}_x$  as follows:

$$d = 2(\dot{\epsilon}_x/A)^{1/n}(\rho g)^{-1}$$

where  $\rho$  is the density of ice,  $g$  is the gravitational acceleration, and  $A$  and  $n$  are glacier-ice flow-law parameters. Nye's model predicts the strain rate  $\dot{\epsilon}_x$  (in  $\text{a}^{-1}$ ) that corresponds to the depth within a glacier at which the principal tensile stress approaches zero. We can solve the Nye equation for  $\dot{\epsilon}_x$ , because our GALA measurements provide estimates of  $d$  for a large number of crevasses within the glacier. Solving for  $\dot{\epsilon}_x$  in terms of  $d$  yields:

$$\dot{\epsilon}_x = A(0.5\rho g d)^n.$$

If we assume an  $n = 3$  flow law and using an estimated average temperature for temperate glacier ice in southeastern Iceland of  $\sim 0^\circ\text{C}$ , we can use Paterson (1981) to select a suitable value for  $A$ :  $A = 5.3 \times 10^{-15} \text{ s}^{-1} (\text{kPa})^{-3}$  at  $0^\circ\text{C}$ . If we let  $\rho = 880 \text{ kg m}^{-3}$  for ice and  $g = 9.81 \text{ ms}^{-2}$ , we can then estimate  $\dot{\epsilon}_x$  for any measured value of  $d$  as follows:

$$\dot{\epsilon}_x = 1.34 \times 10^{-5} d^3 \text{ (in } \text{a}^{-1}\text{)}$$

where  $d$  is the maximum crevasse depth in meters. Most of the largest crevasses measured on Breidamerkurjökull were about 20 m deep, suggesting strain rates in the vicinity of  $0.110 \text{ a}^{-1}$ , that is within the range observed (e.g.  $\dot{\epsilon}_x$  are  $0.004\text{--}0.16 \text{ a}^{-1}$ ) by Hambrey and Müller

Table 1. Laser altimetry of glacier crevasses

Glacier name	Data source	Down-glacier slope	Crevasse spatial density	Maximum crevasse depth	Typical crevasse spacing	Nye* strain rate	Dilatation stagnant ice	Dilatation active ice
	sampling	°	crevasses $\text{km}^{-1}$	m	m	$1 \text{ a}^{-1}$	$1 \text{ a}^{-1}$	$1 \text{ a}^{-1}$
Breidamerkurjökull (5/89)	AOL/GPS (dx = 0.5 m)	2.4	45	20.7	43	0.119	0.017	0.060
Skeidarárjökull (9/91)	ATLAS (dx = 1.7 m)	0.8	25	36.1	51	0.630	0.092	0.316
Jakobshavns Isbræ (4/92)	ATLAS/INS (dx = 3.2 m)	1.6**	19	45.6	41	1.27	0.185	0.637
Jakobshavns Isbræ (4/92)	ATLAS/GPS (dx = 0.5 m)	1.6**	27	50.2	28	1.70	0.247	0.850

\* Nye theory strain rate  $\dot{\epsilon}_x @T = 0^\circ\text{C}$ ,  $\rho(\text{ice}) = 880 \text{ kg m}^{-3}$  for  $n = 3$  (temperate case).

\*\* Varies from 0.5 to  $6.3^\circ$ .

(1978) for White Glacier on Axel Heiberg Island, northern Canada, and also from Lliboutry (1965) for Mer de Glace in France ( $\dot{\epsilon}_x = 0.14 \text{ a}^{-1}$ ). We explored the sensitivity of our results to variations in glacier ice density and temperature. Temperature affects computed  $\dot{\epsilon}_x$  values most severely; a decrease of  $10^\circ\text{C}$  alters  $\dot{\epsilon}_x$  by a factor of 10 (downward), while variations in bulk ice density affect  $\dot{\epsilon}_x$  values by a factor of 1–4 at most (e.g. varying from  $830\text{--}900 \text{ kg m}^{-3}$  increases the strain rates by a factor of 4). Longitudinal strain-rate estimates are based upon the observed maximum depths of the largest crevasses within the GALA profile acquired and may not necessarily be representative.

Given the typical morphometry of glacier crevasses and the fact that most contain snow at the bottom unless newly formed during the ablation season, it is probable that our GALA data systematically underestimated the maximum depths of the crevasses (Fig. 1). We believe, however, that routine GALA surveys of glaciers could provide an extensive database of crevasse depths from which to explore further strain rates within various types of glaciers.

Given the assumptions implicit in the Nye (1957) model that relates maximum crevasse depth to longitudinal strain rate  $\dot{\epsilon}_x$ , one can evaluate the dilatations (in  $\text{a}^{-1}$ ) that can be estimated from crevasse depths using relationships developed by Lliboutry (1965). Lliboutry (1965) suggested that for active temperate glaciers, the dilatation  $|a|$  can be estimated using an equation of the form:

$$|a| = 6.72 \times 10^{-6} d_{\text{crev}}^3$$

given crevasse depth  $d_{\text{crev}}$  in meters. This equation is very similar to the Nye (1957) strain rate model for  $n = 3$ , when the temperature is approximately  $-5^\circ\text{C}$ . When the glacier is in a stagnant state, such as is the case in its lower part after a surge (for example, Skeidarárjökull in the late summer of 1991), Lliboutry (1965) suggests that the dilatation  $|a|$  in  $\text{a}^{-1}$  is:

$$|a| = 1.95 \times 10^{-6} d_{\text{crev}}^3.$$

Table 1 summarizes the ranges of dilatations computed using the Lliboutry equations above as a function of the observed maximum crevasse depths, in comparison with the Nye model strain rates (for  $n = 3$ ,  $T = 0^\circ\text{C}$ ,  $\rho = 880 \text{ kg m}^{-3}$ ). The maximum dilatations are generally a factor of two lower than the Nye model strain rates, as would be expected (Lliboutry, 1965). Thus, our impression is that there is reasonable evidence that the strain rates for highly active or recently surged outlet glaciers in Iceland and Greenland range from  $0.32$  to  $1.7 \text{ a}^{-1}$ , in contrast with less active glaciers such as Breidamerkurjökull.

## SUMMARY

Sub-meter resolution geodetic airborne laser altimeter (GALA) profiles of Breidamerkurjökull and Skeidarárjökull outlet glaciers in Iceland and of Jakobshavns Isbræ in Greenland were analyzed. The maximum crevasse depth ranges from 21 m on Breidamerkurjökull (Fig. 1) to 50.2 m on Jakobshavns Isbræ (Fig. 5c), assuming no snow

in the crevasses. Typical crevasse spacings and density for Breidamerkurjökull were 43 m from power-spectral density analysis ( $45 \text{ crevasses km}^{-1}$ ), 46–51 m on Skeidarárjökull ( $25 \text{ crevasses km}^{-1}$ ), and 20–41 m for the lower part of Jakobshavns Isbræ and 50–80 m for its upper part ( $18$  to  $20 \text{ km}^{-1}$ ). Surface slopes were  $\sim 2^\circ$  for the lower 11 km of Breidamerkurjökull and  $\sim 0.8^\circ$  for the lower 10 km of Skeidarárjökull. Jakobshavns Isbræ has a  $1.55^\circ$  slope over its lower 28 km but the final 17 km has a slope of  $0.55^\circ$ . The steeper central part has a surface slope of  $\sim 6.3^\circ$  (Fig. 4).

Nye model strain rates range from  $0.12 \text{ a}^{-1}$  for the Breidamerkurjökull outlet glacier to as high as  $0.63 \text{ a}^{-1}$  for the recently surged Skeidarárjökull outlet glacier. In contrast are the strain rates inferred for the heavily crevassed regions of Jakobshavns Isbræ in West Greenland, where values in excess on  $1.2 \text{ a}^{-1}$  were computed. Dilatations computed using the Lliboutry (1965) model for active glacier ice are generally a factor of two lower than the Nye strain rates, and range from  $0.06 \text{ a}^{-1}$  for Breidamerkurjökull to  $0.85 \text{ a}^{-1}$  for Jakobshavns Isbræ. Both the strain-rate and dilatation values derived from our GALA measurements of maximum crevasse depths for the three outlet glaciers suggest deformation rates that exceed those previously summarized in the literature (e.g. Lliboutry, 1965; Paterson, 1981). Glacier-flow rates can also be inferred from our results; e.g.  $3910 \text{ m a}^{-1}$  for Jakobshavns Isbræ and  $900 \text{ m a}^{-1}$  for Skeidarárjökull. Sub-meter-scale topographic cross-section are required to characterize accurately difficult-to-measure surface features of glaciers, such as crevasses. Future Earth-orbiting altimeter systems, including those under study as part of NASA's Earth Observing System (EOS) Project, will not be able to provide the spatial resolution necessary (e.g.  $\leq 10 \text{ m}$ ) to characterize salient surface textures on glaciers for the purpose of characterizing aspects of glacier surface deformation and flow. GALA methods may be the only practical and reliable means for measuring accurately the depth and morphometry of glacier crevasses.

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