NOTE ON THE DEFINITION OF THE INTERNATIONAL ATOMIC TIME TAI

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ABSTRACT. Relations among the three concepts (TAI, coordinate time, and proper time) are discussed and comments of the definition of the TAI are given.

1. Introduction

Since the establishment of the atomic time scale in 1956 at the BIH and the presence of its notation TAI in 1971 (Guinot, Seidelmann, 1988), two possible definitions of the TAI(Guinot, 1990) are being concerned from time to time: a coordinate time in a non-rotating geocentric reference system; or the proper time at a specified location on the Earth. Although TAI was defined as a coordinate time in 1980 by the CCDS, discussions between different opinions are still carrying on frequently. It is quite possible that a clearer understanding of the TAI is still needed in nowadays. The purpose of this paper is to make a further explanation on the relations among the three concepts: TAI, coordinate time, and proper time, then some comments on the definition of the TAI are given.

2. Relation between the proper time τ of a Earth fixed clock and the coordinate time T in a non-rotating reference system

The relation between the proper time r_i of a clock i, which is fixed on the Earth, and the coordinate time T of the non-rotating geocentric reference system is:

$$dr'_{i} = \left(1 - \frac{2\Phi_{i}}{c^{2}}\right)^{\frac{1}{2}}dT,$$
 (1)

 Φ_i is the sum of gravitational and centrifugal potential of the i clock. If the same clock is moved to the geoid, the proper time τ_i of the corresponding fictitious clock can be related to the coordinate time T as:

$$d\tau_i = (1 - \frac{2\Phi_o}{c^2})^{\frac{1}{2}} dT,$$
 (2)

 Φ_{o} represents the corresponding potential of the fictitious clock.

So, we have:

e:
$$d\tau_{i} / d\tau'_{i} = (1 - \frac{\Phi_{o} - \Phi_{i}}{c^{2}})$$
(3)

From (3), the time scale established by the i clock can be changed to the fictitious clock's. The existence of equation (2) demonstrates that τ_i is not the coordinate time T itself, but it is one of the members of the coordinate time family and relates to T with a constant factor k. Let us use t to express it and the following relation can thus be written:

$$dt = kdT = d\tau_i$$
 (4)

It can be concluded here, the proper time τ_i of a Earth fixed clock, after being reduced to the fictitious clock's on the geoid, has its duality: it is the proper time τ_i of the fictitious clock on the geoid, but it is also the coordinate time t of the non-rotating geocentric reference system at the same time.

Relations among TAI, proper time τ, and the coordinate time t

As a result from the proper times τ_i (i=1,2,...,n) of a certain number of fictitious clocks on the geoid, what is the nature of the TAI now?

In practice, we can not establish the TAI without using the concept of coordinate synchronization. So, it is easy to believe that TAI is only a coordinate time now, and not a proper time anymore (Huang et al., 1989). Is it the only answer that we can have?

We first discuss the case in which only real clocks are concerned. Let us suppose that clocks are fixed on the geoid and located at the same place. Due to the existence of the equation (4), it is clear that the obtained TAI will be no difference whether the concept of coordinate synchronization or the concept of so-called standard synchronization (Huang et al., 1989) is used in comparing clocks. Then, clocks are moved to different locations but still fixed on the geoid. The equivalence of the two kinds of synchronization will still exist. So, if the TAI is obtained from the readings of the fixed real clocks on the geoid, it can be regarded as coordinate time and also proper time as well at the same time even we have only used the concept of coordinate synchronization in comparing clocks.

Now we come to the case that we are actually confronting in establishing TAI. The answer depends on the understanding of the "proper time of fictitious clocks". If it can be regarded as a proper time, why we will then be not able to regard the TAI as a proper time? It should be noticed that the approximation of regarding TAI as a proper time is the same as that of the actual expression which we have used to define the coordinate time.

4. Discussion

- (1) In considering the problem of the definition of TAI, it is possible to understand it in a wider sense in practice. It is not the unique solution that we have to restrict ourself to a unchangeable definition.
- (2) It is correct to define the TAI as a coordinate time in the non-rotating geocentric reference system, but it is not necessary to exclude the TAI absolutely from the concept of proper time at the same time. It depends on the accuracy of the demanding in theory. It is possible to define TAI as a proper time at a specified location on the Earth in practice. Its actual meaning is the mean, or weighted mean, of the readings of fictitious clocks which are fixed on the geoid.

References

Guinot.B., Seidelmann, P.K.: 1988, Astron. Astrophys. 194,304 Guinot.B.: 1990, Circular 2 of the Working Group on Reference System, Sub-Group on Time Huang Tianyi et al.: 1989, Progress in Astronomy, Vol. 7,43