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Finite irreducible monomial groups of small prime degree

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We present a classification of the finite irreducible complex monomial groups of degree p, where $p = 2, 3, 5, 7, 11$. This is done by giving a family of groups indexed by countably many nonnegative integer parameters. Each relevant group is $GL(p, \mathbb{C})$-conjugate to exactly one member of the family, and every member of the family is a relevant group. The family is presented by giving generating matrices for each group in the family, and every generating matrix is given as an explicit function of the indexing parameters.

The study of finite irreducible complex linear groups dates back to the last century. Until recently, authors have been content to describe only the finite primitive linear groups, and those only up to conjugacy in the appropriate projective group. With the advent of computer algebra systems, a more exhaustive classification (as described in the previous paragraph) has become desirable. Conlon [**1**] was the first to move in this direction, classifying the finite irreducible complex linear p-groups of degree p. Recently Flannery [**3**] and Höfling [**4**] classified all monomial and imprimitive non-monomial groups of degree 4. This work complements those papers, and the combined results are to be implemented as a GAP data library.

This thesis explicitly presents the classification for degrees 2, 3, 5, 7, and 11. Most of the results in this document apply to arbitrary prime degree. Our strategy follows the procedure of Conlon [1] and Flannery [**2**], which is apparently due to Szekeres. Broadly speaking, it involves three steps, outlined below.

Every finite irreducible monomial group of degree n is an extension of an abelian group A (the *diagonal subgroup*) by a transitive permutation group T of degree n (the *projection group*). The diagonal subgroup A is a module for T, which acts by conjugation. We proceed on a case by case basis, according to the group T. For each possible T, first determine the possibilities for A (the Submodule Problem). Secondly determine which extensions of A by T are realised as irreducible monomial groups (the Extension Problem). Thirdly, divide these monomial extensions into conjugacy classes, and select exactly one representative from each class (the Conjugacy Problem).

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A prerequisite for this procedure is a list of the possibilities for T; this amounts to a table of contents listing the cases to be addressed. In our context this means a list of the transitive permutation groups of prime degree (up to conjugacy in the relevant symmetric group). Such a list is conveniently available in P.M. Neumann's paper [**5**].

For many primes, every case is covered by this thesis, and all that remains is the mechanical step of explicitly computing generating matrices, using the formulae we provide. There are infinitely many primes which require additional case analysis, but they are rare (even amongst primes). The only prime degrees less than 1000 which exhibit these extra cases are 7 and 11 (for which we have solved the extra cases), and 13, 17, 23, 31, 73, 127, 257, 307 and 757. The methods established in this document should be sufficient to complete the classification in any of these remaining cases as well.

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