

Revisiting Taylor's analysis of the Trinity test

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Abstract

The atomic bomb uses fission of heavy elements to produce a large amount of energy. It was designed and deployed during World War II by the United States military. The first test of an atomic bomb occurred in July 1945 in New Mexico and was given the name Trinity; this test was not declassified until 1949. In that year, Geoffrey Ingram Taylor released two papers detailing his process in calculating the energy yield of the atomic bomb from pictures of the Trinity explosion alone. Many scientists made similar calculations concurrently, although Taylor is often accredited with them. Since then, many scientists have also attempted to calculate a yield through various methods. This paper walks through these methods with a focus on Taylor's method—based on first principles—as well as redoing the calculations that he performed with modern tools. In this paper, we make use of state-of-the-art computer vision tools to find a more precise measurement of the blast radius, as well as using curve fitting and numerical integration methods. With more precise measurements we are able to follow in Taylor's footsteps toward a more accurate approximation.

Impact Statement

The goal of this work is to provide a modern look at the Trinity test using tools that were unavailable to Taylor and other scientists at the time, and to reconduct the same experiments that led Taylor to his famous calculation of the energy yield. We also aim to provide a comprehensive background on the multiple methods that scientists, both modern and past, have used to calculate the energy yield of the Trinity test and provide a comparison between values.

1. Introduction

In most introductory engineering fluid mechanics and thermodynamic courses, the work of Sir Geoffrey Ingram Taylor (TaylorI, 1950, TaylorII, 1950) is presented as an important application of dimensional analysis: using the Buckingham Pi theorem (Buckingham, 1914) to estimate the power of an atomic bomb by studying a few images of the blast. Over the years, there have been numerous surveys of Taylor's work (Deakin, 2011a,b; Díaz, 2021), all emphasizing that Taylor *did not* invoke the dimension reduction analysis he is often incorrectly credited with. Arguably, his work was far more profound; starting with

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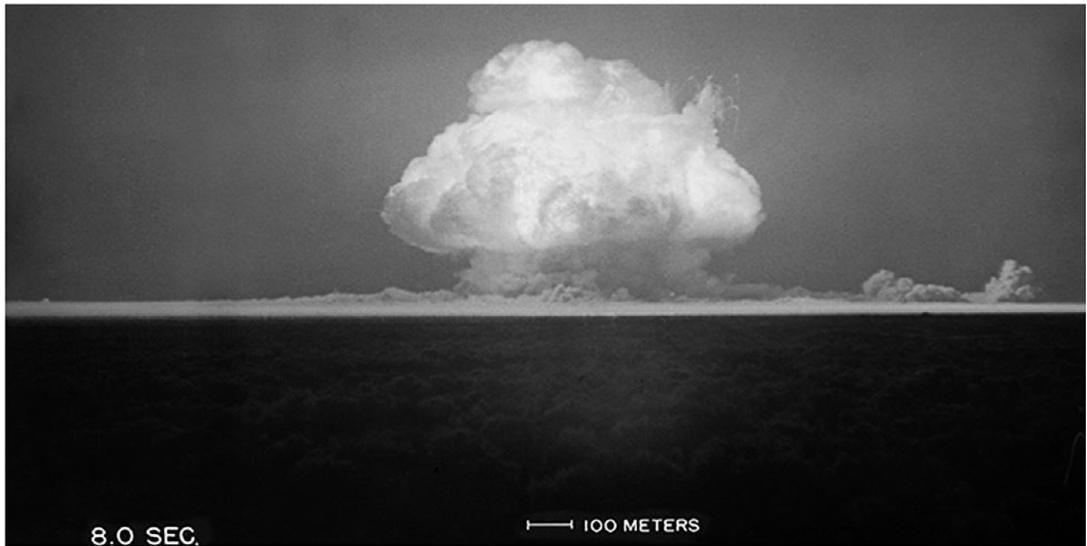


Figure 1. Picture of the Trinity explosion (nps.gov, 2023).

underlying equations and introducing a series of simplifications in concert with observations to work out a chosen quantity of interest. The approach Taylor takes is particularly salient and relevant for the broad (beyond fluids and thermodynamics) data-centric engineering (DCE) audience, as it offers insights into studying data-driven physical processes. In this brief survey article, the work of Taylor is revisited and communicated using a series of illustrations, state-of-the-art computer vision software, and commentary—particularly on uncertainty in equations and data. The aim is to make Taylor’s original papers more accessible, and in doing so highlight how much information regarding a problem can be obtained without resorting to computational simulations. The exposition here is distinct from the aforementioned surveys and is intended for the DCE readership that cuts across engineering, statistics, and machine learning.

2. Background

The Trinity explosion, shown in [Figure 1](#), was the name for the first test of the atomic bomb that occurred on July 16, 1945 in New Mexico, just a month before the bombs were dropped into Hiroshima and Nagasaki. The test bomb released a huge amount of energy, causing radioactive fallout in the area and a blast that could be seen from space, reaching 40,000 feet in only 7 minutes (Szasz, 1995). The atomic bomb would go on to end the war and have devastating consequences for the civilians of Japan, raising profound questions on the very nature of our existence and the use of weapons of such cataclysmic destruction. Those involved in the Manhattan project¹, that is, the name of the team that developed the atomic bomb, as well as others such as Taylor, spent much time actualizing the idea of a fission bomb and understanding the effects of such a device.

There are multiple approaches one can take when calculating the energy of a blast, such as one from an atomic bomb. During the time of the Trinity test and World War II, there were three main minds working on the problem. Taylor, the focus of this paper, is often cited with the calculation as he was the only one who gave proof of the validity of a point source model, as well as calculating the energy within a range of error. Two others, Hungarian-American John von Neumann and Russian Leonid Sedov, also performed similar calculations at the time, and actually achieved better accuracy than Taylor. Taylor’s approach was

¹ We note that our submission coincides with the release of Christopher Nolan’s *Oppenheimer*, which details the life of the chief scientist of the Manhattan project, Robert J. Oppenheimer.

Table 1. Summary of Trinity test calculations

Author	Energy (kilotons TNT)
Taylor (TaylorII, 1950)	16.8
Sedov, von Neumann (Deakin, 2011b)	16.9
Deakin (Deakin, 2011b)	17.5
Hanson (Hanson et al., 2016)	22.1
Truman (Batchelor, 1996)	20
Groves (Groves, 1945)	15–20

to use partial differential forms of the equations for motion, continuity, and state as well as integral forms of energy to arrive at an approximate formula for the energy of the blast wave. Sedov and von Neumann followed similar approaches that began with dimensional analysis. The two then determined integral forms for energy and computed them analytically, although neither directly calculated energy. A more in-depth exploration of their calculations is provided in their own papers (Sedov, 1946; Neumann, 1958) and in the work of Deakin (2011b). All three did not find exact values as the radiative energy was not taken into account (Deakin, 2011b).

Table 1 shows the energy estimates of various authors, including Taylor's estimate. Taylor's, Deakin's, Sedov's, and von Neumann's calculations are all under the assumption that the specific heat ratio γ is constant and equal to 1.4. Hanson's calculations—done more recently—use physical samples from the explosion to calculate the amount of plutonium and other elemental constituents of the bomb, from which the yield is determined (Hanson et al., 2016). President Truman and General Groves, who were involved in the Manhattan project, released the values presented in the table once the information was declassified (Deakin, 2011a,b).

2.1. Notation

Table 2 contains the variables used both in this paper and by TaylorI (1950), TaylorII (1950), and will be used throughout this brief paper.

2.2. Dimensional analysis approach

Prior to detailing Taylor's approach, it will be useful to touch upon dimensional analysis with the Buckingham Pi theorem. This theorem states that with N variables and K fundamental units there are $P = (N - K)$ dimensionless quantities (Buckingham, 1914). By using this theorem, we can identify the unit-less quantities that will enable us to calculate the energy of the explosion. To complete such an analysis, the three scientists, Taylor, Sedov, and von Neumann, reduced the explosion to starting from a point source and expanding outward in a spherical shock wave, rapidly releasing large amounts of energy.

Table 2. Nomenclature

Symbol	Quantity	Symbol	Quantity
p	Pressure	p_0	Atmospheric pressure
ρ	Density	ρ_0	Atmospheric density
u	Radial velocity	t	Time
E	Energy	γ	Heat capacity ratio
R	Shock radius	r	Radial position

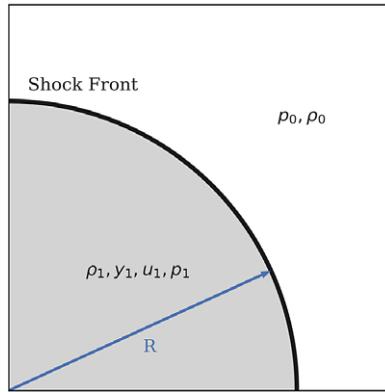


Figure 2. Flow properties inside and outside a spherical shock front.

The use of a shock wave suggests that the state of the air can be expressed in two categories: internal shock wave properties and external conditions of the undisturbed atmosphere. Therefore, we can devise seven variables: E the energy, R the radius, t the time, p the static pressure of the wave, p_0 the pressure of the undisturbed atmospheric pressure, ρ the static density of the shock wave, and ρ_0 the undisturbed density of the atmospheric density (see Figure 2). All variable units, when broken down into their base units, contain three fundamental units: mass, length, and time. In terms of the Buckingham Pi theorem, there are $N = 7$ variables and $K = 3$ fundamental quantities; therefore, this problem contains $P = 4$ dimensionless quantities (Deakin, 2011b).

We wish to focus on a quantity that involves the target variable of energy, as well as easily measurable quantities, which in this case are radius, time, and the density of undisturbed air. To find the exact form of the dimensionless quantity, we can use dimensional analysis to solve for energy. In SI units, energy is recorded in Joules (J) which is equivalent to a Newton-meter ($N \cdot m$) or kilogram-meter-squared-per-second-squared ($kg \cdot m^2/s^2$). Therefore, an equation for energy must contain units of mass (kg), length (m), and time (s). Mass is not directly used within the context of the problem, instead density (units of kg/m^3) takes its place. In Taylor’s paper, he is interested in the time since the blast, the density of the undisturbed atmosphere, and the radius of the shock wave. Thus, the equation for energy has the form

$$E = K\rho_0^a R^b t^c \tag{1}$$

where a, b, c , and K are the constants (Díaz, 2021).

There are a few important points to note regarding the above. First, energy is per second squared, and since neither density nor radius contain units of time, the equation for energy must contain time, leading to the negative second power, or $c = -2$. Second, density is the only variable that contains units of mass, and since both energy and density contain mass only to the first power, then a must equal 1. However, the inclusion of density adds an additional m^{-3} to the equation that must be corrected in order to get m^2 in the final form. Therefore, to fix this, we need a factor of m^5 , which can be provided by the radius, meaning $b = 5$ and we have that

$$E = K\rho_0 R^5 t^{-2}, \tag{2}$$

where K is a constant and proven to be a function of γ by Taylor (1950). This dimensional analysis is taken from Díaz (2021).

2.3. Taylor’s first principles approach

While many believe that Taylor used the above approach (Deakin, 2011b), he instead used first principles and similarity assumptions to arrive at an equation for energy. He eventually arrived at an expression of the

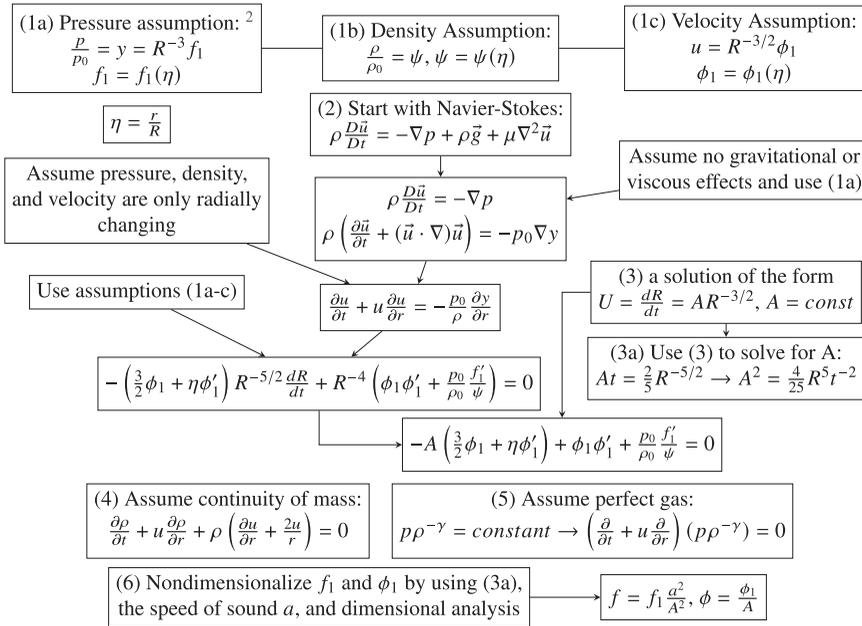


Figure 3. Introduction of nondimensional pressure, density, velocity, and first principles analysis.

same form as equation (2), identifying the constant K . Taylor’s process and assumptions are captured in Figures 2–4; a detailed derivation is provided in the Data Availability Statement section.

Taylor provides the similarity assumptions in Figure 3 (1a-c) based on his earlier works in 1946 and 1950 on spherical detonation and expansion; he comes to the conclusion these are the appropriate forms for the expansion of a blast, proving that they obey continuity and motion as outlined above. In Figure 3, Taylor uses Navier–Stokes under the assumption of incompressible flow and no gravitational or viscous effects, resulting in the incompressible Euler equations. This process lead him to three equations for the derivatives of the non-dimensional ratios $f, \psi,$ and ϕ of pressure, density, and velocity, respectively, where $f' = df/d\eta, \phi' = d\phi/d\eta,$ and $\psi' = d\psi/d\eta$. These are expressed as

$$f' \left((\eta - \phi)^2 - \frac{f}{\gamma} \right) = f \left(-3\eta + \phi \left(3 + \frac{1}{2}\gamma \right) - 2\frac{\gamma\phi^2}{\eta} \right) \tag{3}$$

$$\phi' (\eta - \phi) = \frac{1}{\gamma} \frac{f'}{\psi} - \frac{3}{2} \phi. \tag{4}$$

$$\frac{\psi'}{\psi} = \frac{\phi' + \frac{2\phi}{\eta}}{\eta - \phi} \tag{5}$$

Using equations (3)–(5), Taylor performs a discretized step (first-order) calculation for the values of $f, \phi,$ and ψ using

$$\begin{aligned} f(\eta_i) &\approx f(\eta_{i-1}) + f'(\eta_{i-1}) \cdot (\eta_i - \eta_{i-1}), \\ \phi(\eta_i) &\approx \phi(\eta_{i-1}) + \phi'(\eta_{i-1}) \cdot (\eta_i - \eta_{i-1}), \\ \psi(\eta_i) &\approx \psi(\eta_{i-1}) + \psi'(\eta_{i-1}) \cdot (\eta_i - \eta_{i-1}), \end{aligned} \tag{6}$$

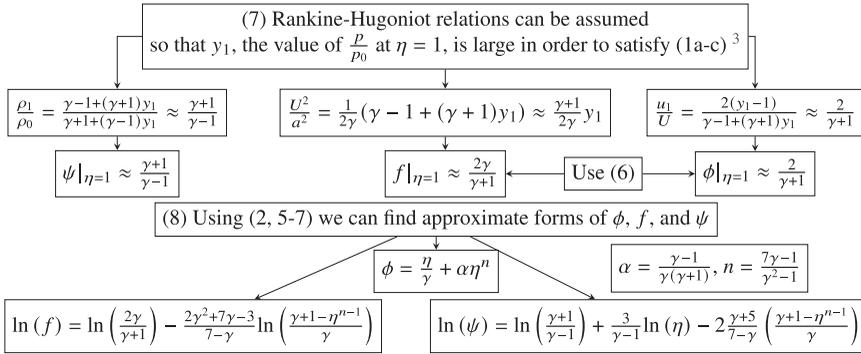


Figure 4. Approximate forms of nondimensional parameters.

where $\eta_i - \eta_{i-1}$ represents a small decrement in the nondimensional radius. Taylor performs his step calculation over the nondimensional radius range $\eta \in [0.5, 1]$ with intervals of $\eta_i - \eta_{i-1} = -0.02$ for the values of f , ϕ , and ψ . He sets the initial conditions in equation (6) to be at $\eta = 1$, found by the Rankine–Hugoniot solutions and calculated in box (7) of Figure 4. In this case, f , ψ and ϕ can be expressed solely as functions of γ (see top half of Figure 4). From this calculation and the processes outlined in Figures 3 and 4, Taylor finds approximate forms of f , ϕ , and ψ . Note that while his exact procedure is difficult to follow, we performed a similar analysis, plotted as a black line (labeled step calculations) in Figure 5.

To solve for analytical forms of the three nondimensional quantities, Taylor substitutes equation (5) into (3), to arrive at

$$\frac{f'}{f}(\eta - \phi) = \gamma\phi' - 3 + \frac{2\gamma\phi}{\eta}. \tag{7}$$

However, this equation cannot be directly solved unless f' is assumed small, such that the f' term can be ignored. This assumption can be made at early values of η where ϕ is approximately linear as f minimally increases in a lower η range, as shown in Figure 5, at η values of <0.6 and $f' < 0.035$. Using this assumption, he evaluates the initial approximate form as $\phi = \frac{\eta}{\gamma}$. He notes this form, shown as the blue dashed line (labeled initial approximation) in Figure 5, does not agree well with the step calculations above $\eta \approx 0.7$, where the f' small assumption breaks down. He suggests the approximate form would not be linear—containing a term of order n . This form is shown in box (8) of Figure 4. Taylor calculates the values of the constants α and n using the initial conditions set forth by the Rankine–Hugoniot solutions at $\eta = 1$. From the corrected approximate form of ϕ , he used equations (3)–(5) to arrive at approximate forms for ψ and f as well (Taylor, 1950), as shown in step (8) of Figure 4. These corrected approximation forms are shown as red dashed lines (labeled approximate form) in Figure 5. One can observe that the step calculations (black) have relatively good agreement with the corrected approximate forms (red), with almost complete agreement for ψ . Once Taylor found these approximate forms, he was able to calculate the constant K in his energy equation.

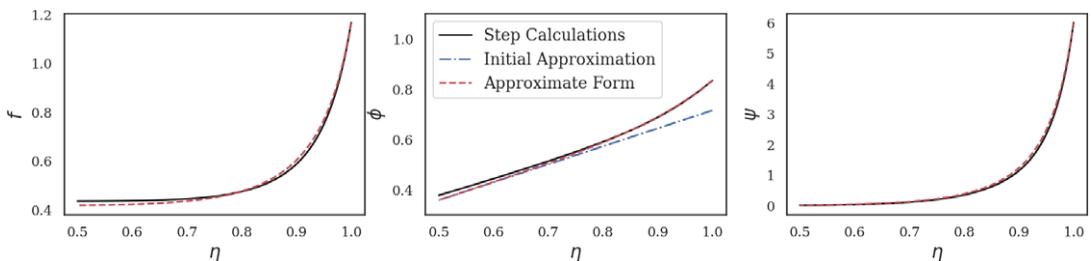


Figure 5. f , ϕ , and ψ versus η .

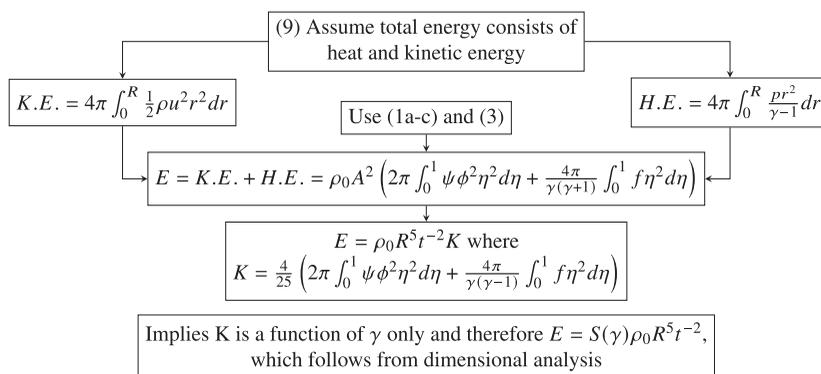


Figure 6. Energy calculations.

In Figure 6, we show his process for formulating the equation for energy. This marks the end of what he accomplished in his first paper with regard to calculating the energy of the blast (TaylorI, 1950), as values for radius and time are still needed. Once the Trinity test was declassified, he obtained those values, and in his second paper (TaylorII, 1950), he linearly interpolates data on the log scale plot of $R^{5/2}$ versus t and uses the intercept value for $R^5 t^{-2}$ in his energy calculations.

3. Revisiting Taylor’s analysis

In this paper, we take the same approach as Taylor but make use of modern techniques to improve upon his calculation. We started by reviewing his work, as documented in the previous section, to get a full understanding of his approach and assumptions.

To emulate his radius measurements, we made use of the open source Computer Visualization Annotation Tool (Sekachev et al., 2020) to annotate the sets of photos by Mack, the U. S. Atomic Energy Commission (1946), and others presented by Taylor in his second paper (TaylorII, 1950), though we could not find an image for $t = 3.26$ ms. With this tool, we made an outline of the blast for earlier times as well as annotating two circles that represent the smallest enclosing circle and the largest enclosed circle for all times. Later times do not include a blast outline as the entire blast is not visible within the photographs. For later time stamps, an arrow is provided by Mack and U. S. Atomic Energy Commission (1946) that designates the shock front, and this is used as the maximum circle for these times. The minimum enclosing circle was checked using the Welzl method (Welzl, 2005) and the maximum enclosed circle by testing circles centered at Voronoi points (Voronoi, 1908) of the polygon created by the blast outline. An example of this method along with the radii that Taylor provided for $t = 0.36$ and $t = 3.53$ ms is shown in Figure 7a,b. The larger of the two circles is closer to the approximation that Taylor made, and is the more likely radius as the shock wave is the radius we are interested in, which occurs in front of the fire blast.

The assumption that Taylor makes based on his first principles analysis, shown in Figure 3 step (3a), is that the radius and time are related by $At = \frac{2}{5}R^{-5/2}$ where A is a constant. Therefore, the radius and time of the blast should correspond to:

$$\frac{5}{2} \log(R) = \log(t) + \log(n) \tag{8}$$

where n is a constant and n^2 corresponds to the value of $R^5 t^{-2}$. Using the radii found in the previous paragraph, we plot the logarithmic-scale graph of $R^{5/2}$ versus t as Taylor did in his second paper (TaylorII, 1950); this is shown in Figure 7c. This figure contains three lines with a slope of one whose y-intercepts correspond to values of $R^5 t^{-2}$ for the minimum, maximum, and average radii based on equation (8). These values are calculated using standard linear least-squares interpolation to fit a linear approximation. Figure 7 also displays a zoomed-in view of the calculations from $t \in [3.53, 4.61]$ ms as the difference

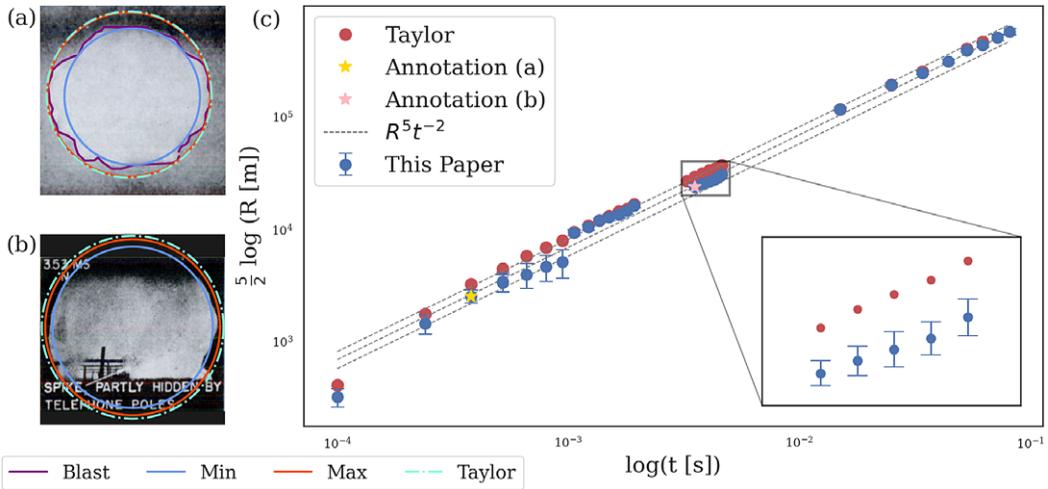


Figure 7. Estimating the blast radius from images taken at different times and plotting the data on a logarithmic scale.

Table 3. K Values computed using (9) contrasted with those reported by Taylor

γ	This paper	Taylor
1.2	1.720	1.727
1.3	1.145	1.167
1.4	0.852	0.856
1.667	0.495	0.487

between our values and Taylor’s lead to a large discrepancy in the comparison of energy values shown in the next section.

To calculate K in the energy equation shown in Figure 6, we used the equations for ϕ , ψ , and f put forth by Taylor in his first paper (TaylorI, 1950) and shown in Figure 3 step (8). These integrals do not have an analytical solution, so the open-source code equadratures (Seshadri et al., 2021) was used to computationally estimate these integrals, that is,

$$K \approx \frac{4\omega_j}{25} \left(2\pi \sum_{j=1}^{J=20} \psi(\eta_j) \phi^2(\eta_j) \eta_j^2 + \frac{4\pi}{\gamma(\gamma-1)} \sum_{j=1}^{J=20} f(\eta_j) \eta_j^2 \right) \quad (9)$$

where $\{\eta_j, \omega_j\}_{j=1}^{J=21}$ are Gauss–Legendre quadrature points and weights of order 20 across the support $\eta \in [0, 1]$. This order was found to be sufficiently high for numerical precision (to nine decimals), a convergence analysis can be found in the GitHub link. We calculated K using (9) for 10 values of γ in the range $[1.2, 1.667]$. The values calculated using this method show minor differences from those Taylor calculated; see Table 3. It is difficult to ascertain what approach Taylor took in calculating the integrals, as he simply states in TaylorI (1950) that he evaluates the integrals and uses a step calculation. Based on the differences in the K calculation alone (not considering differences in radius/time), this would put our energy ranges within a 1.89% difference from Taylor.

Using the values for $R^5 t^{-2}$ and K calculated in (8) and (9), respectively, the total energy was estimated using the same equation as Taylor, shown in Figure 6 step (9). This was done for both the minimum and maximum radii. These values are contrasted to those that Taylor presents in his paper (TaylorII, 1950).

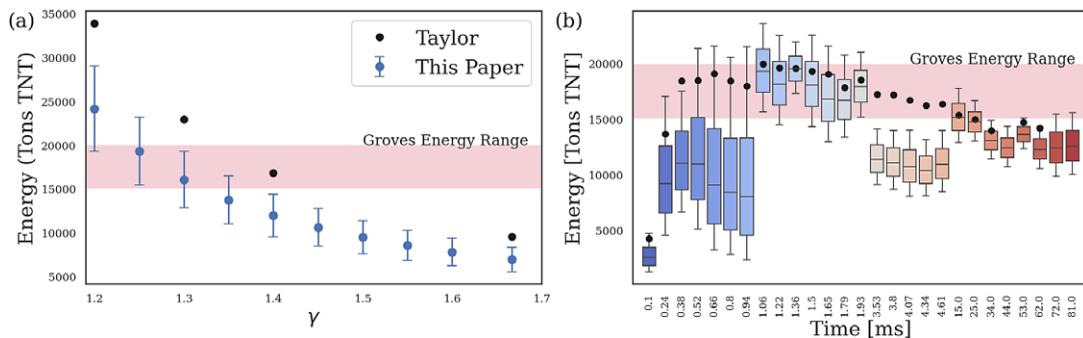


Figure 8. Energy graphs: (left) energy estimates from this paper (blue error bars) across different values of γ ; (right) energy estimates from this paper (box plot) for $\gamma = 1.4$ at different times.

4. Discussion

Figure 8 shows the results laid out in the previous section. Figure 8a shows 10 energy values at $\gamma \in [1.2, 1.667]$ where the R^5t^{-2} value is taken from the average of the two circles representing the blast, with the blue error bars encompassing the standard deviation, assuming a uniform distribution between the minimum and maximum radius values. This uncertainty in radius was propagated to the uncertainty in energy via a standard Monte Carlo run. The red shaded rectangle in both (a) and (b) shows the energy range that was given by General Groves in Table 1; his values were used as the range because he was directly involved in the Manhattan Project, and therefore the Trinity test. Figure 8b shows the energy value using the R, t pairs and the box plot of the energy for those pairs. The scale of the x-axis is not linear and is set to have equal spacing between time measurements to have a better perspective of the box plots. The agreement with Groves' energy estimate is best between 1 and 3 ms. This is as expected as the point source solution is invalid at earlier times, such as the first time stamp of 0.10 ms. Higher uncertainties at earlier times are seen as earlier times have a rougher blast outline, making the minimum and maximum radii encompass a larger interval than later times, which have fewer outlying points on the blast cloud. Additionally, the spherical model is no longer valid at later times; approximations at these times can be seen in Figure 9.

From Figure 8, we can see that our calculations, specifically for the larger of the two radii, are in very good agreement with those of Taylor. As well as being within the acceptable range of Energy values for $\gamma \approx [1.2, 1.4]$, which Taylor stated was a likely range for the value of γ . In (b), however, between 3.53 and

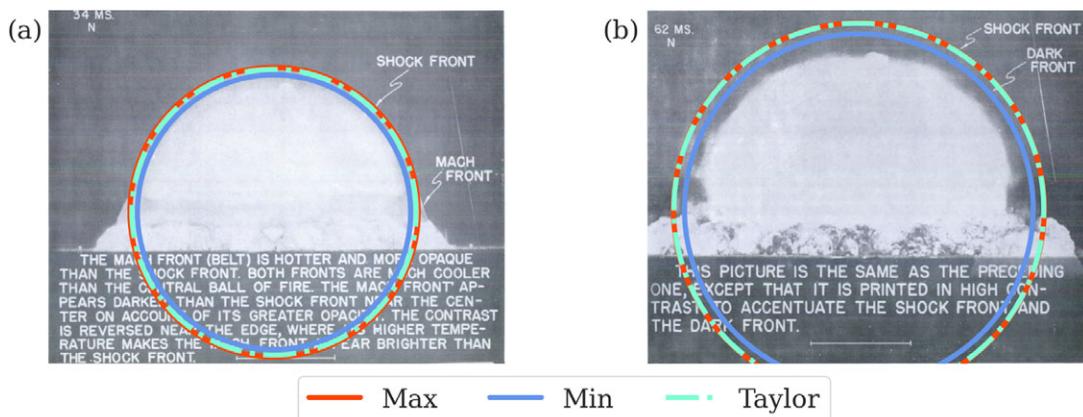


Figure 9. Computer Visualization Annotation Tool (CVAT) annotations compared with Taylor's values at later times.

4.61 ms, the energy values we calculated are different from those calculated using Taylor's radius and time stamps. This is due to differences in the radii measured as these images are unclear and the window is not wide enough to view the entire blast; therefore, a certain radius has been assumed, which must have been different for us and Taylor. All annotated blast images are provided on the GitHub link.

Additionally, small differences in radii can lead to large differences in energy as the radii is raised to the fifth power. Thus, where the radii difference between Taylor and this paper for $t = 3.53$ ms differs by only 2.3 m, the energy difference is around 3000 tons TNT. Moreover, despite the uniform uncertainty in radius, Figure 8 shows nonuniform uncertainty in energy, seen better at earlier times, this is due to the energy equation being dependent on R^5 .

5. Conclusions

Taylor's approximation was, for the time and even today, a very good approximation of the energy. His work built upon certain governing equations and a series of assumptions to yield relatively simple calculations. More broadly, the fusion of governing equations with data—inferred from a series of images—for estimating a physical quantity of interest is a task that likely underscores many DCE efforts. Our hope in writing this relatively short paper is that the detailed methods serve as a useful example of how one can: (i) leverage governing equations without necessarily resorting to costly numerical simulations and (ii) utilize dimensional analysis to gain valuable quantitative insight into a problem.

6. Lessons learned

- While dimensional analysis provides a simple process for finding the structure of a driving equation, it cannot account for constant values as Taylor's first principles approach can.
- While not exact, using reliable assumptions (like those pursued by Taylor) can provide one with an easier process to calculate a value within a margin of error.
- Revisiting the analysis with modern tools assisted in providing a value range, more accurately representing the photographic data, and quantifying the uncertainty, an aspect Taylor failed to include.
- This is very useful in situations where a back-of-the-envelope-type calculation is required, and where perhaps expensive simulations are not feasible or necessary.
- Attempting a recreation of historical works and accessing the same references is difficult with the differences in technology and an inability to contact the author.

Author contribution. Conceptualization: P.S.; Data curation: E.M., P.S.; Formal analysis: E.M.; Investigation: E.M., P.S.; Methodology: P.S.; Project administration: P.S.; Resources: P.S.; Software: E.M., P.S.; Supervision: P.S.; Validation: P.S.; Visualization: E.M.; Writing—original draft and review: E.M., P.S.

Competing interest. The authors declare none.

Data availability statement. The GitHub with all the code/annotation files is published here <https://github.com/03emone/Trinity-Taylor-Files>.

Ethical standard. The research meets all ethical guidelines, including adherence to the legal requirements of the study country.

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