Mr. Whipple was elected Chairman; Professor Nunn and the retiring Chairman, Mr. Abbott, were elected Vice-Chairmen. The Treasurer and Secretaries were re-elected.

Dr. Dyson delivered an address dealing with the uses that might be made of astronomical facts in teaching elementary mathematics and giving some account of recent astronomical research.

Mr. Abbott read a paper on "Practical Work in Elementary Mathematics." This was illustrated by models actually made by boys in the Polytechnic School. There was also an exhibit of models and apparatus in the Mathematical Laboratory. A discussion followed, in which Mr. Daniell, Professor Bickerton, Mr. Boon, and Mr. Ellis took part. Mr. R. F Davis distributed some graphed copies of useful geometrical illustrations of a formula in differential calculus. It is hoped that other members will follow this excellent example.

CORRESPONDENCE.

THE EDITOR OF THE *Mathematical Gazette.*

DEAR SIR,—Is it not time that public opinion should refuse any longer to permit the time-honoured laxity in the use of the words "because" and "therefore" in mathematical text-books?

Thus, in books on elementary geometry, the proof that the opposite angles of a parallelogram $ABCD$ are equal is usually given somewhat as follows. Join BD. Then *because* BD meets the parallels AB and DC, the angles ABD and CDB are equal. Again, *because* BD meets the parallels AD and BC, the angles CBD and ADB are equal. *Therefore* the whole angle at B equals the whole angle at D.

This definitely asserts that the angles B and D are equal because BD is drawn, while presumably we merely wish to say that we are able to detect the equality of the angles because we have drawn BD.

To take another illustration, here is a passage from a book on algebra (the best with which I am acquainted): "By the square-root process we can show that $\sqrt{4{\cdot}053}=2{\cdot}0132\ldots$;

$$2{\cdot}0132<\sqrt{\sqrt{5}+\sqrt[3]{6}}."$$

It would surely be better to avoid the statement that one number is smaller than another because we can use the square-root process.

While instances as marked as these are to be found everywhere, still more numerous are statements which, though less strikingly indiscreet, are yet questionable.

Thus, in the course of a proof we are hardly entitled to say that two triangles are congruent *because* we find certain parts in one equal to certain parts in the other, and would do better to content ourselves with saying that we are aware of their congruence on that account; and yet it is the first assertion that is almost invariably found even when an alternative proof on the opposite page asserts equally firmly that something else *causes* the triangles to be equal.

Another tolerated statement which is equally unhappy is that one proposition or one property of a figure *depends* upon another, when it is merely meant that *one proof* of the one depends on the other, since the two propositions if true at all are both true simultaneously, and are co-equal, neither coming before or after the other.

One possibility would be to state clearly that in mathematics "because" and "therefore" are to be unemphatic, and to refer to our processes of perception, not to the facts perceived, but this seems undesirable. If, for instance, we have to prove something about a square $ABCD$, we may (I suppose) assert with any emphasis we like that *because* $ABCD$ is a square, *therefore* $AB=BC$, or if x is defined as $=2$, we may say $x^2=4$ without restricting to mean "therefore we perceive that."

The only other alternative would seem to be to deny ourselves the conventional laxity and to write "hence we see" instead of "therefore" wherever the latter word is too strong a conjunction.

Even in such an argument as

$$6x=30, \quad .. \; 3x=15, \qquad x=5,$$

the 's appear rather to overdo it, for we might equally well have

$$6x=30, \qquad x=5, \quad .. \; 3x=15,$$

and consequently the rule of self-denial suggested would be tiresome. Does not, however, the spirit of the time demand it?—Yours, etc.,

CHARLES HARDINGHAM.

7 Queen Anne Terrace, Cambridge,
February 2nd, 1914.

The Editor, *Mathematical Gazette.*

DEAR SIR,—In your December issue you published a notice of my book on "School Algebra." I should be very grateful if you would allow me the favour of a reply to the more important criticisms.

Your reviewer's main criticism is that my treatment of algebraic theory does not conform to the requirements of formal Algebra. I would submit that the methods of formal Algebra, in which, for example, the rules of signs are matters of definition, are not suitable for beginners, and that the problem which confronts the writer of an elementary text-book is to devise suitable explanations of algebraic processes based in the first place on arithmetical conceptions.

The rules of signs can be demonstrated without difficulty in the case of algebraical expressions in which all the letters represent positive quantities and all the signs have their strictly arithmetical meaning. Presumably this was how the rules were first derived. In any case I think this line of advance is the correct one for teaching purposes, and for this reason practically the whole of my theoretical work in Chapters I.–XIII. assumes that the letters represent positive quantities, and that the minus sign indicates arithmetical subtraction: §§ 19-22 are a digression, as I have indicated in § 21.

In Chapter XIV I am not dealing with the negative numbers of formal Algebra, but with negative quantities such as occur in Mechanics, Trigonometry and Coordinate Geometry

The fact that the rules of signs are matters of definition in pure Algebra implies that the negative numbers of Algebra may not be used to represent the negative quantities of other branches of mathematics until we have verified that these negative quantities will conform to the rules of signs.—I am, dear Sir, yours faithfully,

A. G. CRACKNELL.

I cannot agree with Mr. Cracknell that the methods of formal Algebra are not suitable for beginners: the method of presentation is, I believe, a question of taste simply, and was not to any great extent in my mind when reading the "School Algebra."

Mr. Cracknell professes to base his reasoning on arithmetical conceptions. So far, very good. But his arithmetical conceptions include many algebraical ideas. Thus, on page 173 it is stated that "$(+3)+(-7)+(-6)+(+2)$ *corresponds* exactly to the arithmetical process

$$+3-7-6+2=+5-13=-8."$$

There is no arithmetical process here, for the first step $+3-7$ is impossible.

Similarly, on page 94, we have "$x^2=\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$" (presumably quoting an "Arithmetical Rule"): this is a very good "rule," but unsatisfactory as a foundation for the "Laws of Algebraical Fractions." For this rule cannot be proved or even "explained." What would be the result of sending a boy to a shop for 6 lbs. of sugar $1\frac{1}{2}$ times?

If, as Mr. Cracknell says, Chapter XIV only refers to quantities such as occur in Mechanics, Trigonometry and Coordinate Geometry, the note at the foot of page 173, where it is stated that "the signs inside the brackets indicate positive and negative *numbers*," should be deleted as misleading. Besides, what is the use of this chapter, for no operations are performed by negative quantities? The sign of a "moment" does not materialise out of the sign of "the force" and "the distance," but is added afterwards by a totally distinct consideration, at any rate in Elementary work: nor does a conscientious teacher allow the usual definition, "moment = force × distance," to pass without an emphatic warning that only the *numbers* representing the measures are concerned. If so, we should have 3 oranges × 2 apples = 6 orange-apples. In Coordinate Geometry all the letters stand for numbers, or else x^4 has no meaning. All that is required in the way of interpretation is the meaning to be attached to the negative sign as a sign of *quality*: the whole of the calculation is concerned with abstract numbers alone.

Even the operation of counting *things* has no relation to the things counted. There are two mental processes going on simultaneously, one noting the thing and distinguishing it, the other counting: and these are independent.

Similarly, if we require the result of three twists, two right-handed and one left-handed, whose magnitudes are 20°, 30°, 60° respectively; we may *interpret* a right-handed twist as positive and the left-handed as negative, or *vice versa*. The mind makes a note of this and of the unit employed, but the calculation is with abstract numbers—either

$$(+20)+(+30)+(-60)=(-10) \quad\text{or}\quad (-20)+(-30)+(+60)=+10$$

—and the result in each case is interpreted as a *left-handed-twist in degrees* (supplied by the mental note), whose magnitude is *10* (supplied by the calculation). I think the main points of my criticism are founded on the wide divergence between the conceptions held by Mr. Cracknell and myself as to the boundary line between Arithmetic and Algebra: and I hold that if he assumes that a boy is "familiar with" such a result as $3-7+5=8-7$ in Arithmetic, he must *prove* that this is justifiable before he founds a theory of Algebra on it: similarly, for fractions it is necessary to *prove* that in $\frac{3}{4}$, as a multiplier, the 3 and the 4 are separable, and much more, before it can be used as a foundation for an Algebra that is to be convincing. My opinion is that Arithmetic is concerned only with positive integers, whether expressed numerically or literally, as far as theoretical processes are concerned.

X. Y. Z.

7 Queen Anne Terrace, Cambridge,
February 12*th*, 1914.

The Editor, *Mathematical Gazette.*

Dear Sir,—I thank you very much for your kindness in allowing me not only to reply to your reader's criticisms, but also to see his reply to mine.

With regard to the second paragraph in his reply, I would point out that there is a strictly arithmetical interpretation of the equation $+3-7=-4$,

https://doi.org/10.2307/3603577 Published online by Cambridge University Press

viz. that to add 3 to any number (not less than 4) and then subtract 7 gives the same result as to subtract 4 from that number.

As for the statement $\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$—if in dealing with beginners I am not allowed to assume it from arithmetic, nor to explain it, nor to prove it except by the rigorous treatment of the Theory of Number, I fear I must abandon the unfortunate equation in despair.

For the rest I should like to discuss briefly the application of + and − signs in connection with the moments of forces about points.

If two forces have moments $(+3)$ and (-7), we apply the rule of signs for algebraical addition, and infer that their combined moment is (-4). To justify this it would not be sufficient to say that the + and − signs are convenient symbols for counter-clockwise and clockwise. It would be sufficient to show from dynamical considerations that the two moments we are calling $(+1)$ and (-1) when combined give a resulting moment of zero: for then moment $(+3)$ cancels with moment (-3) and leaves moment (-4). Extending this idea, I have shown in § 119 that where a negative unit can be defined as that which cancels a positive unit, the quantities concerned will obey the rules of signs for addition and subtraction.

Again, we may apply the rule of signs for multiplication in the formula :—

$$\text{moment} = \text{force} \times \text{arm}.$$

To justify this it is not sufficient to say that the + and − signs are convenient symbols for opposite directions. We must show from dynamical considerations that if either force or arm be reversed in direction, then the moment is reversed: which corresponds accurately to the algebraic rule, that if the sign of either factor is reversed the sign of the product is reversed. This is in essence the account of the matter which I have given on p. 180.—I remain, dear Sir, yours faithfully, A. G. CRACKNELL.

In answering Mr. Cracknell's second letter, I will take the points in the order he puts them.

(1) As Mr. Cracknell interprets $+3-7=-4$, it is first of all not an equation but a statement of equivalence of *operations*, and therefore in no way corresponds to the "sum of a set of positive and negative *numbers*" (quoted from ll. 1, 2 on page 173). Even as a statement of equivalence of operations it is not complete, for Mr. Cracknell has to accept numbers less than 4. As I read it from the context quoted above, I made it an equation, *i.e.* a statement that $+3-7$ and -4 stood for the same number; hence I said the first step of $+3-7-6+2$, namely "*take* the number 3 and subtract 7" was impossible.

(2) There is no reason why Mr. Cracknell should "abandon the unfortunate equation in despair." He has all the material for a correct *proof* in his preliminary chapters. He has only to prove that $\frac{a}{b}\times\frac{c}{d}$ works out the same as $\frac{ac}{bd}$, when $\frac{a}{b}$, $\frac{c}{d}$ are *fractional forms denoting integers*, and then define the operation $\times\frac{c}{d}$ by the "equation" (sic) $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$. Perhaps it would be better to state it as a convention, explaining that there cannot, for convenience' sake, be *two* laws for fractional forms, one for the case when the form happened to be an integer, the other when it was not: in other words, precisely as he would "find a meaning for a^0, $a^{\frac{1}{2}}$, etc." on the assumption that they obeyed the index laws for positive integral indices. I again put my conundrum, "6 lbs. of sugar $1\frac{1}{2}$ times = ?"

(3) This paragraph simply states in a clearer fashion the point I made, that in calculations with moments the numbers representing the *measures* are alone involved.

(4) I do not quite understand this. Surely Mr. Cracknell does not intend to state that one actually multiplies "a force" by "an arm" to get a moment.

I think in (3) and (4) he emphasizes the wrong word. If he wrote "it is not sufficient to *say* that the + and − signs are convenient symbols for opposite directions, I should agree with him. It must be *shown* that this convention never gives a contradictory result. But once it is shown, that interpretation holds good; the rest of the work, the calculation, is with pure number.

Thus he is quite wrong in saying in the formula he gives on page 179,

$$\text{height of balloon above the sea} = h + dt \text{ feet},$$

that h and dt (on page 180) represent *distances*, d represents a velocity, and t a time; for the symbols h, d, t all represent *numbers*.

But his second letter seems to be quite beside the mark as having any reference to the original criticism. I took exception to his sweeping statement, inferred rather than strictly stated, that because he has shown that $+3$ is the "reverse" of -3 as a *number* that $(+3)$ and (-3) are "reverses" as *operators*. One might as well say then, that a^{-3} is the "reverse" of a^{+3}, and therefore is a cube root.

X. Y. Z.

NAPIER TERCENTENARY CELEBRATION, JULY 1914.

John Napier's *Logarithmorum Canonis Mirifici Descriptio* was published in 1614; and it is proposed to celebrate the tercentenary of this great event in the history of mathematics by a Congress, to be held in Edinburgh on Friday, 24th July, 1914, and following days.

The Celebration is being held under the auspices of the Royal Society of Edinburgh, on whose invitation a General Committee has been formed, representing the Royal Society of London, the Royal Astronomical Society, the Town Council of Edinburgh, the Faculty of Actuaries, the Royal Philosophical Society of Glasgow, the Universities of St. Andrews, Glasgow, Aberdeen, and Edinburgh, the University College of Dundee, and many other bodies and institutions of educational importance.

The President and Council of the Royal Society of Edinburgh have now the honour of giving a general invitation to mathematicians and others interested in this coming Celebration.

The Celebration will be opened on the Friday with an Inaugural Address by Lord of Appeal Sir J. Fletcher Moulton, F.R.S., LL.D. (Edin.), etc., followed by a Reception given by the Right Honourable the Lord Provost, Magistrates and Council of the City of Edinburgh. On the Saturday and Monday the historical and present practice of computation and other developments closely connected with Napier's discoveries and inventions will be discussed.

A Memorial Service will be held in St. Giles' Cathedral on the Sunday.

Among many who have expressed a warm interest in the Celebration, and who hope to take part in the Congress, may be mentioned Professor Andoyer, Paris; Professor J. Bauschinger, Strassburg; Professor Hume Brown, Historiographer Royal for Scotland; Professor F Cajori, Colorado, U.S.A.; Professor G. A. Gibson, Glasgow; Dr. J. W. L. Glaisher, Cambridge; Professor Lang, St. Andrews; Professor Macdonald, Aberdeen; Professor E. Pascal, Naples; Professor Karl Pearson, London; Professor Eugene