1

Introduction

En un lugar de La Mancha, de cuyo nombre no quiero acordarme . . .

— Miguel de Cervantes Saavedra, *Don Quixote*

Modern portfolio theory started with Harry Markowitz's 1952 seminal paper "Portfolio Selection" (Markowitz, 1952), for which he would later receive the Nobel Prize in Economic Sciences[1] in 1990. He put forth the idea that risk-averse investors should optimize their portfolio based on a combination of two objectives: expected return and risk. Until today, that idea has remained central to portfolio optimization. In practice, however, the vanilla Markowitz portfolio formulation does not perform as anticipated. Consequently, most practitioners either combine it with various heuristics or refrain from using it altogether.

Over the past 70 years, researchers and practitioners have reconsidered the Markowitz portfolio formulation, leading to numerous variations, enhancements, and alternatives. These include robust optimization methods, alternative risk measures, regularization through sparsity, improved covariance matrix estimators via random matrix theory, robust estimators for heavy tails, factor models, mean models, volatility clustering models, risk parity formulations, and more.

This book explores practical financial data modeling and portfolio optimization, covering a wide range of variations and extensions. It systematically starts with mathematical formulations and proceeds to develop practical numerical algorithms, supplemented with code examples to enhance understanding.

- The financial data modeling considered herein moves away from the unrealistic Gaussian assumption and delves into more realistic models based on heavy-tailed distributions. It encompasses an array of topics, ranging from basic time series models, making extensive use of Kalman filtering methods, to state-of-the-art techniques for estimating financial graphs.
- The portfolio formulations covered in this book span a wide range, from the original 1952 Markowitz's mean–variance portfolio and 1966 maximum Sharpe ratio portfolio, to more sophisticated formulations such as Kelly-based portfolios, utility-based portfolios, high-order portfolios, downside risk portfolios, semi-variance portfolios, CVaR portfolios,

[1] To be exact, what is usually referred to as the Nobel Prize in Economic Sciences is actually the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.

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drawdown portfolios, risk parity portfolios, graph-based portfolios, index tracking portfolios, robust portfolios, bootstrapped portfolios, bagged portfolios, pairs trading portfolios, statistical arbitrage portfolios, and deep learning portfolios, among others.

The primary focus and central theme of this book is on practical algorithms for portfolio formulations that can be effortlessly executed on a standard computer.

1.1 What is Portfolio Optimization?

Suppose you observe a random variable X with mean $\mu = \mathbb{E}[X]$ and variance $\sigma^2 = \mathbb{E}[(X-\mu)^2]$; for example, a normal (or Gaussian) random variable $X \sim \mathcal{N}(\mu, \sigma^2)$. The mean μ is the value you expect to obtain, whereas the variance σ^2 gives the variability or randomness around that value. The ratio μ/σ gives a measure of the deterministic-to-random balance. In finance, X may represent the *return* of an investment and the ratio μ/σ is called *Sharpe ratio*. In signal processing, it is more common to use the *signal-to-noise ratio (SNR)* measured in units of power and defined as μ^2/σ^2.

Now suppose that for each time t, a different (independent) value of the random variable is observed (called a random process or random time series): $X_t \sim \mathcal{N}(\mu, \sigma^2)$. In finance, these represent the returns of the investment, and the cumulative return is the accumulation of the previous returns, which reflects the accumulated wealth of the investment. Figure 1.1 shows a realization of such return random variables as well as the cumulative returns.

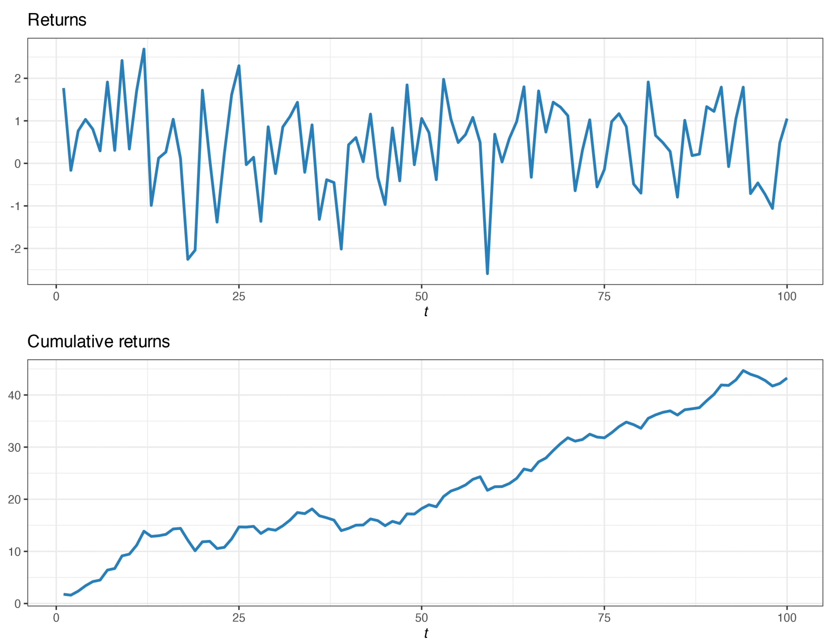

Figure 1.1 Illustration of random returns and cumulative returns.

The evolution of the cumulative returns or wealth over time, albeit random, is strongly determined by the value of the Sharpe ratio, μ/σ, as illustrated in Figure 1.2 for high and low

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values. If the Sharpe ratio is high, the cumulative return will have some fluctuations but with a consistent growth. On the other hand, if the ratio is low, the fluctuations become larger and one may even end up losing everything, leading to bankruptcy.

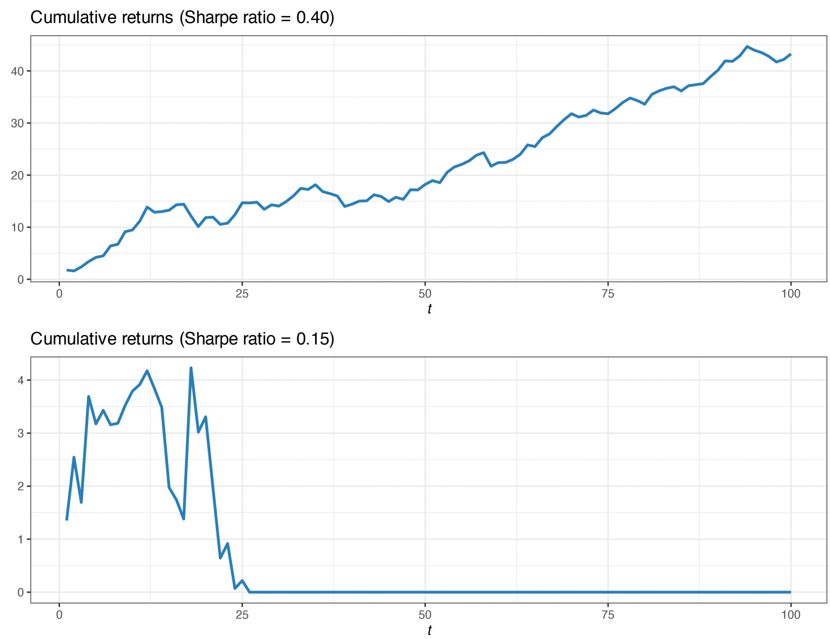

Figure 1.2 Illustration of cumulative returns with different values of Sharpe ratio.

What can an investor do to improve the cumulative return? While the random nature of the investment assets themselves cannot be changed, there are at least two dimensions that can be potentially exploited: the temporal dimension and the asset dimension.

- *Temporal dimension*: It may be the case that the distribution of the random return X_t changes with time, leading to time-varying μ_t and σ_t^2. In that case, a smart investor will adapt the size of the investment to the current value of μ_t/σ_t. In order to do that, one needs to develop an appropriate time series model, that is, a data model at time t given the past observations. This is called *data modeling* and it is explored in Part I of this book.

- *Asset dimension*: In general, an investor has a choice of N potential assets in which to invest, with returns X_i for $i = 1, \ldots, N$. Suppose they are all independent and identically distributed (i.i.d.): $X_i \sim \mathcal{N}(\mu, \sigma^2)$. It follows from basic probability that the average of such returns, $\frac{1}{N}\sum_{i=1}^{N} X_i$, preserves the mean μ but enjoys a reduced variance of σ^2/N. In finance, this average is achieved by distributing the capital equally over the N assets (the popular $1/N$ portfolio precisely implements this). In practice, however, the $1/N$ factor in the reduction of the variance cannot be achieved because the random returns X_i are correlated among the assets, that is, the assumption of uncorrelation does not hold. Over the decades, academics and practitioners have proposed a multitude of ways to properly allocate the capital, as opposed to the baseline $1/N$ allocation, in order to try to circumvent the inherent correlation of the assets and minimize the risk or variance. This is called

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portfolio optimization (also known as *portfolio allocation* or *portfolio design*) and it is covered in detail in Part II of this book.

1.2 The Big Picture

The two main components for portfolio design are *data modeling* and *portfolio optimization*. Figure 1.3 illustrates these two building blocks for the case of mean–variance portfolios (i.e., based on the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$) to produce the optimal portfolio weights $\boldsymbol{w}$.

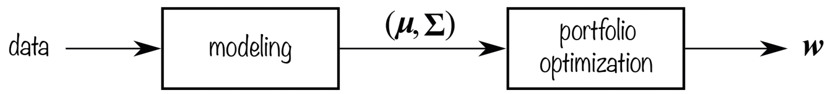

Figure 1.3 Block diagram of data modeling and portfolio optimization.

Part I of this book examines the data modeling component in Figure 1.3. The main purpose of this block is to characterize the statistical distribution of future returns, primarily in terms of the first- and second-order moments, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, which will be utilized by the portfolio optimization block later on.

Part II fully explores a wide variety of formulations for the portfolio optimization component in Figure 1.3. These portfolio formulations can be classified according to different criteria leading to a diverse *taxonomy of portfolios* as follows.

- Taxonomy according to the data used:
  - *second-order portfolios*: portfolios based on the mean and the variance, such as Markowitz mean–variance portfolio, maximum Sharpe ratio portfolio, index tracking portfolios, and volatility-based risk parity portfolios;
  - *high-order portfolios*: portfolios based directly on high-order moments as well as approximations of utility-based portfolios; and
  - *raw-data portfolios*: these include portfolios that require the raw data, such as downside risk portfolios, semi-variance portfolios, conditional variance-at-risk (CVaR) portfolios, drawdown portfolios, graph-based portfolios, and deep learning portfolios.
- Taxonomy according to the view on the *efficient-market hypothesis*:[2]
  - *active portfolios*: most of the portfolio formulations that attempt to beat the market; and
  - *passive portfolios*: index tracking portfolios which simply track the market, avoiding frequent portfolio rebalancing.
- Taxonomy according to the myopic nature of the portfolio formulation:
  - *single period portfolios*: most of the formulations considered here are based on a single step into the future; and

[2] The efficient-market hypothesis states that asset prices reflect all information and, therefore, it should be impossible to outperform the overall market through expert stock selection or market timing.

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– *multi-period portfolios*: more involved formulations that consider several steps into the future so that the long-term effect of current actions is better taken into account; this is not covered in this book, see (Boyd et al., 2017) for a monograph on multi-period portfolio optimization.

1.3 Outline of the Book

This book is organized into two main parts, comprising a total of 16 chapters, along with two appendices at the end. The content of each of the chapters is outlined next.

Part I. *Financial Data*: This part focuses on financial data modeling, which is a necessary component before the portfolio design.

- Chapter 2 discusses *stylized facts* in financial data. These unique characteristics differentiate financial data from other types of data. Some of these characteristics include lack of stationarity, volatility clustering, heavy-tailed distributions, and strong asset correlation. This chapter provides a concise and visual overview of these stylized facts to help readers better understand and analyze financial data.

- Chapter 3 focuses on *i.i.d. modeling* in financial data. Although the i.i.d. model is a simplistic approximation, it is still widely used in practice. However, challenges arise due to non-Gaussian distributions and noise, which are often ignored in financial literature. To address these challenges, robust and heavy-tailed estimators for the mean vector and the covariance matrix are necessary, and this chapter provides detailed explanations for these estimators. Furthermore, incorporating prior information through techniques such as shrinkage, factor modeling, and Black–Litterman fusion can significantly improve the accuracy of estimates. Due to the breadth of topics covered in this chapter, the length is rather long, but it provides readers with a comprehensive understanding of i.i.d. modeling for financial data.

- Chapter 4 explores the application of *time series models* to financial data to capture temporal dependencies for both mean modeling and variance modeling. While mean models provide debatable improvement over the i.i.d. approach, variance models, including GARCH-related models and stochastic volatility models, have been shown to be effective in capturing the volatility of financial data (the latter showing improved results but at a higher computational cost). This chapter presents a unified modeling approach through state-space modeling with special emphasis on the use of the efficient Kalman filter, which notably allows the approximation of stochastic volatility models with low computational cost.

- Chapter 5 focuses on *financial graphs* and their applications in financial data analysis. While graphical modeling of financial data originated in 1999, many methods have since been proposed. Among these methods, sparse Gaussian models are suitable for providing basic insights, low-rank formulations can be used to cluster assets, and heavy-tailed models are appropriate for accounting for non-Gaussian data. Graph-based techniques can provide valuable visual and analytical tools for financial data analysis. This chapter provides an overview of cutting-edge techniques for graph modeling of financial assets, allowing readers to gain a deeper understanding of the applications and benefits of financial graphs in data analysis.

Part II. *Portfolio Optimization*: This part contains a wide range of chapters covering various portfolio formulations with corresponding algorithms and examples.

- Chapter 6 provides a comprehensive introduction to *portfolio basics*. The chapter covers fundamental topics such as portfolio notation, cumulative return calculation, transaction costs, portfolio rebalancing, practical constraints, measures of performance, simple heuristic portfolios, and basic risk-based portfolios. While the chapter covers the basics, it also includes an interesting nugget on the interpretation of the heuristic quintile portfolio, widely used by practitioners, as a formally derived robust portfolio. This chapter serves as an excellent starting point for readers new to portfolio management, providing them with the foundational knowledge necessary to understand and build portfolios.

- Chapter 7 delves into the topic of *modern portfolio theory*, which is the main focus of the majority of textbooks on portfolio design. In this book, this chapter serves as a starting point for exploring a wide range of different portfolio formulations. The chapter begins with an introduction to the basic mean–variance portfolio and then moves on to the often-ignored maximum Sharpe ratio portfolio, for which several practical numerical algorithms are presented in detail (such as bisection, Dinkelbach, and Schaible transform-based methods). The Kelly portfolio and utility-based portfolios are also introduced. The chapter concludes with a discussion of a recently proposed universal algorithm that can be utilized to solve portfolios based on any trade-off between the mean and variance. Overall, this chapter provides readers with a comprehensive understanding of modern portfolio theory and its practical applications.

- Chapter 8 focuses on *portfolio backtesting*, which is essential in strategy evaluation. Many biases, such as overfitting, can invalidate backtesting results, making it a challenging task. As a consequence, published backtests should not be trusted blindly. This chapter delves into the common pitfalls and dangers of backtesting, which are often ignored in textbooks, and puts forward the approach of multiple randomized backtests to help mitigate risks. The chapter also discusses the benefits of stress testing with resampled data to complement the backtesting results. By providing readers with a comprehensive understanding of the challenges of backtesting and suggesting practical solutions to overcome them, this chapter serves as an essential guide for portfolio assessment.

- Chapter 9 explores *high-order portfolios*, which introduce high-order moments in the mean–variance formulation. This idea dates back to the beginning of modern portfolio theory, but until recently it was impractical due to difficulties in parameter estimation, excessive memory requirements, and the complexity of optimization methods for a realistic number of assets. This chapter covers all the basics of high-order portfolios and introduces recent advances that make this approach practical.

- Chapter 10 considers *portfolios with alternative measures of risk*. While variance is the most commonly used measure of risk in portfolio optimization, many advanced risk measures, such as downside risk, semi-variance, CVaR, and drawdown, can also be incorporated. These measures can be formulated in convex form, allowing for the use of efficient algorithms. This chapter provides an overview of these sophisticated alternatives to Markowitz's seminal mean–variance formulation.

- Chapter 11 presents *risk parity portfolios*, which aim to diversify risk allocation beyond equal capital allocation. These portfolios were proposed by practitioners and rely on using granular asset risk contributions rather than overall portfolio risk. This chapter presents risk parity portfolios progressively, starting from a naive diagonal formulation and progressing to sophisticated convex and nonconvex formulations. It also covers a wide range of numerical algorithms, including newly proposed techniques.

- Chapter 12 gives an overview of *graph-based portfolios*, which utilize graphical representations of asset relationships learned from data to improve the portfolio design. Graph-based portfolios enable hierarchical clustering and novel formulations that account for asset interconnectivity, enhancing portfolio construction. This chapter provides a comprehensive overview of all existing graph-based portfolios, presenting a unified view of the different approaches.

- Chapter 13 covers *index tracking portfolios*, which are designed to mimic an index under the assumption that the market is efficient and cannot be beaten. Sparse index tracking further improves this approach by using few assets, posing a sparse regression problem. This chapter provides a state-of-the-art overview of the existing methodologies and introduces new formulations for index tracking portfolios. It also includes a cutting-edge algorithm that automatically selects the right level of sparsity, making index tracking more efficient and effective.

- Chapter 14 gives an overview of *robust portfolios*, which aim to address the inevitable parameter estimation errors that can lead to meaningless or catastrophic results if ignored. While optimal portfolio solutions may seem ideal in theory, practical implementation requires techniques like robust optimization and resampling methods. This chapter covers these standard techniques, providing readers with a comprehensive understanding of robust portfolios and how to optimize them.

- Chapter 15 explores *pairs trading* or *statistical arbitrage portfolios*, which are market-neutral strategies designed to be orthogonal to the market trend. These strategies trade on the oscillations among different assets, making them a popular technique in advanced portfolio management. This chapter provides an overview of the basics of pairs trading and statistical arbitrage, as well as exploring the more sophisticated use of Kalman filtering.

- Chapter 16 presents the concept of *deep learning portfolios*, which utilize deep learning techniques to analyze financial time series data and optimize portfolios. While deep learning has revolutionized fields like natural language processing and computer vision, its potential in finance remains uncertain due to challenges such as limited availability of nonstationary data and the weakness of the signal buried in noise. This chapter provides a standalone account of deep learning and the current efforts in the financial arena, acknowledging the risk of becoming quickly obsolete but still providing a good starting point.

Appendices A and B. *Preliminaries on Optimization*: This final part provides an overview of basic concepts in optimization theory (Appendix A) and a concise account of practical algorithms (Appendix B) used throughout the book.

1.4 Comparison with Existing Books

The financial literature on data modeling and portfolio design is extensive and diverse. This book aims to provide a unique perspective on these topics, and it is instructive to compare it with some of the existing textbooks.

- *Financial data modeling*: Many excellent textbooks cover financial data modeling, such as Campbell et al. (1997), Meucci (2005), Tsay (2010), Ruppert and Matteson (2015), Lütkepohl (2007), Tsay (2013), Fabozzi et al. (2007), Fabozzi et al. (2010), and Feng and Palomar (2016). In this book, Chapters 3 and 4 provide a succinct overview of i.i.d. models and models with temporal structure, respectively. Particular emphasis is placed on heavy-tailed models and estimators (as opposed to the more traditional methods based on the Gaussian assumption), stochastic volatility models (usually not receiving their deserved attention), and the use of state-space models with Kalman filtering as a unified approach with efficient algorithms.
- *Modern portfolio theory*: Traditional books that focus primarily on portfolio foundations and mean–variance portfolios include Grinold and Kahn (2000), Meucci (2005), Cornuejols and Tütüncü (2006), Fabozzi et al. (2007), Prigent (2007), Michaud and Michaud (2008), Bacon (2008), and Fabozzi et al. (2010). In this book, Chapters 6 and 7 cover this material with an optimization perspective, including utility-based portfolios, a recent derivation of the otherwise heuristic quintile portfolio as a robust solution, and particularly delving in detail into the nonconvex formulation of the maximum Sharpe ratio portfolio. It also provides a recently proposed universal algorithm for all these portfolios based on different trade-offs of the mean and variance.
- *Risk parity portfolios*: Roncalli's book (Roncalli, 2013) provides a detailed mathematical treatment (see also Feng and Palomar (2016)), while Qian's book (Qian, 2016) covers the fundamentals. In this book, Chapter 11 covers risk parity portfolios from an optimization perspective, progressively covering the naive solution, the vanilla convex formulations, and the more practical and general nonconvex formulations, with emphasis on the numerical algorithms.
- *Backtesting*: López de Prado's book (López de Prado, 2018) covers backtesting and its dangers in great detail from the perspective of machine learning, while Pardo (2008) focuses on the walk-forward backtest. In this book, Chapter 8 explores the many dangers of backtesting and the different forms of executing backtesting based on market data, as well as synthetic data, with abundant figures.
- *Index tracking*: The topic of index tracking is treated in detail in Prigent (2007) and Benidis et al. (2018), with shorter treatments in Cornuejols and Tütüncü (2006) and Feng and Palomar (2016). In this book, Chapter 13 provides a concise yet broad state-of-the-art exposure, offering new formulations and a cutting-edge algorithm that automatically selects the right level of sparsity.

- *Robust portfolios*: Robust optimization is widely explored within the context of portfolio design, with standard references including Fabozzi et al. (2007) and Cornuejols and Tütüncü (2006) (see also Feng and Palomar (2016)). In this book, Chapter 14 gives a concise presentation of these techniques for obtaining robust portfolios with illustrative numerical experiments.
- *Pairs trading*: The standard reference to this topic is Vidyamurthy (2004); see also Feng and Palomar (2016). In this book, Chapter 15 provides full coverage of the basics and presents a more sophisticated use of the Kalman filter for better adaptability over time.
- *High-frequency trading*: High-frequency data and trading based on the limit order book require a completely different treatment than what is covered in this book. Some key references include Abergel et al. (2016), Lehalle and Laruelle (2018), Bouchaud et al. (2018), and Kissell (2020).
- *Machine learning in finance*: Recent textbooks that give a broad account of the use of machine learning in financial systems include López de Prado (2018) and Dixon et al. (2020). In this book, Chapter 16 briefly discusses machine learning and deep learning techniques in the context of portfolio design.

1.5 Reading Guidelines

This book has been written under the premise that each chapter can be read independently. For example, a reader who is already familiar with portfolio optimization can jump directly to Chapter 16 on deep learning portfolios or to Chapter 15 on pairs trading.

Some suggested ways to read the book include the following approaches:

- A "reader in a rush" can go directly to Chapter 6 for portfolio basics and Chapter 7 for modern portfolio theory, perhaps also taking a quick look at Chapter 2 on stylized facts of financial data, and then jump to any other chapter, for example Chapter 14 on robust portfolios or Chapter 9 on high-order portfolios.
- A "reader with a bit more time," apart from the basic Chapters 2, 6, and 7, could also read Chapter 3 on i.i.d. data modeling and Chapter 8 on portfolio backtesting to get a better grasp of the fundamentals.
- For full coverage of all the different portfolio designs, a reader can go over any chapter in Part II; that is, apart from the fundamental Chapters 6–8, one can explore (in any particular order):
  - high-order portfolios (Chapter 9);
  - portfolios with alternative risk measures (Chapter 10);
  - risk parity portfolios (Chapter 11);
  - graph-based portfolios (Chapter 12);
  - index tracking portfolios (Chapter 13);
  - robust portfolios (Chapter 14);
  - pairs trading or statistical arbitrage portfolios (Chapter 15); and
  - deep learning portfolios (Chapter 16).

- To complete the financial data modeling, one should go over all the chapters in Part I: apart from Chapters 2 and 3; Chapter 4 covers time series modeling, and Chapter 5 explores the more recent topic of graph modeling of financial assets.
- In order to gain a more solid understanding of the portfolio optimization formulations and algorithms, a reader may want to go over Appendices A and B, that is, the basics of convex optimization theory in Appendix A and optimization algorithms in Appendix B.

1.6 Notation

Notation differs depending on the research area and on the personal taste of the author. This book mainly follows the notation widely accepted in the statistics, signal processing, and operations research communities.

To differentiate the dimensionality of quantities we employ lowercase for scalars, boldface lowercase for (column) vectors, and boldface uppercase for matrices, for example, x, $\boldsymbol{x}$, and $\boldsymbol{X}$, respectively. The ith entry of vector $\boldsymbol{x}$ is denoted by x_i and the (i, j)th element of matrix $\boldsymbol{X}$ by $X_{i,j}$. The elementwise product (also termed the Hadamard product) and elementwise division are denoted by $\odot$ and $\oslash$, respectively, e.g., $\boldsymbol{x} \odot \boldsymbol{y}$ and $\boldsymbol{x} \oslash \boldsymbol{y}$ ($\boldsymbol{x}/\boldsymbol{y}$ abusing notation); similarly, the Kronecker product is denoted by $\otimes$. The transpose of a vector $\boldsymbol{x}$ or a matrix $\boldsymbol{X}$ are denoted by $\boldsymbol{x}^\mathsf{T}$ and $\boldsymbol{X}^\mathsf{T}$, respectively. The inverse, trace, and determinant of matrix $\boldsymbol{X}$ are denoted by $\boldsymbol{X}^{-1}$, $\mathrm{Tr}(\boldsymbol{X})$, and $|\boldsymbol{X}|$ (or $\det(\boldsymbol{X})$), respectively. The norm of a vector is written as $\|\boldsymbol{x}\|$, which can be further specified as the ℓ_2-norm $\|\boldsymbol{x}\|_2$ (also termed the Euclidean norm), the ℓ_1-norm $\|\boldsymbol{x}\|_1$, and the ℓ_∞-norm $\|\boldsymbol{x}\|_\infty$. The operator $(\boldsymbol{x})^+$ denotes the projection onto the nonnegative orthant, that is, $(\boldsymbol{x})^+ \triangleq \max(\boldsymbol{0}, \boldsymbol{x})$. We denote by $\boldsymbol{I}$ the identity matrix of appropriate dimensions.

For random variables, $\Pr[\cdot]$ denotes probability, and the operators $\mathbb{E}[\cdot]$, $\mathrm{Std}[\cdot]$, $\mathrm{Var}[\cdot]$, and $\mathrm{Cov}[\cdot]$ denote expected value, standard deviation, variance, and covariance matrix, respectively.

The set of real numbers is denoted by $\mathbb{R}$ (nonnegative real numbers by $\mathbb{R}_+$ and positive real numbers by $\mathbb{R}_{++}$). The set of $m \times n$ matrices is denoted by $\mathbb{R}^{m\times n}$, the set of symmetric $n \times n$ matrices by $\mathbb{S}^n$, and the set of positive semidefinite $n \times n$ matrices by $\mathbb{S}^n_+$. By $\boldsymbol{a} \geq \boldsymbol{b}$ we denote elementwise inequality (i.e., $a_i \geq b_i$). The matrix inequalities $\boldsymbol{A} \succeq \boldsymbol{B}$ and $\boldsymbol{A} \succ \boldsymbol{B}$ denote that $\boldsymbol{A} - \boldsymbol{B}$ is positive semidefinite and positive definite, respectively. The indicator function is denoted by $1\{\cdot\}$ or $I(\cdot)$.

Table 1.1 lists the most common abbreviations used throughout the book, and Table 1.2 provides some key financial mathematical symbols.

Table 1.1 Common abbreviations used in the book.

Abbreviation	Meaning
AI	Artificial intelligence
AR	Autoregressive
ARCH	Autoregressive conditional heteroskedasticity
ARIMA	Autoregressive integrated moving average

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Table 1.1 Common abbreviations used in the book. *(continued)*

Abbreviation	Meaning
ARMA	Autoregressive moving average
B&H portfolio	Buy and hold portfolio
BCD	Block coordinate descent
CAPM	Capital asset pricing model
CCC	Constant conditional correlation
CP	Conic problem/program
CVaR	Conditional value-at-risk
DCC	Dynamic conditional correlation
DD	Drawdown
DL	Deep learning
DR	Downside risk
ES	Expected shortfall
EWMA	Exponentially weighted moving average
EWP	Equally weighted portfolio (a.k.a. $1/N$ portfolio)
FP	Fractional problem/program
FX	Foreign exchange
GARCH	Generalized autoregressive conditional heteroskedasticity
GICS	Global Industry Classification Standard
GMRP	Global maximum return portfolio
GMVP	Global minimum variance portfolio
GP	Geometric problem/program
HRP	Hierarchical risk parity
i.i.d.	independent and identically distributed
IPM	Interior-point method
IVarP	Inverse variance portfolio
IVolP	Inverse volatility portfolio
LFP	Linear fractional problem/program
LP	Linear problem/program
LPM	Lower partial moment
LS	Least squares
MA	Moving average
MDecP	Maximum decorrelation portfolio
MDivP	Most diversified portfolio
ML	Maximum likelihood or machine learning (depending on context)
MM	Majorization–minimization
MSRP	Maximum Sharpe ratio portfolio
MVolP	Mean–volatility portfolio
MVP	Mean–variance portfolio
MVSK	Mean–variance–skewness–kurtosis
NAV	Net asset value
P&L	Profit and loss
QCQP	Quadratically–constrained quadratic problem/program
QP	Quadratic problem/program
QuintP	Quintile portfolio
RPP	Risk parity portfolio
S&P 500	Standard & Poor's 500
SCA	Successive convex approximation
SDP	Semidefinite problem/program
SOCP	Second-order cone problem/program

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Table 1.1 Common abbreviations used in the book. *(continued)*

Abbreviation	Meaning
SR	Sharpe ratio
SV	Stochastic volatility
TE	Tracking error
VaR	Value-at-risk
VARMA	Vector autoregressive moving average
VECM	Vector error correction model

Table 1.2 Mathematical notation used in the book.

Term	Meaning
$\boldsymbol{w}$	Normalized portfolio weight vector
$\boldsymbol{w}^{\text{cap}}$	Portfolio capital allocation vector (e.g., in units of US dollar)
$\boldsymbol{w}^{\text{units}}$	Portfolio unit allocation vector (e.g., in units of shares for stocks)
$\boldsymbol{p}_t$	Price vector of assets at time t
$\boldsymbol{y}_t$	Log-price vector of assets at time t
$\boldsymbol{r}_t$ ($\boldsymbol{x}_t$)	Return vector of assets at time t (linear or log-returns, depending on context)
$\boldsymbol{r}_t^{\text{lin}}$	Linear returns vector of assets at time t
$\boldsymbol{r}_t^{\text{log}}$	Log-returns vector of assets at time t
$\boldsymbol{\mu}_t$	Vector of expected value of returns $\boldsymbol{r}_t$
$\boldsymbol{\Sigma}_t$	Covariance matrix of returns $\boldsymbol{r}_t$
N	Number of financial assets in the considered universe
T	Number of temporal observations $t = 1, \dots, T$
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Normal or Gaussian multivariate distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$

1.7 Website for the Book

The book is supplemented with a variety of additional materials, including slides, sample code, exercises with solutions, and videos. These supplementary resources can be accessed on the companion website at:

portfoliooptimizationbook.com

1.8 Code Examples

This book is supplemented with a large number of code examples in R and Python that can reproduce all the figures in the book. These supplementary resources are available on the companion website for the book.

Generally speaking, the resolution of all the portfolio optimization formulations covered in the book can be approached in a variety of ways, namely:

- Use a software package or library specifically designed to optimize portfolios under a wide variety of formulations and constraints. Examples include the popular R package

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fPortfolio (Wuertz et al., 2023) and the Python packages Riskfolio-Lib (Cajas, 2023) and PyPortfolioOpt (Martin, 2021).

- Utilize a modeling framework like CVX, which automatically calls upon a solver behind the scenes, available for programming languages including Python, R, and Julia (Fu et al., 2020, 2022; Grant & Boyd, 2008, 2014).
- Directly invoke an appropriate solver.
- Develop ad hoc efficient algorithms for specific formulations, as done in the packages developed by the ConvexFi group.[3]

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