Distance Off by Vertical Sextant Angle

J. W. Crosbie

DISTANCE off by measuring the elevation of a mountain top above the horizon has been a popular means of finding position at sea ever since Captain Lecky published his famous tables. This popularity persists into the radar age because distance off can often be found from a vertical sextant angle long before a ship is within effective radar range of the coast. In view of this it is surprising that the standard textbooks on navigation give scant attention to this method and any seaman finding himself without Lecky's tables would find little guidance elsewhere.

Wing Commander E. W. Anderson in *The Principles of Navigation* indicates that the whole angle subtended by a mountain at a ship can be found by adding half the estimated distance off in miles to the visible angle in minutes, having first corrected this angle for dip and refraction. This can be proved by simple geometry by showing that the angle subtended by the part of a mountain hidden to an observer at sea-level is equal to half the distance off. Once the whole angle has been calculated the distance can be found from the formula $D = 0{\cdot}565h \times \frac{1}{\theta}$, and it is worth noting that the error in the distance found is at most a half of the error in the estimated distance off. Thus the exact distance off can be arrived at by a series of closer and closer approximations.

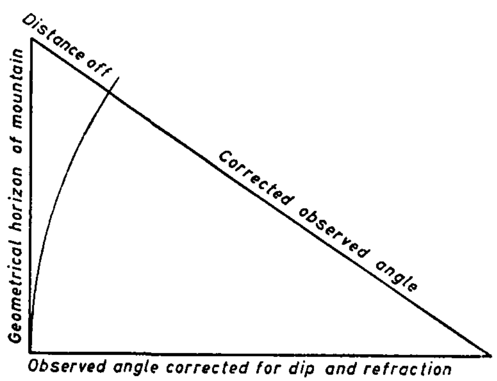

FIG. 1. Mnemonic showing relationship between height of mountain peak, angle observed and distance off.

Alternatively the above formula can be written:

$D = 0{\cdot}565h \div (\theta + \frac{1}{2}D)$, where θ = visible angle, and then transposing, $D^2 + 2\theta D = 1{\cdot}13h$, and completing the square, $D = \sqrt{(1{\cdot}13h + \theta^2)} - \theta$, so that the true distance off can be calculated directly from the observed angle, corrected for dip and refraction, and the height of the mountain.

Navigators seeking an even easier solution may write the formula as $D = \sqrt{\{(1{\cdot}06\ \sqrt{h})^2 + \theta^2\}} - \theta$, from which it can be seen that the part $\sqrt{\{(1{\cdot}06\sqrt{h})^2 + \theta^2\}}$ is the hypotenuse of a right-angled triangle in which θ and $1{\cdot}06\ \sqrt{h}$ are the other two sides. θ is, of course, the observed angle

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and 1·06 $\sqrt{h}$ is the 'rising' distance of the mountain top where refraction has been ignored. 1·06 $\sqrt{h}$ can be found from a specially constructed table or else taken from any book of nautical tables giving distance of the sea horizon, but in this case, one twelfth of that distance should be subtracted from itself to compensate for the allowance for refraction. The traverse table can now be entered with 1·06 $\sqrt{h}$ as D. Lat. and θ as Dep and a corresponding distance found. The observed angle θ, when subtracted from this distance, gives the distance off.

DISTANCE TO THE GEOMETRICAL HORIZON

(For use when solving vertical sextant angles by traverse tables.)

feet	miles	feet	miles	feet	miles	feet	miles	feet	miles
100	10·6	550	24·9	1600	42·4	3700	64·5	5800	80·7
120	11·6	600	25·9	1700	43·7	3800	65·4	5900	81·4
140	12·5	650	27·0	1800	44·9	3900	66·2	6000	82·1
160	13·3	700	28·0	1900	46·2	4000	67·0	6100	82·8
180	14·2	750	29·0	2000	47·4	4100	67·9	6200	83·4
200	14·9	800	30·0	2100	48·6	4200	68·7	6300	84·1
220	15·7	850	30·9	2200	49·7	4300	69·5	6400	84·7
240	16·4	900	31·8	2300	50·8	4400	70·3	6500	85·4
260	17·1	950	32·6	2400	51·9	4500	71·1	6600	86·1
280	17·7	1000	33·6	2500	53·0	4600	71·9	6800	87·4
300	18·3	1050	34·4	2600	54·0	4700	72·6	7000	88·7
320	19·0	1100	35·1	2700	55·0	4800	73·4	7200	90·0
340	19·5	1150	35·9	2800	56·0	4900	74·1	7400	91·2
360	20·1	1200	36·7	2900	57·1	5000	74·9	7600	92·4
380	20·6	1250	37·4	3000	58·1	5100	75·6	7800	93·6
400	21·2	1300	38·3	3100	59·1	5200	76·4	8000	94·8
420	21·7	1350	39·1	3200	60·0	5300	77·1	8500	97·7
440	22·2	1400	39·8	3300	60·9	5400	77·9	9000	100·6
460	22·7	1450	40·4	3400	61·8	5500	78·6	9500	103·3
480	23·2	1500	41·0	3500	62·7	5600	79·3	10000	106·0
500	23·7	1550	41·7	3600	63·6	5700	80·0	11000	111·0

On very large ships it should be borne in mind that for strict accuracy the observer should subtract his own height of eye from the height of the mountain to obtain h. This is because when correcting the observed angle for dip he 'cuts' a piece out of the mountain as his horizontal is higher than an observer at sea-level.

S. M. Burton's *The Art of Astronomical Navigation*, incidentally, on p. 136 refers to the use of the traverse tables for this type of problem.

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EXAMPLE USING TRAVERSE TABLE

The 5228 ft. peak of the island Anjouan was observed from m.v. *British Willow* in a D.R. position 60 miles off. The height of eye was 56 ft. and the sextant angle read 28′·8

Index error on the arc:	0′·8	Height of peak	5228 ft.
	28′·0		
Dip	7′·3	Height of eye	56 ft.
	20′·7		
Refraction (60 ÷ 12)	5′·0	*h*	5172 ft.
True angle of elevation	15′·7	From table $1{\cdot}06\sqrt{h} = 76{\cdot}2$ n.m.	

From traverse table	Distance	D. Lat	Dep
under 12°	77·9	76·2	16·2
under 11°	77·6	76·2	14·8
Interpolating	77·8	76·2	15·7
	15·7		
True distance off	62·1 n.m.		

It should be noted that the interpolation required in the traverse table is that carried out by ocean navigators when calculating currents.

Captain Mário Gama's Direct Method for Star-sight Reduction

Charles H. Cotter

In a very interesting paper which appeared recently[1] the author, Capitão Mário Gama of the Portuguese Merchant Navy, describes a direct method for computing position lines from star (or planet) observations.

The principal feature of Captain Gama's method is the systematic manner in which he arrives at an intercept to the extent, not only of saving time in sight reduction, but also in reducing the possibility of blundering.

The method employs the *Computed Tables of Altitude and Azimuth* (HD 486) and involves timing a series of star-sights, the ship making headway meanwhile, by means of a stop watch which is set at zero at a noted chronometer time shortly before the observations commence. The following example, given in the original paper, in reducing sights of Vega and Denebola observed during morning twilight of 25 Feb. 1959, in D.R. position Lat. 10°10′ S., Long. 73°48′ E., will serve to illustrate the method.

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