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Alignment as an Indicator of Changes to Modal Structure within the Roberts Flow

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Abstract. Alignment of the magnetic and velocity fields has previously been shown to play a role within nonlinear dynamo theory (Cameron and Galloway 2006), MHD turbulence (Matthaeus *et al.* 1980) and mean field theory (Yokoi 2013). What has not been previously examined is whether it is beneficial to examine alignment within kinematic dynamo theory. I show how measurements of alignment within kinematic dynamo theory for the Roberts flow can indicate a change in the structure of the magnetic field.

Keywords. MHD, Kinematic Dynamo Theory

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1. Introduction

The evolution of an incompressible, electrically conducting, fluid and the magnetic field that it couples with is governed by the equation of motion for the fluid (1.1) and the induction equation (1.2). At the start of the dynamo the magnetic field is weak. The Lorentz force term ($(\mathbf{B}\cdot\nabla)\mathbf{B}$) within equation (1.1) can thus be neglected and the fluid evolves independently from the magnetic field. The initial growth of a seed magnetic field can therefore be examined by solving (1.2) using a prescribed flow $\mathbf{u}$. As the induction equation is linear solutions will be of the form $\mathbf{B}(x, y, z)\exp(\sigma t)$. Amplification of the seed magnetic field therefore requires $\Re\{\sigma\} > 0$.

One example of a flow that produces dynamo action and has been studied extensively is the Roberts flow (Roberts 1972) which is shown in equation (1.3). The flow amplifies the magnetic field by exponentially stretching along the null lines of the flow. The flow results in a magnetic field whose structure depends upon the wavenumber in the z direction, k_z, and it is for this reason that I use it to examine alignment.

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla P + (\mathbf{B}\cdot\nabla)\mathbf{B} - (\mathbf{u}\cdot\nabla)\mathbf{u} + \nu\nabla^2\mathbf{u} + \mathbf{F} \tag{1.1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = (\mathbf{B}\cdot\nabla)\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{B} + \eta\nabla^2\mathbf{B} \tag{1.2}$$

$$\mathbf{u} = \sqrt{\frac{3}{2}}\left[\cos(y), \sin(x), \sin(y) + \cos(x)\right] \tag{1.3}$$

The quantity that I use to measure alignment is shown within equation (1.4) and is the volume average of $|\cos(\theta)|$ where θ is the angle between $\mathbf{u}$ and $\mathbf{B}$ at each point. $\mathbf{AH}_c$ therefore measures alignment over the entire domain.

$$\mathbf{AH}_c = \frac{k_z}{8\pi^3}\int_0^{\frac{2\pi}{k_z}}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{|\mathbf{u}\cdot\mathbf{B}|}{|\mathbf{u}||\mathbf{B}|}dV \tag{1.4}$$

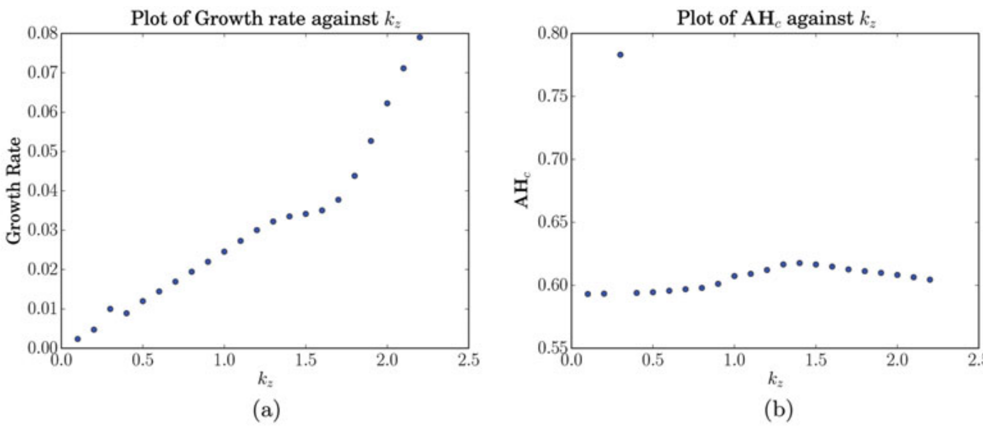


Figure 1. $\mathbf{AH}_c$ highlights the difference between $k_z = 0.3$ and the other k_z which is not obvious from the plot of the growth rate

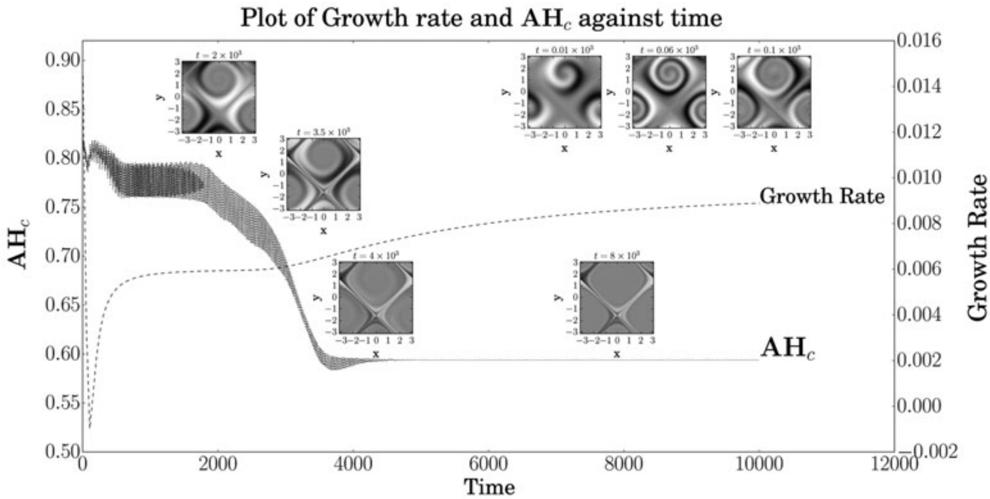


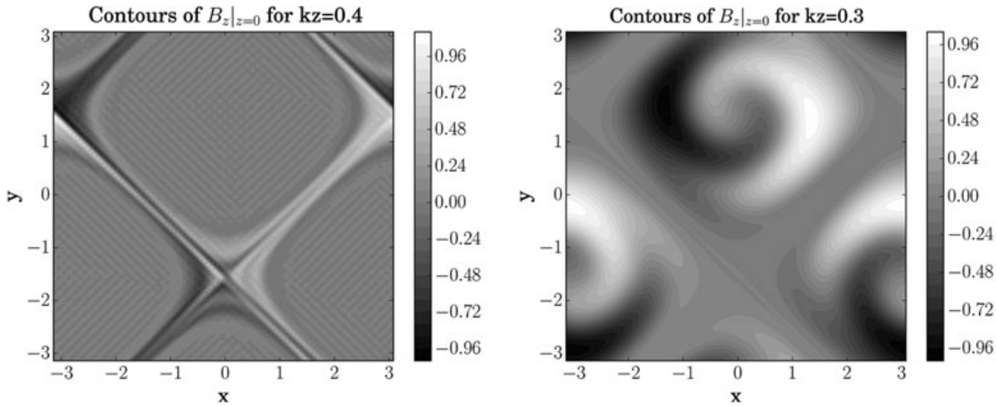
Figure 2. $k_z = 0.4$ run showing how $\mathbf{AH}_c$ changes in response to a change in the structure of $\mathbf{B}$. Light indicates strong positive field dark is strongly negative. Position indicates time where the plots are taken except those in the top right which are at times $t = 10, 60, 100$ respectively.

2. Overview

Due to the z independence of $\mathbf{u}$ and the linear nature of (1.2) it can be shown that the magnetic field may be rewritten in the form shown within equation (2.1). The wavenumber in the z direction, k_z is therefore a free parameter.

$$\mathbf{B}(x, y, z, t) = 2\Re\{\tilde{\mathbf{B}}(x, y)\exp(ik_z z + \sigma t)\} \qquad (2.1)$$

I solve (1.2) within a doubly periodic geometry for prescribed flow (1.3) and $\eta = 1/1000$. I use a spectral code which makes use of an Adams-Bashforth: Adams-Moulton predictor-corrector scheme for timestepping. I begin at $k_z = 0.1$ with $\mathbf{B}$ as a random seed field and then increase k_z in steps of 0.1. Each subsequent k_z is then initialised using the rescaled magnetic field of the previous k_z for computational efficiency. Figure 1(a) shows a plot of the growth rate vs k_z and is in good agreement with published results (Roberts 1972). Figure 1(b) shows a plot of the time asymptotic $\mathbf{AH}_c$ against k_z. Figure 2 shows



(a) $z = 0$ contour for $k_z = 0.4$ (b) $z = 0$ contour for $k_z = 0.3$

Figure 3. Figure shows that the structure of B_z for $k_z = 0.3$ is different to that of all other k_z

one individual, $k_z = 0.4$, with the subplot windows being the $z = 0$ contours of the B_z component of the magnetic field at various times.

3. Implications

Figure 3(a) shows the z component of the magnetic field evaluated at $z = 0$ for $k_z = 0.4$ which is similar in structure to all k_z other than $k_z = 0.3$ which is shown within Figure 3(b) and consists of large scale modes rather than elongated structures. The different modal structure that $k_z = 0.3$ has compared to all other k_z examined is clearly seen by the large change in value of $\mathbf{AH}_c$ shown within Figure 1(b) whereas examination of the growth rate alone would not perhaps highlight the special nature of the magnetic field structure of $k_z = 0.3$. This suggests that the calculation of $\mathbf{AH}_c$ may be a useful method to identify differing magnetic field structures as k_z is varied.

In Figure 2 we see that as time evolves the magnetic field leaves the center region of the flow and becomes exponentially stretched along the null lines of the flow forming into a seperatrix like structure. Correspondingly as the magnetic field structure transitions from a large scale modal structure to one that is more seperatrix like the value of $\mathbf{AH}_c$ drops to a value in line with that obtained for other seperatrix structures. Time series of $\mathbf{AH}_c$ thus enable us to see that the magnetic field is restructuring itself without needing to examine plots of the magnetic field directly.

In Summary I have shown via numerical simulations that the measure of alignment $\mathbf{AH}_c$ between the flow and magnetic field within kinematic dynamo simulations can be a useful method for highlighting differences in magnetic field structure. This suggests that $\mathbf{AH}_c$ may be a useful diagnostic tool within kinematic dynamo theory.

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