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Residual properties of free groups

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In recent years there has been some research done on the following problem. Given a non-cyclic free group F determine those sets C of groups for which F is residually C. To tackle such a problem it is of course sensible to put restrictions on the type of set C one considers. A restriction which is of interest, and which has received the attention of several authors, is that C consist of an infinite number of pairwise non-isomorphic known finite non-abelian simple groups. The first major theorem of this thesis asserts that if the simple groups in C are taken from the set of projective unimodular groups $\{\mathrm{PSL}(m, p) : p \text{ a prime}\}$ for a fixed odd integer $m > 1$, then any non-cyclic free group is residually C.

Another reasonable restriction which can be imposed on C is that it consist of only one group G, where G has a presentation on at least two generators with one defining relator. The case when G has non-trivial elements of finite order is treated here. It is shown that, except when G can be presented in the form $\langle a, t; [a, t]^n \rangle$ $(n > 1)$, a free group F is residually $\{G\}$ if and only if the rank of F is at least as great as the minimal number of generators of G. On the other hand, if $G = \langle a, t; [a, t]^n \rangle$ $(n > 1)$ then F is residually $\{G\}$ if and only if the rank of F is at least *three*.

The verification that if $G = \langle a, t; [a, t]^n \rangle$ $(n > 1)$ then a free group of rank 2 is not residually $\{G\}$ makes use of the fact that any two generating pairs of G are, in a certain sense, equivalent ("Nielsen equivalent"). The third major work of this thesis gives a proof of this result. In fact, a similar result is proved for a much wider class of

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groups, namely those groups G which can be presented in the form $\left\langle a, t; \left(a^{\alpha_1}t^{-1}a^{\beta_1}t \ldots a^{\alpha_s}t^{-1}a^{\beta_s}t\right)^n\right\rangle$ where $n > 1$, all the α_i are non-zero and have the same sign, all the β_i are non-zero and have the same sign. Moreover, it is shown that there is an algorithm to decide for any pair of words (U, V) in a, t whether or not U and V generate G. This theorem has several interesting corollaries, amongst which is a counterexample to the converse of Corollary 4.13.1 of Magnus, Karrass, Solitar [1]. In order to prove the above result it is found necessary to use part of the theory of HNN groups, and in particular, to develop a method for reducing pairs of elements in certain types of HNN groups.

Several of the results of this thesis appear in papers [2], [3], [4], [5] listed below.

References

[1] Wilhelm Magnus, Abraham Karrass, Donald Solitar, *Combinatorial group theory* (Interscience [John Wiley & Sons], New York, London, Sydney, 1966).

[2] Stephen J. Pride, "Residual properties of free groups", *Pacific J. Math.* **43** (1972), 725-733.

[3] Stephen J. Pride, "Residual properties of free groups II", *Bull. Austral. Math. Soc.* **7** (1972), 113-120.

[4] Stephen J. Pride, "Residual properties of free groups. III", *Math. Z.* **132** (1973), 245-248.

[5] Stephen J. Pride, "On the Nielsen equivalence of pairs of generators in certain HNN groups", *Proc. Second Internat. Conf. Theory of Groups*, Canberra, 1973 (Lecture Notes in Mathematics. Springer-Verlag, Berlin, Heidelberg, New York, to appear).