NON-LINEAR EVOLUTION OF INHOMOGENEITIES IN UNIVERSES DOMINATED BY DARK MATTER

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ABSTRACT. The astronomical data show that most of the mass in the Universe is dark. At present there are a few models of the dark matter: (i) the standard neutrino model, (ii) the model with unstable neutrinos, (iii) axion model. All models can be modified with Λ - term which plays role of the homogeneous component of the dark matter. In this paper the non-linear processes of gravitational instability are briefly discussed.

1. INTRODUCTION

The problem of dark matter has many different aspects, both observational and theoretical. In this report I consider one that concerns the non-linear evolution of density perturbations in their course to the formation of large scale objects: galaxies, clusters and superclusters of galaxies. It is assumed that most of the dark matter consists of some sort of weakly interacting particles: neutrinos, photinos, axions..., and the background universe model is based on the well known inflation scenario (for review see, for example [1,2]).

The theory of the non-linear processes must be an essential part of future full theory of the formation of structure in the Universe, because the observed objects like clusters and super-clusters are the inhomogeneities of density in the non linear regime of evolution: $\delta\rho/\bar{\rho} > 1$. In recent years the accuracy of theoretical predictions became so high that some models (for example the universe dominated by stable neutrinos with rest mass about 30 ev) are claimed to be rejected because they cannot match the observations within a factor only about 2 or 3. To make predictions of such quality one needs to possess the quantitative theory of non-linear processes. At present such theory is not fully developed. However the understanding of the non-linear stage of density perturbation growth has much improved recently. For this symposium the theoretical prediction that non-linear evolution of density perturbations goes different ways in the different models of dark matter is of particular interest, hence coupled with observations it can be a probe for these models.

J. Kormendy and G. R. Knapp (eds.), Dark Matter in the Universe, 369–377.

2. EVOLUTION OF PERTURBATIONS

The failure to measure angular variations in the temperature of relic radiation (except the dipole component [3,4]) is interpreted as the absence of any noticeable density perturbations at decoupling. On the other hand at present time the Universe is highly inhomogeneous on scales smaller than superclusters of galaxies $M \leqslant 10^{15}\ M_\odot$. Thus there must be considerable amplification of density fluctuations probably present at decoupling. Remembering the fact that most of the mass in the Universe consists of particles interacting essentially only due to gravity one arrives to the conclusion that amplification must be due to gravitational instability. There is a rather long stage of linear growth of inhomogeneities when baryons, representing a small portion of the total mass $\rho_b \sim (0.1 \rightarrow 0.01)\rho_t$ move in the gravitational field of dark matter. Baryons can gravitationaly influence dark matter when their perturbations at some place reach the strongly non-linear regime: $\delta\rho_b/\rho_b \sim (\delta\rho_b/\rho_b)$ on the homogeneous background of dark matter or $\delta\rho_b/\rho_b \sim (\delta\rho_\nu/\rho_\nu)(\rho_\nu/\rho_b)$ (here density of the dark matter is conventionally marked by index "ν"). The latter probably takes place in the central regions of galaxies but is hardly possible on scales of clusters and superclusters of galaxies.

When density perturbations reach the non linear stage evolution can go in two principally different ways, depending on the spectrum of initial perturbations at the linear stage: they are called "fragmentation"[5] and "hierarchical clustering"[6]. It is worth noting that in Nature the structure formation can possess the features of both processes, though it is useful to discuss them separately.

3. FRAGMENTATION SCENARIO

The standard inflation scenario of early universe predicts a Zeldovich spectrum of primordial adiabatic fluctuations, which in long wavelength limit (small k) has the form $\delta_k^2 \propto k^n$ (here δ_k^2 is spectrum of density perturbations) independently of the type of collisionless particles dominating in the dark matter. The short wavelength part of the spectrum evolves differently depending on the kind of particles, that enables Dick Bond to introduce the terms "hot", "warm" and "cold" particles. These terms specify the largest scale of perturbations suffering the free streaming dissipation.

The typical "hot" particles are electronic neutrinos with rest mass about 30 eV. The dissipation scale in this case is about $M\nu \sim 10^{15}\ M_\odot$ and approximately coincides with the masses of superclusters[7,8]. In the "hot" particle model of dark matter the fragmentation scenario is typical. Owing to the sharp cutoff in the spectrum on scales smaller than M_ν, the objects formed first have just this size and the very specific shapes of irregular pancakes. This was predicted by Zeldovich in 1970[9]. Later it was found that other kinds of objects, both elongated and compact ones form as well as pancakes[10]. They can be classified as typical (generic) singularities in accordance with catastrophe theory. Numerical simulations (2D[11,12] as well as 3D[13-16]) have shown that flattened, elongated and compact objects form a distinct cellular or network structure depending on the density contrast level.

In the frame of this scenario it is natural to suppose that galaxies form later by fragmentation of the baryon component of the pancakes and/or filaments and compact clusters[5]. However at present there is no good quantitative theory for this process.

What we can be sure of is our understanding of causes of the cellular structure formation[17-19]. The cellular structure originates inevitably at the non-linear stage of gravitational instability if the spectrum of density perturbations at linear stage has a sharp cutoff on short wavelengths and the scale of the cutoff λ_c is much greater than the Jeans length λ_J ; $\lambda_c >> \lambda_J$. As the smaller the temperature of the medium the less the Jeans scale becomes this inequality is easier to satisfy in cold medium at $T \to 0$. Thus one who likes paradoxes can say the cellular structure originates in the cold medium of "hot" particles.

Considering different scenarios one usually tries to find the stage in the evolution which is the best fit to the present Universe. In the fragmentation scenario the cellular structure is quite distinct at this stage. What will happen to it later? The answer to this question will help us to understand the process of hierarchical clustering better.

Evolution of density perturbations in a cold medium ($\lambda_c >> \lambda_J$) can be expressed with the Zeldovich formalism[9]

$$r_i = a(t)\cdot(q_i - b(t)\ \nabla_q\Phi(q)) \qquad (1)$$

here r_i and q_i are Eulerian and Lagrangian coordinates of particles, a(t) is a scale factor of Friedmann universe, b(t) is the growing solution of the linear density perturbations (if $\Omega_o = 1$, $\Lambda = 0$, $P = 0$, then $b(t) \propto a(t) \propto t^{2/3}$); $\Phi(q)$ is a function conserving full information about the growing mode of perturbations. In the linear regime $\delta\rho/\rho = b(t)\Delta\Phi$. At the non-linear stage

$$\rho/\bar{\rho} = (1 - b\cdot\alpha)^{-1}\ (1 - b\cdot\beta)^{-1}\ (1 - b\cdot\gamma)^{-1} \qquad (2)$$

here α, β, γ are the eigenvalues of the tensor $\partial^2\Phi/\partial q_i \partial q_k$. The important result of the Zeldovich formalism is that the structure forming at the beginning of the non-linear stage is determined by the spatial structure of the functions α, β, γ (mostly by the largest one)[5]. Unfortunately this formalism is not applicable at the non-linear stage when the cellular structure begins to disrupt.

4. HEIRARCHICAL CLUSTERING

The traditional approach[6] to the heirarchical clustering process is based on the calculation of the density perturbations at the scale $M \sim \rho\cdot k^{-3}$

$$\frac{\delta\rho}{\rho}(M) \propto \left(\int_0^k \delta_k^2\cdot k^2\cdot dk\right)^{1/2} \propto M^{-\frac{3+n}{6}} \qquad (3)$$

https://doi.org/10.1017/S0074180900150557 Published online by Cambridge University Press

if $\delta_k \propto k^n$ and $n > -3$. Combining this with the linear law of perturbation growth and assuming that after reaching the non-linear stage ($\delta\rho/\rho \sim 1$) the perturbation on the scale M virializes and ceases growing one easily finds the typical mass of the objects at time t ($\Omega_o = 1$, $b(t) = a(t)$)

$$M(t) \propto a^{\frac{6}{n+3}} \propto t^{\frac{4}{n+3}} \qquad (4)$$

If $n > 4$ one must take into account the non-linear generation of long wave perturbations with spectrum $\delta_k \propto k^4$ that results in limit law $M \propto a^{6/7} \propto t^{4/7}$ even at $n > 4$.

However this approach is not fully non-linear, because it actually considers generation of long waves only "once" at some moment t_g and later they are assumed to grow in agreement with the linear law. But it cannot be true, because non-linear generation continues to work later at $t > t_g$ as well as at t_g.

Recently a rather simple but fully non-linear model has been developed (based on the Burgers equation, well known in the theory of turbulence) for the evolution of cellular structure at a stage when it disrupts[20]. Mass flows from the walls of the cells (originated as pancakes) to the ribs of the cells and from ribs to the compact concentrations in the apicies of the cells. Soon most of the mass concentrates in clusters whose mean mass grows continuously by the merging of clusters. This process is in fact hierarchical clustering.

Without going into details of the model I just enumerate the main features of the process. In contrast to the initial period of the non-linear stage when pancakes and the whole cellular structure is determined by the structure of the eigenvalues α, β and γ in Lagrangian space (2) the process of hierarchical clustering is determined by the structure of potential $\Phi(q)$ (1). The main result of this is a much stronger influence of the long wavelength part of the spectrum on the process of clustering - the so called long distance correlations. This becomes quite clear if one remembers that the spectrum $\Delta_k{}^2$ of Φ (q) is $\Delta_k \propto \delta_k{}^2 \cdot k^{-4}$.

This non-linear theory of hierarchical clustering predicts a growth of the typical mass of clusters which coincides with the standard linear theory in the case of rather flat spectra

$$\delta_k^2 \propto k^n,\ M \propto a^{\frac{6}{n+3}} \quad \text{if } -1 \leqslant n \leqslant 1. \qquad (5)$$

At $n > n_{cr} = 1$ there is a limit law $M \propto a^{3/2} \propto t$ independently of n. In terms of non-linear generation of long wavelength perturbations it takes place because of continuity of the process. It is interesting that n_{cr}

https://doi.org/10.1017/S0074180900150557 Published online by Cambridge University Press

depends on the dimension of space: $n_{cr}(3D) = 1$, $n_{cr}(2D) = 2$ and $n_{cr}(1D) = 3$. If $n < -1$ the spectrum δ_k^2 must be bent down at some long wave λ_b, otherwise there is too strong divergence in spectrum of $\Phi(q)$ at $k \to 0$; because $\Delta_k^2 \propto k^{n-4}$. At $n < -1$ clustering is not actually pure hierarchical, because even at the time when masses $M \ll M_b \sim \rho \cdot \lambda_b^3$ decouple from Hubble expansion there is an ordering influence of scale λ_b. This influence is the stronger the steeper the spectrum is and at the limit of big negative n ($n < 0$ and $|n| \gg 1$) it becomes pure pancake picture with characteristic scale λ_b.

Spectrum with $n \approx -3$ which naturally occurs in the axion model of the dark matter[22,23] is extremely difficult to analyse and at present there is no good theory for the clustering process in this case. Nevertheless it is clear that influence of the scale at the bend of the spectrum must be rather strong so the formation of the large scale network structure seems to be quite possible, however it is probably a much less distinct one than that in the standard neutrino model.

5. MODELS OF DARK MATTER AND SCENARIOS OF STRUCTURE FORMATION

Now let us briefly discuss present models of dark matter with respect to the structure formation. At present any baryonic model of the dark matter encounters probably unsolvable problems. Therefore I discuss only hypotheses assuming that most of dark matter is in a form of weakly interacting particles or a positive Λ - term.

As it has become clear from the above discussion the character of the large scale structure depends on the type of the spectrum of density perturbations at the linear stage after decoupling. In turn the spectrum of perturbations depends on the type of particles constituting the dark matter.

5.1 Stable electronic neutrinos

The model of the Universe dominated by stable electronic neutrinos with rest mass ~ 30 eV (standard neutrino model[5,7]) seems to be the most economical of modern cosmological models in the sense of the number of ad hoc hypotheses needed. In this picture the formation of the large scale structure is mostly developed.

The spectrum of density perturbations is very simple: $\delta_k^2 \propto k$ on large scales $M > 10^{15} M_\odot$ and very sharp cutoff at smaller scales[21] $\delta_k^2 \propto k^{-\mu}$ ($\mu \gtrsim 12$) $M < 10^{15} M_\odot$. For this reason the structure originating at non-linear stage is very distinct. It is probably more distinct than the real structure observed in the distribution of galaxies.

This model has several difficulties, widely discussed in the literature [2,5,24]. Remembering the uncertainties of astronomical data and poor understanding of the process of galaxy formation none of them seems to be fatal to the model[25]. However at present there is no positive solution to some of them. As the most severe of them I would like to mention: (i) the fast growth of the portion of the mass in the compact clusters[26] and (ii) too great M/L ratio in clusters. The problem of the correlation length becomes not so serious if the new results of Einasto et al.[27] are taken into account.

https://doi.org/10.1017/S0074180900150557 Published online by Cambridge University Press

5.2 Unstable neutrinos

As an attempt to solve the problems of standard neutrino model Doroshkevich and Khlopov[28] proposed the model with unstable neutrinos ($\nu_H \rightarrow \nu_L + f$). This model has succeeded in solving many problems of the standard model including the ones mentioned above[29,30]. However it operates with several additional parameters like: 1) ratio of mean densities of unstable and stable components, 2) mean density of the stable component, 3) ratio of masses of stable and unstable particles. In addition it probably has difficulty with the age of the Universe and appeals to the Λ -term. Compared with the standard model it is much less economic and its basic "investment" is an appeal to "fantastic" particles (term by A. Dolgov): unstable neutrino with rest mass about 100 eV, as well as familon.

Returning to the problem of the non-linear evolution one should say that the process of structure formation in this model is very much similar to that in the standard model. The only difference is that the process of the evolution of cellular structure slows down due to the decay of particles and the influence of a possible Λ-term.

5.3 Axion model

The other alternative to the standard model is the widely discussed axion model[2,22,24]. The spectrum of density perturbations in this model after decoupling in the long waves is the same $\delta_k^2 \propto k$; in the short wavelength limit it is $\delta_k \propto k^{-3} \ln^2 k$[23] with a bend at $M \sim 10^{15}\ M_\odot$. But the bend is very smooth, extended more than an order of magnitude.

The process of structure formation starts from the origin of objects about $10^6\ M_\odot$ (Jeans mass in baryons after decoupling) and extends to $\sim 10^{15}\ M_\odot$ at present time. This model has no principal difficulties with the epoch of galaxy and quasar formation. It is more economic in free parameters than the model of unstable neutrinos. Nevertheless it also appeals to "fantastic" particles and for a better fit of numerical parameters to Λ-term.

As follows from section 4 in this model a dim network structure can naturally originate at the non-linear stage. 3D numerical simulations perhaps give some positive evidence for this[31]. But to make adequate simulations of this model is an extremely difficult task, because in this case one needs to follow evolution of perturbations in very wide range of scales which is beyond the possibilities of modern numerical techniques. Nevertheless on the basis of available numerical data it was reported that in this scenario there is a difficulty with explanation of huge voids in distribution of galaxies in space[32]. This must depend on the ratio of the power in k^{-3} part and the less steep part of the spectrum at linear stage.

6. DISCUSSION

I have considered three of the presently most popular models of dark matter from a dynamical point of view at the non-linear stage of the density inhomogeneities. Comparison with the available astronomical data shows that two of them: 1) model with unstable neutrinos and 2) axion model are in remarkably better positions than the standard neutrino model. However they have "missing" from the eyes of astronomers problems invoking "fantastic" particles like unstable neutrino, familon or axion as well as Λ-term of needed value. Will the "investments" in the form of additional hypotheses in this model bring enough "interest" in the form of better quantitative explanation of observational data to keep their positions of leaders on the "market" of models we shall see in a few years. New neutrino experiments and the Space Telescope will bring answers to many key questions.

Comparison of the structure originating at non-linear stage in different models with observational one seems to show that in neutrino models the structure is too distinct[40] and in axion model too dim[32] than real one. The former problem could be solved if one assumes some kind of explosions at the stage of galaxy formation. Physically the explosions could be similar to ones proposed by Ostriker and Cowie[33] but occuring only inside pancakes. This process can throw some galaxies out of the pancakes, making the structure dimmer, reducing the correlation length and slowing down evolution of the observable structure. In this case the distribution of galaxies would not be the same as that of the dark matter.

It is worth to mention the possibility of non-monotonic spectra of initial perturbations. Recently Kofman and Linde[34] found a mechanism to generate not only flat (in metrics) but also such kind of spectra during inflation. This gives new possibilities in the structure and galaxy formation but again by the price of introducing additional free parameters. The advantage of the spectra with two maxima is advocated by Dekel[35]. To make the structure comparable with the real one, one needs to have a rather steep short wavelength slope (probably $n < -3$) of the long wavelength maximum.

Finally I would like to make a short comment on works about percolation. I proposed[36] to use percolation parameter B_c as a quantity characterizing topological properties of the large scale distribution of galaxies. The application[40] of percolation technique to the data of CfA catalogue has shown its usefulness. Later Bhavsar, Barrow[37] and Dekel, West[38] criticized it as a cosmological test. However their criticism concerns mostly the method of estimation of B_c used firstly in[36,40]. That method was based on the calculation of the length of the largest cluster, which is simple and works well in the case of region of cubic shape analysed in[36,40], but for regions of more complicated shape considered in[37,38] it is not appropriate. Recently Klypin[39] has shown that the percolation parameter B_c can be easily estimated in regions of arbitrary shape using the approach of phase transitions. His analysis of different samples confirms our previous results[36,40]. Percolation analysis provides a new practically independent on correlation analysis parameter of the real distribution of galaxies characterizing its topology and any model of the large scale structure formation can be tested by it as well as by the correlation analysis.

I am grateful to A. Szalay for help.

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