A NON-LINEAR EMISSION MECHANISM FOR PULSAR RADIO RADIATION

E. Asseo
Centre de Physique Théorique, Ecole Polytechnique
G. Pelletier
Observatoire de Grenoble
H. Sol
DARC, Observatoire de Paris-Meudon

Introduction

Among the various plasma instabilities which could be responsible for coherent pulsar radio emission, we investigate the two-stream instability, first introduced by Ruderman and Sutherland (1975) in order to account for the physical situation expected in the environment of neutron stars. They describe how, in a polar cap model, pair creation arises and leads to the formation of a very energetic beam of e^+ (and/or e^-) and of an e^-e^+ plasma, both with relativistic bulk motion along the bundle of dipolar magnetic field lines. The study of their interaction is limited to the cone of open $\boldsymbol{B}$ lines, a site which provides a natural geometry for the radio emission zone, observed as core and/or conal emission by Lyne and Manchester (1988) and Rankin (1983, 1986, 1990). We follow Cheng and Ruderman (1977, 1980), and consider as negligible the contribution of the most energetic e^+ beam to the two-stream instability whereas the contribution of the e^- and e^+ of thé plasma is the most important one. The relative motion acquired by the e^- and e^+ of the plasma, flowing along the curved $\boldsymbol{B}$ lines, gives rise to a rapidly growing beam plasma instability, the slowest component being "the plasma", the fastest one "the beam". We show that linear increase of this instability is insufficient to account for high pulsar radio luminosities but that strong Langmuir turbulence in the magnetosphere would ensure sufficient energy radiated in the radio range and introduce characteristic structures which can be related to the observed microstructures of individual pulses.

Linear treatment

Specific pulsar dispersion relation and beam plasma instability

In the extremely strong pulsar magnetic field the particle motion can be considered as one-dimensional. Locally, at some point on the $\boldsymbol{B}$ line, we approximate particle motion by motion along a circle, located in the plane of the field line itself, with radius, R_c, the radius of curvature of the $\boldsymbol{B}$ line at this point. We first estimate the importance of the curvature and conclude that development of the two-stream instability or nonlinear evolution of strong turbulence are insensitive to curvature effects, the characteristic wavelengths of Langmuir waves or the spatial extent of Langmuir solitons generated by strong turbulence being much shorter than R_c (see Asseo *et al.* 1980, 1983, 1990). Thus, locally, we use a cartesian description. The indices $\|, \perp$ and t respectively refer to the direction along the $\boldsymbol{B}$ line, along $\boldsymbol{R}_c$ and along some axis perpendicular to $\boldsymbol{B}$ and $\boldsymbol{R}_c$. We treat the interaction in the observer's frame and define n_b, v_b and γ_b to characterize the beam density, velocity and Lorentz factor, $n_f(n_s), v_f(v_s)$ and $\gamma_f(\gamma_s)$ to characterize the density, velocity and Lorentz factor of the fastest (slowest) component of the e^-e^+ plasma. We derive the dispersion relation after linearization of the relativistic fluid and Maxwell equations with respect to wave perturbations $f \exp[i(\omega t - k_\| x_\| - k_\perp x_\perp - k_t z)]$ imposed on the initial equilibrium state. One primary simplification follows from the extremely high value of $|\boldsymbol{B}|$. Another one arises from quasi-neutrality. We define $n^r = n/\gamma^3$. Then, with physical pulsar parameters $n_f^r/n_s^r \approx 10^{-2}$ and $n_b^r/n_f^r \approx 4 \times 10^{-14}$. The beam contribution is negligible while terms proportional to the e^- and e^+ gyrofrequency have opposite signs and cancel. From this, a simplified linearized system of equations for $E_\|, E_\perp$ and E_t is obtained. In the general case $k_\perp, k_t \neq 0$, the three electric field components are coupled. If both $k_\perp \ll k_\|$ and $k_t \ll k_\|$, and if terms beyond second order, in $k_\perp$ or k_t, are negligible, the dispersion relation becomes

$$\varepsilon_\| \left[\omega^2/c^2 k_\|^2 - 1\right] = k_\perp^2 + k_t^2 . \qquad (1)$$

Here

$$\varepsilon_\| = 1 - \omega_s^{r2} \frac{1 + 3k_\|^2 \lambda_D^2}{\left(\omega - k_\| v_s\right)^2} - \frac{\omega_f^{r2}}{\left(\omega - k_\| v_f\right)^2}$$

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and $\omega^{r}_{s,f} = n^{r}_{s,f} 4\pi e^2/m$ is the relativistic "plasma" or "beam" frequency, $v_{th} = (2kT/m)^{1/2}$ the thermal velocity, $\lambda_D = v_{th}/\omega_s$ the associated Debye length. We assume that the velocity dispersion expected in the "plasma" is well described by the inclusion of the classical thermal term in $\varepsilon_{\parallel}$. Eq.(1) is very specific to the pulsar e^-e^+ plasma immersed in a strong $\boldsymbol{B}$. When $k_t(k_\perp) = 0, E_\parallel$ and $E_\perp(E_t)$ are coupled while $E_t(E_\perp)$ is completely decoupled. Then two different modes of polarization for the perturbed waves are possible.

1. a purely transverse mode with $E_t(E_\perp) \neq 0$ but $E_\parallel = E_\perp(E_t) = 0$;
2. a quasi-longitudinal mode with $E_\parallel, E_\perp(E_t) \neq 0$ but $E_t(E_\perp) = 0$ with dispersion relation eq.(1).

In what follows we focus on the properties of mode 2 and derive the characteristics of the beam plasma instability from eq.(1). We define

$$\begin{aligned} \omega_s^* &= \omega_s^r(1+\Omega), \\ \Omega &= 3k_\parallel^2\lambda_D^2/2, \\ \omega_f^* &= \omega_f^r, \\ 2g^3 &= \omega_f^*/\omega_s^*, \\ \Delta v &= (v_s - v_f) \ll c. \end{aligned}$$

Unstable solutions with maximum growth rate, $\mathrm{Im}\,\omega = \pm\omega_s^* g\sqrt{3/2}$, have very narrow instability windows when beams are relativistic,

$$\begin{aligned} \Delta k_\parallel &\lesssim \omega_s^*(g+\Omega), \\ \Delta k_\perp &\approx \Delta k_t \approx k^{obs}_{\perp max} \approx \omega_s^* g/(c\Delta v)^{1/2}. \end{aligned}$$

Since $\Delta k_\perp(\Delta k_t) \ll \Delta k_\parallel$, quasi-parallel propagation in the direction of $\boldsymbol{B}$ lines is effective for unstable waves.

Electromagnetic energy radiated in the linear stage, P^{L}

When the source of coherent radio emission is attributed to the beam plasma instability, it is possible to relate the electromagnetic energy available from the beam and the expected pulsar radio luminosity. From Asséo *et al.* (1980, 1983), we know that for the whole beam and plasma system, flowing along the cone of open curved $\boldsymbol{B}$ lines in a strong $|\boldsymbol{B}|$ and surrounded by external plasma, the growth rate is unchanged as long as the extent of the emission region is sufficient for the beam and the plasma to be treated as thick *i.e.* as infinite and homogeneous. Thus P^{L}, the electromagnetic energy radiated through the boundaries of the radio emission zone, can be directly evaluated by averaging, over frequencies and wave vectors, the flux of the Poynting vector, $\boldsymbol{\pi}$, through a surface S characteristic of the emission zone.

$$P \approx \langle \boldsymbol{\pi} \rangle \cdot \boldsymbol{S} \tag{2}$$

Since $\boldsymbol{\pi}$ depends on the Fourier transforms of the fluctuating electromagnetic fields,

$$\begin{aligned} E_\perp &\approx -k_\perp k_\parallel c^2 E_\parallel / \left(\omega^2 - c^2 k_\parallel^2\right), \\ E_t &\approx k_t E_\perp / k_\perp, \\ B_\perp &\approx -\omega E_t / c k_\parallel, \\ B_t &\approx \omega E_\parallel / c k_\parallel, \end{aligned}$$

it is clear that the most important contribution to P^{L} comes from the resonant domain where these components can reach very high values. Then $P^{L}_\perp = P^{L}_t = 0$, $\pi_\perp$ and π_t being odd functions of $k_\perp$ or k_t, while

$$p^{L}_\parallel = \langle \pi_\parallel \rangle S_\perp \approx 3 \times 10^{26} (\delta n/n_s)^2 \,\mathrm{erg/sec} \tag{3}$$

with $S_\perp$ estimated using Lyne and Manchester (1988) data. The electromagnetic energy flows only in the direction of the strong $\boldsymbol{B}$ but, if reasonable density fluctuations $(\delta n/n_s)^2 \approx 10^{-2}$ to 10^{-3} arise, the linear development of the beam plasma instability in the pulsar magnetosphere is insufficient to reach the level of observed radio luminosity.

Nonlinear treatment

Derivation of the nonlinear system of equations

The nonlinear stage of the beam plasma interaction is treated in the plasma rest frame. Nonlinear charge density perturbations δn^{NL} appear in the plasma, on scales $> \lambda_D$ and on time scales $> 2\pi\omega_s^{-1}$, through the so-called ponderomotive force $\boldsymbol{F}$. One simple way to estimate $\boldsymbol{F}$ is to analyze "plasma" particle motion in the high frequency Langmuir wave field, $\boldsymbol{F} = -\omega_s^2 \nabla |E_\parallel|^2 / 16\pi\omega^2$. Then "plasma" particles drift towards the gradient of lower energy density in the Langmuir waves. Relativistic "beam" particles are almost unaffected by the ponderomotive effect, $\boldsymbol{F}$ being decreased by the factor γ_f^3. Thus "plasma" particles drift out of the intense wave field region and afterwards, pull out the "beam" particles through a charge separation electric field. This results in the creation of a density cavity which can trap the Langmuir energy and leads to self-modulation instability of the wave packets over short scales. We estimate $\delta n^{NL}/n_s = -e^2|E_\parallel|^2/8mT\omega^2$ leading to changes in the plasma frequency and dielectric constant. ω_s^{*2}

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becomes $\omega_s^{*2}(1-\delta n/n_s)$, and $\varepsilon_\parallel$ is changed into $\varepsilon_\parallel^{\rm NL}$. From this, a nonlinear system of equations is obtained for $E_\parallel$ and $E_\perp$. Analysis of this system in Asseo *et al.* (1990) shows that we can treat separately the equations governing the evolution of $\varepsilon_\parallel$ and $\varepsilon_\perp$, the $\parallel$ and $\perp$ electric field envelopes varying slowly with time and space. While $\varepsilon_\perp$ follows a classical wave equation (see below), the evolution of $\varepsilon_\parallel$, in both cases of resonant and nonresonant interaction between the "beam" and the "plasma", can be derived from the same "model equation". This "model equation" is very similar to the modified nonlinear Schrödinger equation (hereafter MNLS equation) studied in Pelletier *et al.* (1988)

$$\Delta_\parallel \left[i\partial_t + \frac{1}{2}\Delta_\parallel + |\psi|^2 \right] \psi = \frac{1}{2} r \Delta \psi . \quad (4)$$

To write eq.(4) we introduce dimensionless quantities $t_0 = \omega_s^{-1}$ and $\ell_0 = \sqrt{3}\lambda_{\rm D}$, and define ψ by $|\psi|^2 = |\varepsilon_\parallel|^2/32\pi n_s T$. The parameter characteristic of $|\boldsymbol{B}|$ is $r = \omega_c^2/(\omega_s^2 - \omega_c^2)$ (but $\omega_s \neq \omega_c$). In the strong pulsar $\boldsymbol{B}$, $r \to -1$.

Conditions for the development of strong turbulence

The problem is to know whether $\varepsilon_\parallel$ will evolve through the quasi-linear process or whether wave-wave interactions will lead to the development of strong turbulence. In fact the more rapid process should dominate, but for relativistic beams, the condition for quasi-linear processes is never fulfilled. Comparing the dispersive and nonlinear terms in eq.(4) and assuming that the wave electrostatic energy, W, is much smaller than the thermal energy in the background plasma, $n_s T$, the conditions for development of strong turbulence are $W/n_s T > \lambda_{\rm D}^2 \Delta k_\parallel^2$ and $W/n_s T > \lambda_0^2 \Delta k_\perp^2/8\pi^2$ (λ_0 is the resonant wavelength). Such conditions can be fulfilled for pulsar physical parameters, $\Delta k_\parallel$ and $\Delta k_\perp$ being very narrow. We thus expect that development of strong turbulence in the pulsar magnetosphere, associated with $\varepsilon_\parallel$, will lead to self-modulation instability, creation of localized wave packets and generation of solitons.

Behavior of the soliton-like solution of the MNLS equation

The MNLS eq.(4) admits one-dimensional, nonlinear, Schrödinger (NLS) soliton-like solutions,

$$\psi_s(x_\parallel, t) = a \,\mathrm{sech}\, a(x_\parallel - x_0) \exp i(u(x_\parallel - x_0) - \theta) . \quad (5)$$

a is the soliton amplitude, a^{-1} the soliton width, x_0 localizes the center of the soliton, $u = \dot{x}_0$ characterizes the velocity of its center of mass, θ is related to the frequency. The evolution of the four parameters of eq.(5), namely a, θ, u and x_0, is obtained by means of the perturbation method of the inverse scattering transform developed by Karpman and Maslov (1977). We consider in Pelletier *et al.* (1988) that these parameters evolve adiabatically with time and $\perp$ coordinate as a result of the combined dissipative and perturbative effects, including beam plasma instability on large scale λ_0, Landau absorption on short scale $\lambda_{\rm D}$ and $\perp$ deformations. The resulting set of four coupled equations is reduced, when $u = 0$, to two coupled equations for a and θ. This system admits stationary solutions, $\tilde{a}$ and $\tilde{\theta}$. Linearization around $\tilde{a}$ and $\tilde{\theta}$ and analysis of the corresponding dispersion relation, shows that the solitons are always stable in the case of a strong $\boldsymbol{B}(r < 0)$. We propose to call such solutions "Langmuir microstructures". Such structures should be present in the whole region of open magnetic field lines.

Characteristics of the Langmuir microstructures

Considering dissipative effects alone, we show in Pelletier *et al.* (1988) that the soliton amplitude increases until it reaches the value a_0 at time t_0. a_0 results from competition between instability and Landau absorption. Estimates with pulsar physical quantities yield $a_0 \sim (5\lambda_{\rm D})^{-1}$, longitudinal soliton extent $L_\parallel^{\rm (pl)} \approx 5\lambda_{\rm D}$ and associated temporal width, as seen from the observer's frame, $\tau_\parallel^{\rm obs} = \gamma_{\rm f} L_\parallel^{\rm (pl)}/c \approx 10^{-7}$ sec. A lower limit for soliton spatial extent in the $\perp$ direction $L_\perp^{\rm min} = 2\pi/k_{\perp\rm max}^{\rm (pl)}$ is associated with $\tau_\perp^{\rm obs} \geq \tau_{\perp\rm min} \approx L_\perp^{\rm min}/v_\perp \approx$ a few 10^{-6} sec ($v_\perp$ is the emission region velocity $\perp$ to the line of sight). From this we predict the existence of a lattice of standing solitons generated in the plasma, all with the same fixed amplitude a_0 and longitudinal extent, regularly distributed in space and separated by a few $\lambda_{\rm D}$. Recent numerical simulations by Larroche and Pesme (1990) suggest the same. Once the amplitude a_0 is reached, it keeps this value but the $\perp$ deformations it contains slowly damp and diffuse while giving rise to radiation. We show in Asseo *et al.* (1990) that the characteristic diffusion time for inhomogeneities with size $L_\perp^{\rm min}$ is the longest characteristic time, so that they have sufficient lifetime to persist in the $e^- e^+$ plasma and radiate. An estimate for the number of Langmuir microstructures in a region of extent L yields $N = L/2L_\parallel^{\rm (pl)} \approx$ a few 10^3.

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Electromagnetic energy radiated by Langmuir microstructures

In spite of the fact that Langmuir solitons are not expected to radiate, being stable in the strong pulsar $\boldsymbol{B}$, they can directly emit electromagnetic radiation when suffering $\perp$ deformations. We study the evolution of $E_\perp$ from the original equation

$$\left(c^{-2}\partial_t^2 - \partial_\parallel^2\right) E_\perp(\boldsymbol{x},t) = -\partial_\parallel \boldsymbol{\nabla}_\perp E_\parallel(\boldsymbol{x},t), \quad (6)$$

similar to a classical wave equation with a source term related to $E_\parallel$ which acts as a source for $E_\perp$. Using the Green's function method,

$$E_\perp = E_\perp^+ + E_\perp^-,$$

where

$$E_\perp^\pm \propto (\lambda_D \boldsymbol{\pi}_\perp a/a) \cos\omega_s(t \mp (x_\parallel - x_0)/c).$$

We then deduce the associated B_t and $\pi_\parallel$. Since $E_\perp$ and B_t have a propagative character, they transport energy through the plasma. We estimate $P^{NL}_{\parallel,N}$, the electromagnetic energy associated with the set of N Langmuir microstructures generated in the plasma of the emission region. Assuming that the microstructures radiate independently and are not modified by their own radiation. This seems reasonable since the diffusion time associated with their $\perp$ deformations is slow enough. Then

$$P^{NL}_{\parallel,N} = NS_\perp \langle \pi_\parallel^{\text{observer}} \rangle \approx 8 \times 10^{27}\,\text{erg/sec}, \quad (7)$$

sufficient to account for the observed 10^{26} to 10^{30} erg/sec pulsar radio luminosities.

Comments

Consequently, nonlinear evolution of the beam plasma interaction suggests an efficient mechanism for conversion of the kinetic energy of the relativistic beam into electromagnetic energy sufficient to account for pulsar radio radiation. This process, being directly linked to $(\boldsymbol{\nabla}_\perp a/a)^2$ through the combination of induced perpendicular electric and magnetic fields, and in this way to the perpendicular deformations of the microstructure itself, appears as a direct consequence of the obliquity effect. This radiation mechanism is locally narrowband but can account for global broadband emission. The excited modes are 100% linearly polarized $\perp$ to $\boldsymbol{B}$, but coupling to the transverse mode 1 could induce circular polarization. Also, depending on local conditions of excitation in the magnetosphere, there is a possibility for orthogonal modes. The probable existence of a lattice of radiating Langmuir microstructures, with associated characteristic spatial and temporal widths, can be related to the observed microstructures. In fact, $\tau_\perp^{\text{obs}} \approx$ a few 10^{-6} sec for the observed microstructures arises naturally from the angular structure characteristics of the radiated power.

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