Note on Mental Division by Large Numbers.

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Since $\frac{A}{B} = \frac{nA}{nB}$, it is possible to divide mentally by many numbers, integral or fractional.

Examples: $\frac{3275}{125} = \frac{8.3275}{8.\ 125} = \frac{26200}{1000} = \mathbf{26 \cdot 2}$;

$\frac{4579}{14\frac{2}{7}} = \frac{7.4579}{7.\ 14\frac{2}{7}} = \frac{32053}{100} = \mathbf{320 \cdot 53}.$

In 1901, when I was drawing up notes on Mental Arithmetic, I looked into many text-books in search of a simple method for dividing by such numbers as 19, 29, 99, 87, etc., but found none. The following method, viz., *that of using a multiple of ten as divisor instead of a given divisor*, was then discovered by me, and I think it simple enough to be learned and practised by any one.

If it be required to divide A by D, let the quotient at any step be Q, then the product at that step will be DQ, and the remainder, $R = A - DQ$, where R cannot exceed D nor be less than zero.

Instead of working with D as divisor, take as divisor d, *that multiple of ten which is nearest to D*, and if, at each step, care be taken with Q, so that R never exceeds D nor is less than zero, the required quotient Q will be obtained.

Dividing by d, the remainder at any step is $r = A - dQ$,

and since $\quad R = A - DQ$, and $r = A - dQ$,

$$R = r + (d - D)Q.$$

Thus R, the remainder which would have been obtained on dividing by D, is obtained at every step by adding $(d - D)Q$ to r, the remainder obtained on dividing by d. The importance of obtaining R correctly at each step is so great that I would suggest that the method of obtaining it in each sum be made the *key-word of that sum*.

For example: In dividing by 19, 29, 39, 49, or 99, the divisor used is 20, 30, 40, 50, or 100, where $d - D = 1$, so that $R = r + Q$, and the key-word in such examples would be: *Add once Q*.

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In dividing by 67, 87 or 97, the divisor used is 70, 90 or 100, where $d - \mathrm{D} = 3$, and $\mathrm{R} = r + 3\mathrm{Q}$, and the key-word would be: *Add three times Q.*

Again, in dividing by 31, 61 or 71 the divisor used would be 30, 60 or 70, where $d - \mathrm{D} = -1$, and $\mathrm{R} = r - \mathrm{Q}$, and the key-word would be: *Subtract once Q.*

In 43, 53, 73 or 83 the key-word would be: *Subtract three times Q.*

The divisors used in working the following examples are printed in heavier type.

20 19 | 451360 = **$23755\frac{15}{19}$** Key-word: *Add once Q.*

Divide by 20	Quot.	$r + \mathrm{Q}$	$= \mathrm{R}$
45	2	5 + 2	7
71	3	11 + 3	14
143	7	3 + 7	10
106	5	6 + 5	11
110	5	10 + 5	15

100 99 | 223134 = **$2253\frac{87}{99}$**

80 79 | 763794 = **$9668\frac{22}{79}$**

1000 999 | 2341527 = **$2343\frac{870}{999}$**

120 119 | 2785857 = **$23410\frac{67}{119}$**

70 67 | 4462182 = **$66599\frac{49}{67}$** Key-word: *Add three times Q.*

Divide by 70	Quot.	$r + 3\mathrm{Q}$	$= \mathrm{R}$
446	6	26 + 18	44
442	6	22 + 18	40
401	5	51 + 15	66
668	9	38 + 27	65
652	9	22 + 27	49

60 57 | 312623 = $5484\frac{35}{57}$

100 97 | 236582 = $2438\frac{96}{97}$

90 87 | 5628435 = $64694\frac{57}{87}$

400 397 | 2847928 = $7173\frac{247}{397}$

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Key-word : *Subtract once Q*, $R = r - Q$.

10 11 | 37636876
$3421534\frac{2}{11}$

30 31 | 285670357
$9215172\frac{25}{31}$

Key-word : *Subtract three times Q* ; $R = r - 3Q$.

70 73 | 6249683
$85612\frac{7}{73}$

50 53 | 1286494
$24273\frac{25}{53}$.

In the following examples, the figures in large type show where special care has to be taken with Q, so that the remainder R may neither exceed the divisor D nor be less than zero.

10 11 | 478265
$43478\frac{7}{11}$

40 43 | 23572864
$548206\frac{6}{43}$

70 67 | 376745
$5623\frac{4}{67}$

30 29 | 149386
$5151\frac{7}{29}$

10 9 | 32876
$3652\frac{8}{9}$

100 96 | 327654
$3413\frac{6}{96}$

70 68 | 156675
$2304\frac{3}{68}$

1000 998 | 568976
$570\frac{116}{998}$

$\frac{11}{41} = \cdot\dot{2}682\dot{9}$

40 41 | 11·
·26829

$\frac{5}{17} = \cdot\dot{2}9411764$
$7058823\dot{5}$

20 17 | 5·
·29411764

45243 farthings = £47 " 2 " $6\frac{3}{4}$

1000 960 | 45243
£45 + 2043 farthings
= £45 + £2 + 123 farthings
= £47 " 2 " $6\frac{3}{4}$

367234 farthings = £382 " 10 " $8\frac{1}{2}$

1000 960 | 367234
£382 + 514 farthings

46236 lb. = 412 cwt. 92 lb.

3724562 lb. = 1662 tons 15 cwt. 2 lb.

110 112 | 46236
412 cwt. 92 lb.

110 112 | 3724562
33255 cwt. 2 lb.

It is evident that sums in Long Division may be much simplified by the adoption of this method. The remainder $R = r + (d - D)Q$ may be obtained mentally

```
397 ) 284792857 ( 717362 143/397
400 ) 2800
      ----
        689
        400
        ---
        2922
        2800
        ----
         1438
         1200
         ----
          2475
          2400
          ----
            937
            800
            ---
            143
```

```
479 ) 1848479 ( 3859 18/479
500 ) 1500
      ----
        4114
        4000
        ----
         2827
         2500
         ----
          4329
          4500
          ----
            18
```