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The destruction of ^{3}He by Rayleigh-Taylor instability on the first giant branch

Peter P. Eggleton[1,3], David S. P. Dearborn[2,3] and John C. Lattanzio[4]

[1]Institute of Geophysics and Planetary Physics
[2]Physics and Allied Technologies Division
[3]Lawrence Livermore National Laboratory, 7000 East Ave, Livermore, CA94551, USA
email: ppe@igpp.ucllnl.org, dearborn2@llnl.gov

[4]Centre for Stellar and Planetary Astrophysics,Monash University, Australia
email: john.lattanzio@sci.monash.edu.au

Abstract. Low-mass stars, $\sim 1-2$ solar masses, near the Main Sequence are efficient at producing ^{3}He, which they mix into the convective envelope on the giant branch and distribute into the Galaxy by way of envelope loss. This process is so efficient that it is difficult to reconcile the observed cosmic abundance of ^{3}He with the predictions of Big Bang nucleosynthesis. In this paper we find, by modeling a red giant with a fully three-dimensional hydrodynamic code and a full nucleosynthetic network, that mixing arises in the supposedly stable and radiative zone between the hydrogen-burning shell and the base of the convective envelope. This mixing is due to Rayleigh-Taylor instability within a zone just above the hydrogen-burning shell. In this zone the burning of the ^{3}He left behind by the retreating convective envelope is predominantly by the reaction $^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H}$, a reaction which, untypically for stellar nuclear reactions, *lowers* the mean molecular weight, leading to a local minimum. This local minimum leads to Rayleigh-Taylor instability, and turbulent motion is generated which will continue ultimately up into the normal convective envelope. Consequently material from the envelope is dragged down sufficiently close to the burning shell that the ^{3}He in it is progressively destroyed. Thus we are able to remove the threat that ^{3}He production in low-mass stars poses to the Big Bang nucleosynthesis of ^{3}He.

Some slow mixing mechanism has long been suspected, that connects the convective envelope of a red giant to the burning shell. It appears to be necessary to account for progressive changes in the ^{12}C/^{13}C and ^{14}N/^{12}C ratios on the First Giant Branch. We suggest that these phenomena are also due to the Rayleigh-Taylor-unstable character of the ^{3}He-burning region.

Keywords. Stars: nucleosynthesis, stars: evolution, stars: red giants, instabilities, cosmology: nucleosynthesis

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1. Introduction

Stellar evolution has long shown rather clearly that in the Main-Sequence (MS) region stars burn hydrogen in their cores by a combination of the pp chain and the CNO tricycle. The former is the more important in low-mass stars, $\lesssim 1.5\,M_\odot$, and the latter in more massive stars. In the low-mass stars much ^{3}He is produced in a region outside the main H-burning core, and because the convective core is small or absent in such stars this ^{3}He survives and is mixed (Iben 1967) into the Surface Convection Zone (SCZ) as the star ascends the First Giant Branch (FGB). The initial abundance of ^{3}He may be increased above its primordial value, taken to be 2×10^{-4}, by a factor of nearly 10.

Fig. 1a illustrates the distribution of ^{3}He (and other isotopes) in a $0.8\,M_\odot$ Pop II star towards the end of its MS life. The Figure is the result of a 1D (i.e. spherically symmetric) calculation. ^{3}He is enriched above its initial value (the same as its surface value since this star has only a slight convective envelope) in a broad peak extending over nearly half the mass of the star. The peak abundance is a factor of ~ 30 larger than the initial value.

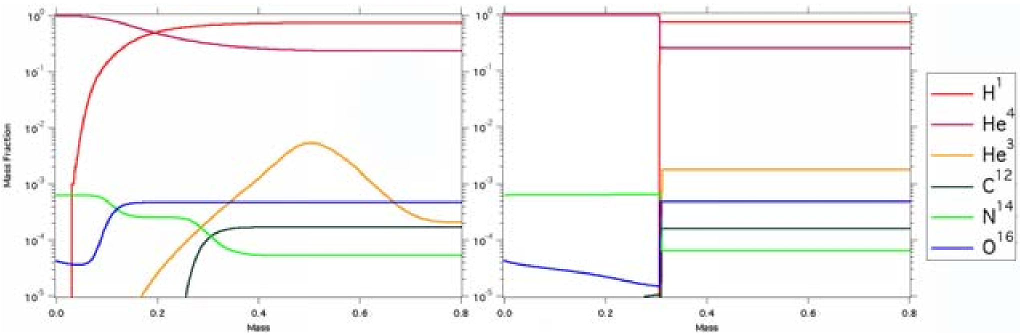

Figure 1. (a) Profiles of the abundances of certain nuclei in a star which has evolved to roughly the end of the MS. Although we only model a Pop II star here, the enrichment of ^{3}He is just as considerable for relatively metal-rich Pop I stars like the Sun. ^{3}He shows a major peak where the abundance reaches ~ 30 times the initial (surface) abundance. (b) The same star later, when the SCZ reaches its maximum inward extent. The ^{3}He peak has been homogenized, to a factor of 9 larger than its initial value. The inert, H-depleted core is about $0.3\,M_\odot$.

In later evolution, a large SCZ develops which mixes and homogenises the outer $\sim 0.5\,M_\odot$ (Fig. 1b). The surface ^{3}He abundance is raised from the initial 2.10^{-4} to $\sim 1.8.10^{-3}$, i.e. by a factor of ~ 9. Later still, as the star climbs the FGB, the SCZ is diminished by (a) nuclear burning below its base, in a zone that marches outwards, and (b) stellar-wind mass loss from its surface. The evidence for the latter is that the next long-lived stage after the FGB is the Horizontal Branch (HB), and HB stars appear to have masses that are typically $0.5 - 0.6\,M_\odot$, substantially less than the masses of stars capable of evolving to the FGB in less than a Hubble time (Faulkner 1966, 1972). Process (b) leads to enrichment of the interstellar medium (ISM) in ^{3}He (Steigman *et al.* 1985, Dearborn *et al.* 1986, 1996).

Only relatively low-mass stars contribute to this enrichment, because in massive stars the convective core becomes a large enough fraction of the total stellar mass that it mixes the ^{3}He peak to the center, where the ^{3}He is burnt to ^{4}He. Roughly, we expect that stars in the mass range $0.8 - 2\,M_\odot$ are the ones that contribute to ^{3}He enrichment; but this is a substantial majority, by combined mass as well as by number, of all stars capable of substantial evolution in the Galaxy's lifetime. Yet the ISM's abundance of ^{3}He, at $\sim 2.10^{-4}$ by mass, is little different from that predicted by Big Bang nucleosynthesis. This is a major problem (Hata *et al.* 1995, Olive *et al.* 1995): either the Big Bang value is too high, or the evolution of low-mass stars is wrong.

In this paper we identify a mechanism by which low-mass stars destroy (on the FGB) the ^{3}He that they initially produce during their MS evolution. Amusingly, this mechanism is driven by the ^{3}He itself, in a narrow zone just above the main hydrogen-burning shell that is characteristic of FGB stars.

2. A local molecular-weight inversion

Once the SCZ has reached its deepest extent, part-way up from the base of the FGB, it retreats, and can be expected to leave behind a region of uniform composition with the ^{3}He abundance of $\sim 1.8.10^{-3}$ as seen in Fig. 1b. This region is stable to convection according to the usual Schwarzschild criterion, and is quite extensive in radius although small in mass. The H-burning front moves outwards into the stable region, but preceding the H-burning region proper is a narrow region, usually thought unimportant, in which the ^{3}He burns. The reaction that mainly consumes it is

$$^3\mathrm{He}\ (^3\mathrm{He}, 2\mathrm{p})^4\mathrm{He}\ , \tag{1}$$

which is an unusual reaction in stellar terms because it *lowers* the mean molecular weight: two nuclei become three nuclei, and the mean mass per nucleus decreases from 3 to 2. The molecular weight being the mean mass per nucleus, but including also the much larger abundances of ^{1}H and ^{4}He that are already there and not taking part in this reaction, this leads to an inversion in the molecular-weight gradient. The effect is tiny (see Fig. 2a, from the same 1D simulation as Fig. 1): it is in about the fourth decimal place. But our 3D modeling shows it to be hydrodynamically unstable, as we should expect from the classic Rayleigh-Taylor instability.

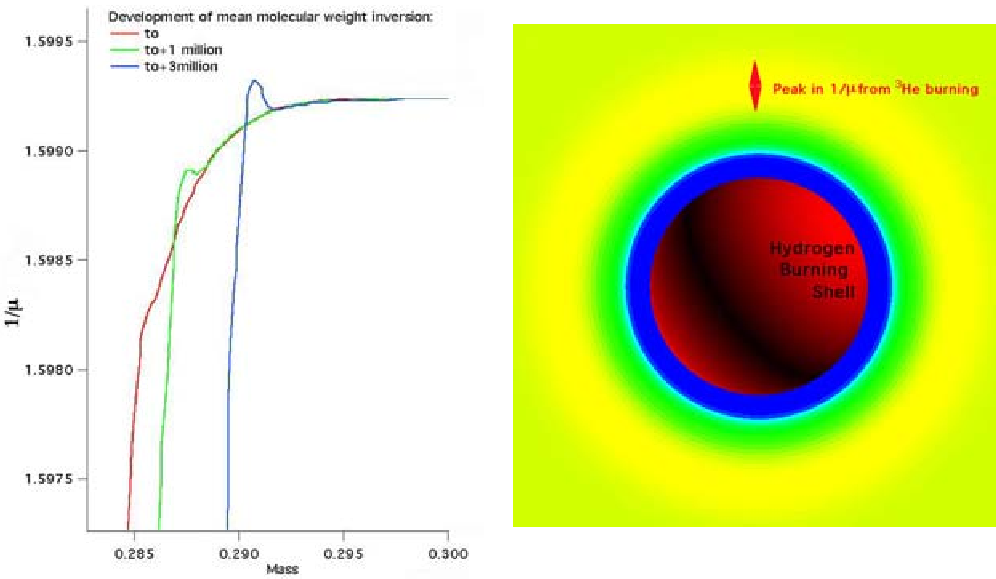

Figure 2. (a) The profile of (reciprocal) molecular weight, on a greatly exaggerated vertical scale, as a function of mass coordinate. Once the burning shell has burnt outwards to the region where the major composition has been homogenized by the previous action of the SCZ, a small peak due to ^{3}He-burning, just outside the main burning region, begins to stand out. The peak exists because ^{3}He-burning *decreases* the mean molecular weight of the interacting particles. (b) A cross-section through the central part of the star at the beginning of the 3D run. Mean molecular weight is color-coded, except that the main hydrogen-burning shell is replaced by a red sphere. The μ-inversion is the marked yellow band.

Fig. 2a shows the $1/\mu$ profile plotted against mass coordinate. At a relatively early stage (red curve) there is no bump, but just a slight distortion at about $0.286\,M_\odot$. This is because the ^{3}He consunption is taking place in a region where there is still a substantial

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μ gradient left over from earlier history. But as the H-burning shell moves out (in mass), the ^{3}He-burning shell preceding it moves into a region of more uniform ^{1}H/^{4}He ratio, and so the peak in $1/\mu$ begins to stand out. By the time the shell has moved to $0.289\,M_\odot$ there is a clear local maximum in $1/\mu$, which persists indefinitely as the H-burning shell advances and the convective envelope retreats.

At this point in the evolution of our 1D star we mapped it on to a 3D model and used the hydrodynamic code 'Djehuty' developed at the Lawrence Livermore National Laboratory (Bazán *et al.* 2003, Eggleton *et al.* 2003, Dearborn *et al.* 2006). This code is described most fully in the third of these papers. Although Djehuty is designed to deal with an entire star, from center to photosphere, we economised on meshpoints by considering only the region below the SCZ. The actual model selected was a Pop I star of $1\,M_\odot$ rather than a Pop II star of $0.8\,M_\odot$, but there is very little difference in regard to the ^{3}He behavior, and the peak in $1/\mu$.

Fig 2b is a color-coded plot of μ on a cross-section through the initial 3D model. The shell where the μ-inversion occurs is the yellow-orange region sandwiched between a pale green and a rather darker green. The inversion is at a radius of $\sim 5.10^7$ m. The base of the SCZ is at $\sim 2.10^9$ m, well outside the frame, and the surface of the star is at $\sim 2.10^{10}$ m. Notwithstanding the red sphere shown in Fig 2b, the entire interior of the the star below the SCZ was in the computational domain.

3. Rayleigh-Taylor instability

Fig 3 shows the early development of the initially-spherical shell on which $1/\mu$ has a constant value near its peak. After only ~ 800 secs, the surface has begun to dimple, and by 2118 secs the dimpling is very marked, and the surface has begun to tear. Some points have moved $\sim 2\%$ radially, ie $\sim 2.10^5$ m, indicating velocities of ~ 100 m/s. The mean velocity decreases slightly in the passage from the second to the fourth panel. Other spherical shells, well away from the inversion on either side, show no such dimpling, at least until the influence of the inversion has spread to them.

The motion appears turbulent, and has the effect of diluting the inverse molecular-weight gradient, but it cannot eliminate it. As the turbulent region entrains more of the normally stable region outside it yet below the normal convective envelope, it brings in fresh ^{3}He, which burns at the base of this mixing region, thus sustaining the inverse molecular-weight gradient. Ultimately this turbulent region will extend to unite with the normally-convective envelope, so that the considerable reservoir of ^{3}He there will also be depleted. If its speed of ~ 100 m/s is maintained the time for processed material to reach the classically unstable SCZ is only about 5 months, while the time to burn through the $\sim 0.02\,M_\odot$ layer is over 10^6 yrs.

Normal convective mixing, as in the usual SCZ, is a rapid process: it would homogenise the surface layers in a matter of weeks, if it were not already homogeneous by this stage in the star's evolution. Our new mixing process might be expected to be very much slower, but in order to modify progressively the composition in the normal convective envelope it need only operate on roughly the nuclear timescale of the star, which is ~ 200 Megayear at this point.

However, small as is the μ inversion that drives our extra mixing, it produces velocities that are surprisingly large, and in fact comparable to the velocity of the normal convection. This is for two reasons. Firstly, although we might expect the inversion to be diluted to a trivial amount by the mixing that it produces, it is in fact sustained because the fresh ^{3}He that is brought in by the mixing is burnt quite rapidly by reaction (1) inside the inversion. If $\Delta\mu$ is the height of the peak, if $t_{\rm mix}$ is the timescale of mixing due to the

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turbulent motion, and if $t_{\rm burn}$ is the timescale of ^{3}He-burning in the inversion, then we expect that $\Delta\mu$ is diluted by a factor $\sim t_{\rm mix}/t_{\rm burn}$. We estimate this below, finding values of $\sim 10^{-4}$. Since $\Delta\mu \sim 10^{-3}$, the strength of our 'engine' is roughly $\sim 10^{-7}$. Secondly, the normal convection of the outer envelope is driven by what is in fact only a very small excess of actual temperature gradient over the adiabatic gradient. The fractional excess of temperature gradient is itself $\sim 10^{-5}$ near the base of the SCZ, and is therefore not, as one might expect, a great deal larger than the fractional deficit in μ that drives our extra mixing. Thus it is reasonable to expect our extra mixing to produce turbulent velocities of perhaps a tenth of the normal convective mixing.

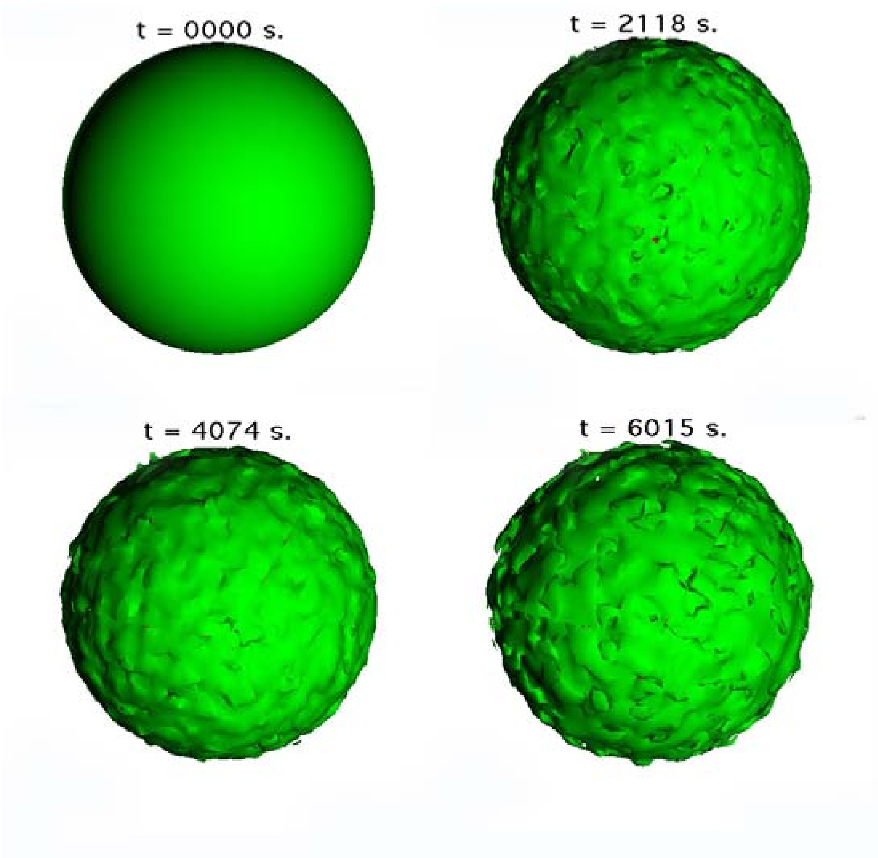

Figure 3. The development with time of a contour surface of mean molecular weight near the peak in Fig. 3. The contour dimples, and begins to break up, on a timescale of only ~ 2000 sec.

As a first approximation, we can expect the effect of buoyancy due to $\Delta\mu$ to generate a velocity of order

$$v^2 \sim gl\,\frac{\Delta\mu}{\mu}, \tag{2}$$

where g is the local gravity and l is the distance an eddy moves before dissolving into the ambient material; l is normally estimated as the pressure scale height. In ordinary convection, near the base of the SCZ, the equivalent estimate is

$$v^2 \sim gl\,\frac{\Delta T}{T}, \tag{3}$$

where ΔT is the excess temperature, relative to the ambient medium, of an eddy that

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expands adiabatically on rising. Near the base of the SCZ we have $g \sim 10^2$ m/s^2, $l \sim 10^{8.5}$ m, $\Delta T/T \sim 10^{-5}$, so that $v \sim 10^{2.8}$ m/s. In our first approximation (2), we ought to modify $\Delta\mu$ by the factor $t_{\rm mix}/t_{\rm burn}$ indicated above, and since we can estimate $t_{\rm mix}$ as l/v, our second approximation is

$$v^3 \sim gl^2 \frac{\Delta\mu}{t_{\rm burn}}. \tag{4}$$

We find $g \sim 10^{4.5}$ m/s^2, $l \sim 10^7$ m, $t_{\rm burn} \sim 10^8$ s and so $v \sim 10^{2.5}$ m/s. Thus the expected velocity driven by the inverted μ-gradient is barely a modest factor of 2 down on the normal convective velocity near the base of the SCZ. This is smaller than the factor of ~ 10 in our first estimate mainly because gl is an order of magnitude larger near the burning zone than near the base of the SCZ.

The above argument establishes that the mixing is extended below the classical Schwarzschild limit, and that it is very fast compared to the nuclear timescales of either the hydrogen-burning shell or the ^{3}He-burning reaction. If the mixing is fast we can make an estimate of the total amount of ^{3}He that will be destroyed, by approximating the mixing region as homogeneous. Beginning with a model near the location where the mechanism starts, and well below the helium flash (top left in Fig. 1) the lifetime of the ^{3}He in a region whose mass coordinate extends from m_1 to m_2 is

$$\frac{1}{t_{33}} = \frac{1}{m_2 - m_1} \int_{m_1}^{m_2} \frac{dm}{\tau_{33}}, \tag{5}$$

where

$$\frac{1}{\tau_{33}} = \frac{d\ln^3 {\rm He}}{dt} = \rho N(^3{\rm He}) < \sigma v > /2, \tag{6}$$

$N(^3{\rm He})$ is the fractional local ^{3}He abundance, and $< \sigma v >$ is the thermally-averaged nuclear cross-section times velocity, i.e. reaction rate. Similarly the timescale for the predominantly CNO-cycling hydrogen-burning shell is

$$\tau_{\rm shell} = \frac{N(^1{\rm H})\epsilon_{\rm CNO}(m_2 - m_1)}{L}, \tag{7}$$

where $N(^1{\rm H})$ is the fractional hydrogen abundance above the shell and ϵ is the local rate of nuclear energy generation. The result of such integrations over some different mass regions shows that for any region much larger than the ^{3}He-burning region the ratio of lifetimes of ^{3}He destruction and core-mass growth is constant and approximately 16. Thus for rapid mixing the ^{3}He will be destroyed in 16 times as much mass as the hydrogen shell burns through.

We believe that the extra mixing that we have discovered gives a satisfactory answer to the problem mentioned in the second-last paragraph of the Introduction that confronts Big Bang nucleosynthesis. Although low-mass stars do indeed produce considerable amounts of ^{3}He on the MS, this will all be destroyed by the substantially deeper mixing that we now expect on the FGB. It is somewhat ironic that this deeper mixing is driven by the ^{3}He itself.

Our deeper mixing can also be relevant to further problems that have troubled stellar modelers for several years. According to the classical models of FGB stars, there is no further modification to the composition in an FGB convective envelope after it has reached its maximum extent early on the FGB. Yet observations persistently suggest that the ratios $^{13}{\rm C}/^{12}{\rm C}$ and $^{14}{\rm N}/^{12}{\rm C}$ both increase appreciably as one goes up the FGB (Suntzeff 1993, Kraft 1994). Both these ratios can be expected to increase only if the material in the envelope is somehow being processed near the H-burning shell. Our model

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makes this very likely. Although the μ-inversion that we find is somewhat above the main part of the H-burning shell, it is not far above and we can expect some modest processing of ^{12}C to ^{13}C and ^{14}N. According to Weiss & Charbonnel (2004), it appears to be necessary for some extra mixing to take place beyond the point on the FGB where the SCZ has penetrated most deeply; that is exactly the point where our mechanism should start to operate.

Correlations between abundance excesses and deficits of various elements and isotopes in the low-mass evolved stars of globular clusters have been discussed thoroughly in (Kraft 1994). The subject is complex, and it is hard to distinguish star-to-star variations that may be due to evolution from those that may be due to primordial variation. Evidence exists for both kinds of variation. However, we expect our mechanism to lead to substantial evolutionary variations.

4. Discussion

The μ-inversion that we investigate has not been noted before, to our knowledge; but even if it has been noted previously it has probably been ignored because it is small, and because traditional 1D models only give turbulent mixing if they are instructed to. We feel that our investigation demonstrates particularly clearly the virtue of attempting to model in 3D, where the motion evolved naturally, and to a magnitude that initially surprised us. 3D modeling is an expensive exercise, but we believe that it is amply justified. The mixing process that we have identified appears to be capable of solving one cosmological problem and two or more stellar problems.

Acknowledgements

We are grateful to R. Palasek for managing the code, and for help with the graphics. This study has been carried out under the auspices of the U.S. Department of Energy, National Nuclear Security Administration, by the University of California, Lawrence Livermore National Laboratory, under contract No. W-7405-Eng-48.

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Discussion

WEISS: A comment: The connection between the carbon anomalies and the ^{3}He-problem was made by Charbonnel (1995) and Weiss, Wagenhuber, and Denissenkov (1995) coming from the empirical evidence for deep mixing.

EGGLETON: Yes: we refer explicitly to you paper with Charbonnel (2004). But you discussed mainly the possibility that the extra mixing might be driven by differential rotation. Our point is that, while rotation and differential rotation might influence the mixing, perhaps accounting for some spread, a different mechanism (the Rayleigh- Taylor mechanism which we describe) is bound to happen, and can account for the ^{3}He problem as well, perhaps, as for ^{13}C and ^{14}N.

ROXBURGH: In order to have the large ^{3}He build up on the main sequence you have to assume that this build up remains in place and is not destroyed by the ^{3}He epsilon-mechanism instability first proposed by D. Gough (Nature, 240, 262, 1972).

EGGLETON: I believe that Gough's 'solar spoon' mechanism was concerned with the much deeper interior. Our mechanism kicks in relatively nearer the surface.