

## THESIS ABSTRACTS

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DODAMGODAGE GIHANEE M. SENADHEERA, *Effective Concept Classes of PAC and PACi Incomparable Degrees, Joins and Embedding of Degrees*, School of Mathematical and Statistical Sciences, Southern Illinois University, Carbondale, IL, USA, 2022. Supervised by Wesley Calvert. MSC: Primary 03D30, Secondary 68T05. Keywords: machine learning, PAC learning, bocomputability, effectively closed sets.

### Abstract

The Probably Approximately Correct (PAC) learning is a machine learning model introduced by Leslie Valiant in 1984. The PACi reducibility refers to the PAC reducibility independent of size and computation time. This reducibility in PAC learning resembles the reducibility in Turing computability. The ordering of concept classes under PAC reducibility is nonlinear, even when restricted to particular concrete examples.

Due to the resemblance to Turing Reducibility, we suspected that there could be incomparable PACi and PAC degrees for the PACi and PAC reducibilities as in Turing incomparable degrees. In 1957 Friedberg and in 1956 Muchnik independently solved the Post problem by constructing computably enumerable sets  $A$  and  $B$  of incomparable degrees using the priority construction method. We adapt this idea to PACi and PAC reducibilities and construct two effective concept classes  $C$  and  $D$  such that  $C$  is not reducible to  $D$  and vice versa. When considering PAC reducibility it was necessary to work on the size of an effective concept class, thus we use Kolmogorov complexity to obtain the size. The non-learnability of concept classes in the PAC learning model is explained by the existence of PAC incomparable degrees.

Analogous to the Turing jump, we give a jump operation on effective concept classes for the zero jump. To define the zero jump operator for PACi degrees the join of all the effective concept classes is constructed and proved that it is a greatest element. There are many properties proven for existing degrees. Thus we can explore proving those properties to PACi and PAC degrees. But if we prove an embedding from those degrees to PACi and



PAC degrees then those properties will be true for PAC<sub>i</sub> and PAC degrees without explicitly proving them.

Abstract prepared by Dodamgodage Gihnee M. Senadheera and taken directly from the thesis

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MIGUEL ANTONIO CARDONA-MONTOYA, *Forcing theory and combinatorics of the real line*, Vienna University of Technology, Vienna, Austria, 2022. Supervised by Jakob Kellner and Diego A. Mejía. MSC: 03E17, 03E15, 03E35, 03E40.

**Abstract**

The main purpose of this dissertation is to apply and develop new forcing techniques to obtain models where several cardinal characteristics are pairwise different as well as force many (even more, continuum many) different values of cardinal characteristics that are parametrized by reals. In particular, we look at cardinal characteristics associated with strong measure zero, Yorioka ideals, and localization and anti-localization cardinals.

In this thesis we introduce the property “*F*-linked” of subsets of posets for a given free filter *F* on the natural numbers, and define the properties “ $\mu$ -*F*-linked” and “ $\theta$ -*F*-Knaster” for posets in a natural way. We show that  $\theta$ -*F*-Knaster posets preserve strong types of unbounded families and of maximal almost disjoint families. These kinds of posets led to the development of a general technique to construct  $\theta$ -Fr-Knaster posets (where Fr is the Frechet ideal) via matrix iterations of  $<\theta$ -ultrafilter-linked posets (restricted to some level of the matrix). The latter technique allows proving consistency results about Cichoń’s diagram (without using large cardinals) and to prove the consistency of the fact that, for each Yorioka ideal, the four cardinal characteristics associated with it are pairwise different. Another important application is to show that three strongly compact cardinals are enough to force that Cichoń’s diagram can be separated into 10 different values. Later on, it was shown by Goldstern, Kellner, Mejía, and Shelah that no large cardinals are needed for Cichoń’s maximum (*J. Eur. Math. Soc.* 24 (2022), no. 11, p. 3951–3967).

On the other hand, we deal with certain types of tree forcings including Sacks forcing, and show that these increase the covering of the strong measure zero ideal  $\mathcal{SN}$ . As a consequence, in Sacks model, such covering number is equal to the size of the continuum, which indicates that this covering number is consistently larger than any other classical cardinal characteristics of the continuum. Even more, Sacks forcing can be used to force that  $\text{non}(\mathcal{SN}) < \text{cov}(\mathcal{SN}) < \text{cof}(\mathcal{SN})$ , which is the first consistency result where more than two cardinal characteristics associated with  $\mathcal{SN}$  are pairwise different. To obtain another result in this direction, we provide bounds for  $\text{cof}(\mathcal{SN})$ , which generalizes Yorioka’s characterization of  $\mathcal{SN}$  (*J. Symbolic Logic* 67.4 (2002), p. 1373–1384). As a consequence, we get the consistency of  $\text{add}(\mathcal{SN}) = \text{cov}(\mathcal{SN}) < \text{non}(\mathcal{SN}) < \text{cof}(\mathcal{SN})$  with ZFC (via a matrix iteration forcing construction).

We conclude this thesis by combining creature forcing approaches by Kellner and Shelah (*Arch. Math. Logic* 51.1–2 (2012), p. 49–70) and by Fischer, Goldstern, Kellner, and Shelah (*Arch. Math. Logic* 56.7–8 (2017), p. 1045–1103) to show that, under CH, there is a proper  $\omega^\omega$ -bounding poset with  $\aleph_2$ -cc that forces continuum many pairwise different cardinal