Preface

In 1959 Regge showed that, when discussing solutions of the Schroedinger equation for non-relativistic potential scattering, it is useful to regard the angular momentum, l, as a complex variable. He proved that for a wide class of potentials the only singularities of the scattering amplitude in the complex l plane were poles, now called 'Regge poles'. If these poles occur for positive integer values of l they correspond to bound states or resonances, and they are also important for determining certain technical aspects of the dispersion properties of the amplitudes. But it soon became clear that his methods might also be applicable in high energy elementary particle physics, and it is in fact here that the theory of the complex angular momentum plane, usually called 'Regge theory' for short, is now most fruitfully employed.

Apart from the leptons (electron, muon and neutrinos) and the photon, all the very large number of elementary particles which have been found, baryons and mesons, enjoy the strong interaction (i.e. the nuclear force which *inter alia* binds nucleons into nuclei) as well as the less forceful electromagnetic, weak and gravitational interactions. Such particles are called 'hadrons', from the Greek ἁδρός meaning large. Some are stable, but most are highly unstable and decay rapidly into other hadrons and leptons. They can be classified according to their various quantum numbers such as baryon number, charge, strangeness etc., but for a given set of quantum numbers sequences of particles have been found which differ only in their spin. For example resonances similar to the rho-meson (which is an unstable particle and decays into pi-mesons, viz $\rho \to \pi\pi$) occur with spins $\sigma = \hbar, 2\hbar, 3\hbar, \ldots$, the mass increasing with the spin.

If one were to try and 'explain' such resonances as being like bound states produced by a potential $V(r)$ acting between the pions (fig. i (a)), the radial Schroedinger equation would contain an effective potential

$$V_{\text{eff}}(r) = V(r) + \frac{l(l+1)}{r^2},$$

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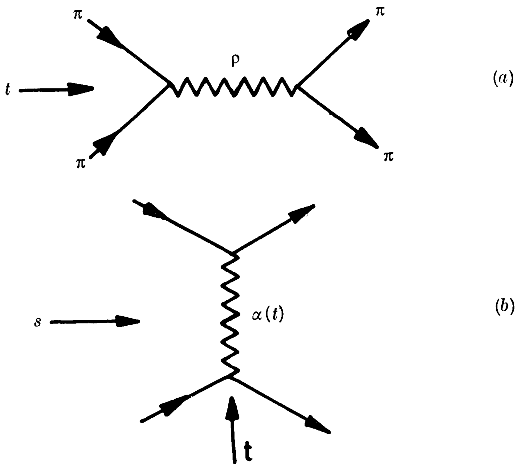

FIG. i (*a*) The binding of two pi-mesons to produce an unstable ρ resonance which subsequently decays into two pi-mesons again. (*b*) The exchange of a trajectory $\alpha(t)$ which gives the high energy behaviour of the scattering amplitude.

which provides less strong binding as the orbital angular momentum of the pions, l, is increased, because of the centrifugal barrier term, $l(l+1)r^{-2}$. So the potential is less effective for high l, which explains why high-spin resonances have higher masses. In fact one could solve the equation for arbitrary complex values of l, and the eigenvalues would vary continuously along a trajectory in the l plane connecting the various physical solutions which occur for $l = n\hbar$ (n integer). Of course such a non-relativistic model is quite hopeless for high energy physics, but the basic idea, that sequences of composite particles of mass m_i and spin σ_i ($i = 1, 2, 3, \ldots$) will lie on a given Regge trajectory $l = \alpha(t)$, where t is the square of the centre-of-mass energy, such that, for all i, $\alpha(m_i^2) = \sigma_i$, successfully inter-relates many sets of resonances. Indeed it is now widely believed that all the hadrons are composite particles lying on such trajectories, and are not really 'elementary' at all.

Also it is well established that the strong-interaction forces are due to the exchange of particles. This is a generalization of Yukawa's hypothesis that the long-range part of the inter-nucleon force is due to the exchange of pi-mesons. But rather than consider the exchange of individual particles it is more useful to consider the exchange of a complete trajectory of particles. Regge theory predicts that the high energy behaviour of a scattering amplitude $A(s,t)$ will be

$$A(s,t) \sim s^{\alpha(t)}$$

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(where now s is the square of the centre-of-mass energy, and $-t$ is the square of the momentum transferred (fig. i(b)). This is found to hold in a great variety of processes.

So Regge theory is concerned with the particle spectrum, the forces between particles, and the high energy behaviour of scattering amplitudes; in fact with almost all aspects of strong interactions. Hence an understanding of Regge theory has become essential for those who wish to work on high energy physics, and the aim of this book is to provide an introduction to the subject.

In the first chapter we discuss the kinematics of scattering processes, introduce scattering amplitudes, and review their analytic structure as functions of the energy and momentum transfer. In chapter 2 we define partial-wave amplitudes for a given l, and show how and why it is useful to make an analytic continuation in l. We explain why Regge poles in l, which lie on Regge trajectories, correspond to particles. In chapter 3 we examine the occurrence of Regge poles in potential scattering, in field theories, and in other models of strong interactions. Then in chapter 4 we introduce the somewhat more complicated formalism needed to discuss spin problems, before presenting in chapter 5 evidence for the Regge classification of particles on trajectories. Chapter 6 is devoted to a discussion of Regge pole predictions for the high energy behaviour of scattering amplitudes, while in chapter 7 we explore the hypothesis that there exists a 'duality' between resonance poles and Regge-trajectory exchanges. Chapter 8 is concerned with the more complicated effects of Regge cuts, singularities in the angular-momentum plane associated with the simultaneous exchange of two or more Regge trajectories. Then in chapter 9 we look at Regge-theory predictions for the behaviour of many-particle scattering processes, and in chapter 10, those for 'inclusive' reactions in which only a few of the final-state particles are actually detected. This is a field which has provided abundant evidence for the success of Regge theory in recent years. In chapter 11 we examine various models for the behaviour of high energy cross-sections, and the self-consistency of strong interactions under the hypothesis that Regge exchanges provide the binding forces between particles which in their turn generate Regge trajectories: the so-called 'bootstrap' mechanism. The final chapter is devoted to a rather brief discussion of the implications of Regge theory for electromagnetic and weak interactions. There are also mathematical appendices on Legendre functions and rotation functions.

The book is intended mainly for those who are just starting to concern themselves with elementary-particle physics, and as far as possible only a good background of undergraduate physics is assumed; that is, quantum theory and especially scattering theory to the level of, say, Schiff's *Quantum mechanics* (1968), special relativity, and the basic concepts of elementary-particle physics such as resonance scattering and isotopic spin (as in, for example, Bransden, Evans and Major (1973)). Also a knowledge of complex-variable theory and the special functions of mathematical physics is required. But in places some of the ideas of quantum field theory (mainly Feynman diagrams) are employed with only the briefest introduction, and the beginner will either have to accept what is said or consult the reference texts. Similarly a more detailed treatment of the Lie groups SU(2) and SU(3) than we have space for here is desirable. But it is hoped that those who read this book in conjunction with some of the references will not experience too many difficulties. They are strongly advised to skip the most difficult parts at a first reading, and refer back when necessary. (To assist this I have marked with a * sections which might be omitted.) It is also hoped that more experienced research workers may find here a useful compendium of the basic ideas and results of Regge theory.

When writing a book on a subject which is developing so fast it is always hard to guess which aspects will stand the test of time, and which will be found wanting. In the early 1960s it seemed to some people that the whole of Regge theory might fall into the latter category, but now many features seem securely established, and I have tried to concentrate on these, with only occasional excursions to glimpse what is happening near the rapidly moving frontier. The greatest consolidation has been possible with those aspects of the theory which directly pertain to experiment, and so I have included a good deal of 'Regge phenomenology', especially in chapters 5–10, but I have tried not to overlook completely the various hints which Regge theory provides as to the long-sought fundamental theory of strong interactions.

I have not attempted to give complete references to the voluminous literature on the subject. Indeed, except for a few of the historically most important papers, I have not referred much to the original literature on the early developments, but such references can readily be found in the various books and review articles which are mentioned. With more recent material I have attempted to give a wider selection

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of useful references, but only to illustrate the text and certainly not to apportion credit for particular discoveries. I can only apologize to those whose work has been overlooked or inadequately represented.

This book owes much to an earlier work on Regge poles which Professor E. J. Squires and I wrote some years ago (Collins and Squires 1968) and to a review article (Collins 1971), as well as various lecture courses I have given at Durham and elsewhere. But, while I have not changed the presentation just for the sake of it, I have tried to think afresh as to the best way of introducing the subject, stealing ideas from the many excellent review articles and lecture notes which are now available. Also I have tried to simplify as much as possible.

In conclusion I would like to express my indebtedness and gratitude to many people; to Professor G. F. Chew who first introduced me to this subject; to Professor E. J. Squires from whom I have learned many of its intricacies; to my colleagues in Durham who provide a stimulating environment for the study of elementary-particle theory, and much else; to Professor J. C. Polkinghorne, F.R.S. who induced me to write this book; to Professor E. J. Squires, Professor J. C. Polkinghorne, Dr A. D. Martin and Dr W. J. Zakrzewski for many useful comments on the text; to Mr T. D. B. Wilkie and Mr A. D. M. Wright for much help with correcting and improving it; to Margaret and Andrew for providing the rest frame which made it possible; and to Mrs Diana Philpot who has coped wonderfully with a very difficult typescript.

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