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Left-orderable Fundamental Group and Dehn Surgery on the Knot 5_2

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Abstract. We show that the manifold resulting from r-surgery on the knot 5_2, which is the two-bridge knot corresponding to the rational number $3/7$, has a left-orderable fundamental group if the slope r satisfies $0 \leq r \leq 4$.

1 Introduction

A group G is said to be *left-orderable* if it admits a strict total ordering that is left invariant. More precisely, this means that if $g < h$, then $fg < fh$ for any $f, g, h \in G$. The fundamental groups of many 3-manifolds are known to be left-orderable. On the other hand, the fundamental groups of lens spaces are not left-orderable, because any left-orderable group is torsion-free. The notion of an L-space was introduced by Ozsváth and Szabó [12] in terms of Heegaard–Floer homology. Lens spaces and Seifert fibered manifolds with finite fundamental groups are typical examples of L-spaces. Although it is an open problem to give a topological characterization of an L-space, there is a possible connection between L-spaces and left-orderability. More precisely, Boyer, Gordon, and Watson [3] conjecture that an irreducible rational homology sphere is an L-space if and only if its fundamental group is not left-orderable. They give affirmative answers for several classes of 3-manifolds.

It is well known that all knot groups are left-orderable (see [4]), but the resulting closed 3-manifold by Dehn surgery on a knot does not necessarily have a left-orderable fundamental group. For example, there are many knots that admit Dehn surgery yielding lens spaces. By [12], the figure-eight knot has no Dehn surgery yielding L-spaces. Hence we can expect that any nontrivial surgery on the figure-eight knot yields a manifold whose fundamental group is left-orderable, if we support the conjecture above. In fact, Boyer, Gordon, and Watson [3] show that if $-4 < r < 4$, then r-surgery on the figure-eight knot yields a manifold whose fundamental group is left-orderable. In addition, Clay, Lidman, and Watson [6] verified it for $r = \pm 4$ through a different argument.

In this paper, we follow the argument of [3] for the most part to handle the knot 5_2 from the knot table in [14]. This knot is the two-bridge knot corresponding to the rational number $3/7$, and is a twist knot. We believe that this is an appropriate target next to the figure-eight knot. Since 5_2 is non-fibered, it does not admit Dehn surgery

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yielding an L-space [11]. Hence we can expect that any non-trivial Dehn surgery on 5_2 will yield a 3-manifold whose fundamental group is left-orderable.

Theorem 1.1 *Let K be the knot 5_2. If $0 \le r \le 4$, then r-surgery on K yields a manifold whose fundamental group is left-orderable.*

In fact, 0-surgery on any knot yields a prime manifold whose first Betti number is 1, and such manifold has left-orderable fundamental group [4, Corollary 3.4]. Furthermore, the same conclusion holds for 4-surgery on twist knots [16]. Hence, in this paper we will handle the case where $0 < r < 4$.

2 Knot Group and Representations

Let K be the knot 5_2 from the knot table in [14]; see Figure 1. This knot is the two-bridge knot corresponding to the rational number $3/7$. In this diagram, K bounds a once-punctured Klein bottle, as seen from the checkerboard coloring, whose boundary slope is 4. In fact, 4-surgery on K gives a toroidal manifold, and 1, 2, and 3-surgeries give small Seifert fibered manifolds ([5]).

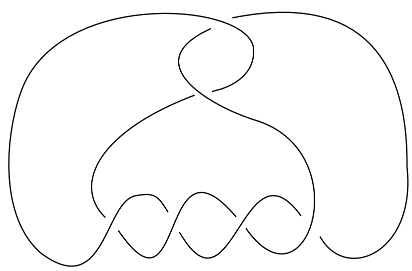

Figure 1

Let M be the knot exterior of K. It is well known that the knot group $G = \pi_1(M)$ has a presentation $\langle x, y \mid wx = yw \rangle$, where x and y are meridians and $w = xyx^{-1}y^{-1}xy$. Also, a (preferred) longitude λ is given by $x^{-4}w^*w$, where $w^* = yxy^{-1}x^{-1}yx$ corresponds to the reverse word of w. (These facts are easily obtained from Schubert's normal form of the knot [15].)

Let $s > 0$ be a real number and let

$$T = \frac{2 + 3s + 2s^2 + \sqrt{s^2 + 4}}{2s}.$$

Then it is easy to see that $T > 4$. Also, let $t = \frac{T + \sqrt{T^2 - 4}}{2}$. Then, $t > 3$ and

$$t = \frac{2 + 3s + 2s^2 + \sqrt{s^2 + 4} + \sqrt{(2 + 3s + 2s^2 + \sqrt{s^2 + 4})^2 - 16s^2}}{4s}. \tag{2.1}$$

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Let $\phi = s(t+t^{-1})^2 - (2s^2+3s+2)(t+t^{-1}) + s^3 + 3s^2 + 4s + 3$. Since $t + t^{-1} = T$, $\phi = sT^2 - (2s^2 + 3s + 2)T + s^3 + 3s^2 + 4s + 3$. If we solve the equation $\phi = 0$ with respect to T, we obtain the expression of T in terms of s as above. Thus $\phi = 0$ holds.

We now examine some limits, which will be necessary later.

Lemma 2.1

(i) $\lim_{s\to+0} t = \infty$.
(ii) $\lim_{s\to+0} st = 2$.
(iii) $t - s > 2$ *and* $\lim_{s\to\infty}(t - s) = 2$.
(iv) $\lim_{s\to\infty} s/t = 1$.
(v) $\lim_{s\to\infty} s(t - s - 2) = 0$.
(vi) $\lim_{s\to\infty} t(t - s - 2) = 0$.

Proof (i) and (ii) are obvious from (2.1). For (iii),

$$t - s = \frac{2 + 3s + \sqrt{s^2+4} + \left(\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2+4})^2 - 16s^2} - 2s^2\right)}{4s}$$

shows us that $t - s > 0$, since $(2 + 3s + 2s^2 + \sqrt{s^2+4})^2 - 16s^2 > 4s^4$. The second conclusion follows from

$$\lim_{s\to\infty} \frac{2 + 3s + \sqrt{s^2+4}}{4s} = 1, \quad \lim_{s\to\infty} \frac{\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2+4})^2 - 16s^2} - 2s^2}{4s} = 1.$$

A direct calculation shows (iv).

For (v),

$$4s(t - s - 2) - 2 = \left(\sqrt{(2 + 3s + 2s^2 + \sqrt{s^2+4})^2 - 16s^2} + \sqrt{s^2+4}\right) - (2s^2 + 5s).$$

Since the right-hand side converges to -2, we have $\lim_{s\to\infty} s(t - s - 2) = 0$.

From (iii), an inequality $s + 2 < t < s + 3$ holds for sufficiently large s. Then $(s+2)(t-s-2) < t(t-s-2) < (s+3)(t-s-2)$. Hence (iii) and (v) imply (vi). ∎

Let $\rho_s\colon G \to SL_2(\mathbb{R})$ be the representation defined by the correspondence

$$\rho_s(x) = \begin{pmatrix} \sqrt{t} & 0 \\ 0 & \frac{1}{\sqrt{t}} \end{pmatrix}, \quad \rho_s(y) = \begin{pmatrix} \frac{t-s-1}{\sqrt{t}-\frac{1}{\sqrt{t}}} & \frac{s}{(\sqrt{t}-\frac{1}{\sqrt{t}})^2} - 1 \\ -s & \frac{s+1-\frac{1}{t}}{\sqrt{t}-\frac{1}{\sqrt{t}}} \end{pmatrix}. \tag{2.2}$$

Here, we continue using the variable t to reduce the complexity. By using the fact that s and t satisfy the equation $\phi = 0$, we can check $\rho_s(wx) = \rho_s(yw)$ by a direct calculation. Hence the correspondence on x and y above gives a homomorphism from G to $SL_2(\mathbb{R})$. In addition, $\rho_s(xy) \neq \rho_s(yx)$, and so ρ_s has the non-abelian image.

Remark 2.2 This representation of G comes from that in [9, p. 786]. The polynomial ϕ corresponds to the Riley polynomial in [13].

Lemma 2.3 *For a longitude λ, $\rho_s(\lambda)$ is diagonal, and its $(1,1)$-entry is a positive real number.*

Proof Note that $\rho_s(x)$ is diagonal and $\rho_s(x) \neq \pm I$. The fact that $\rho_s(x)$ commutes with $\rho_s(\lambda)$ easily implies that $\rho_s(\lambda)$ is also diagonal. (This can also be seen from a direct calculation of $\rho_s(\lambda)$, by using $\phi(s,t) = 0$.)

A direct calculation gives the $(1,1)$-entry

$$\frac{1}{(t-1)^2 t^5}\Big(s\big(1-(2+s)t+t^2\big)\big(s-(2+2s+s^2)t+(1+s)t^2\big)^2 + (1+s-t)^2 t^3 \big(s-(1+s)^2 t + st^2\big)^2 \Big) \tag{2.3}$$

of $\rho_s(\lambda)$. Thus it is enough to show that $1-(2+s)t+t^2 > 0$. This is equivalent to the inequality $T > 2+s$, which is clear from $T = \frac{2+3s+2s^2+\sqrt{s^2+4}}{2s}$. ■

Let $r = p/q$ be a rational number and let $M(r)$ denote the manifold resulting from r-filling on the knot exterior M of K. In other words, $M(r)$ is obtained by attaching a solid torus V to M along its boundaries so that the loop $x^p\lambda^q$ bounds a meridian disk of V.

Clearly, $\rho_s\colon G \to SL_2(\mathbb{R})$ induces a homomorphism $\pi_1(M(r)) \to SL_2(\mathbb{R})$ if and only if $\rho_s(x)^p\rho_s(\lambda)^q = I$. Since both of $\rho_s(x)$ and $\rho_s(\lambda)$ are diagonal, this is equivalent to the equation

$$A_s^p B_s^q = 1, \tag{2.4}$$

where A_s and B_s are the $(1,1)$-entries of $\rho_s(x)$ and $\rho_s(\lambda)$, respectively. We remark that since $A_s = \sqrt{t}$ is a positive real number, so is B_s by Lemma 2.3. Furthermore, equation (2.4) is equivalent to

$$-\frac{\log B_s}{\log A_s} = \frac{p}{q}.$$

Let $g\colon (0,\infty) \to \mathbb{R}$ be a function defined by

$$g(s) = -\frac{\log B_s}{\log A_s}.$$

Lemma 2.4 *The image of g contains an open interval $(0,4)$.*

Proof First, we show that

$$\lim_{s\to+0} g(s) = 0.$$

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Since $\lim_{s\to+0}\log A_s=\infty$, it is enough to show that $\lim_{s\to+0}B_s=1$. We decompose B_s, given in (2.3), as

$$B_s=\frac{s}{t-1}\,\frac{1-(2+s)t+t^2}{(t-1)t}\Big(\frac{s-(2+2s+s^2)t+(1+s)t^2}{t^2}\Big)^2 + \Big(\frac{1+s-t}{t-1}\Big)^2\Big(\frac{s-(1+s)^2t+st^2}{t}\Big)^2.$$

From Lemma 2.1, $\lim_{s\to+0}t=\infty$ and $\lim_{s\to+0}st=2$. These give

$$\lim_{s\to+0}\frac{s}{t-1}=0,\qquad \lim_{s\to+0}\frac{1-(2+s)t+t^2}{(t-1)t}=1,$$

$$\lim_{s\to+0}\frac{s-(2+2s+s^2)t+(1+s)t^2}{t^2}=1,\qquad \lim_{s\to+0}\frac{1+s-t}{t-1}=-1,$$

and

$$\lim_{s\to+0}\frac{s-(1+s)^2t+st^2}{t}=1.$$

Thus we have $\lim_{s\to+0}B_s=1$.

Second, we show

$$\lim_{s\to\infty}g(s)=4.$$

Let N be the numerator of B_s shown in (2.3). Then

$$\frac{\log B_s}{\log A_s}=\frac{2\log N}{\log t}-\frac{2\log(t-1)^2t^5}{\log t}.$$

Claim 2.5 $\lim_{s\to\infty}Nt^{-5}=1$.

Proof of Claim 2.5 From Lemma 2.1, $\lim_{s\to\infty}s/t=1$ and $\lim_{s\to\infty}(1+s-t)=-1$. We have

$$1-(2+s)t+t^2=t(t-s-2)+1,$$

$$\frac{s-(1+s)^2t+st^2}{t}=\frac{s}{t}+s(t-s-2)-1,$$

$$\frac{s-(2+2s+s^2)t+(1+s)t^2}{t^2}=\frac{1}{t}\cdot\frac{s-(1+s)^2t+st^2}{t}-\frac{1}{t}+1.$$

Hence Lemma 2.1 implies

$$\lim_{s\to\infty}(1-(2+s)t+t^2)=\lim_{s\to\infty}\frac{s-(2+2s+s^2)t+(1+s)t^2}{t^2}=1,$$

$$\lim_{s\to\infty}\frac{s-(1+s)^2t+st^2}{t}=0.$$

Combining these, we have $\lim_{s\to\infty}Nt^{-5}=1$. ■

Thus we have $\lim_{s\to\infty}(\log N - 5\log t) = 0$. Then

$$\lim_{s\to\infty} \frac{\log N}{\log t} = 5.$$

Clearly,

$$\lim_{t\to\infty} \frac{\log(t-1)^2 t^5}{\log t} = 7.$$

Hence we have $\lim_{s\to\infty} g(s) = 4$. ■

3 The Universal Covering Group of $SL_2(\mathbb{R})$

Let

$$SU(1,1) = \left\{ \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \;\middle|\; |\alpha|^2 - |\beta|^2 = 1 \right\}$$

be the special unitary group over $\mathbb{C}$ of signature $(1,1)$. It is well known that $SU(1,1)$ is conjugate to $SL_2(\mathbb{R})$ in $GL_2(\mathbb{C})$. The correspondence is given by $\psi\colon SL_2(\mathbb{R}) \to SU(1,1)$, sending

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mapsto \begin{pmatrix} \frac{a+d+(b-c)i}{2} & \frac{a-d-(b+c)i}{2} \\ \frac{a-d+(b+c)i}{2} & \frac{a+d-(b-c)i}{2} \end{pmatrix}.$$

There is a parametrization of $SU(1,1)$ by (γ,ω), where $\gamma = \beta/\alpha$ and $\omega = \arg\alpha$ defined mod 2π (see [1, 10]). Thus $SU(1,1) = \{(\gamma,\omega) \mid |\gamma| < 1, -\pi \le \omega < \pi\}$. The group operation is given by $(\gamma,\omega)(\gamma',\omega') = (\gamma'',\omega'')$, where

$$\gamma'' = \frac{\gamma' + \gamma e^{-2i\omega'}}{1 + \gamma\bar{\gamma}' e^{-2i\omega'}}, \tag{3.1}$$

$$\omega'' = \omega + \omega' + \frac{1}{2i} \log \frac{1 + \gamma\bar{\gamma}' e^{-2i\omega'}}{1 + \bar{\gamma}\gamma' e^{2i\omega'}}. \tag{3.2}$$

Now the universal covering group $\widetilde{SL_2(\mathbb{R})}$ of $SU(1,1)$ can be described as

$$\widetilde{SL_2(\mathbb{R})} = \{(\gamma,\omega) \mid |\gamma| < 1, -\infty < \omega < \infty\}.$$

The group operation is given by (3.1) and (3.2) again, but ω'' is no longer mod 2π. Let $\Phi\colon \widetilde{SL_2(\mathbb{R})} \to SL_2(\mathbb{R})$ be the covering projection. Then it is obvious that $\ker\Phi = \{(0, 2m\pi) \mid m \in \mathbb{Z}\}$.

Lemma 3.1 *The subset* $(-1,1)\times\{0\}$ *of* $\widetilde{SL_2(\mathbb{R})}$ *forms a subgroup.*

Proof From (3.1) and (3.2), it is straightforward to see that $(-1,1)\times\{0\}$ is closed under the group operation. For $(\gamma,0) \in (-1,1)\times\{0\}$, its inverse is $(-\gamma,0)$. ■

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For the representation $\rho_s\colon G \to SL_2(\mathbb{R})$ defined by (2.2),

$$\psi\big(\rho_s(x)\big) = \frac{1}{2\sqrt{t}}\begin{pmatrix} t+1 & t-1 \\ t-1 & t+1 \end{pmatrix} \in SU(1,1).$$

Thus $\psi(\rho_s(x))$ corresponds to $(\gamma_x, 0)$, where $\gamma_x = \frac{t-1}{t+1}$.

Also, for a longitude λ,

$$\psi\big(\rho_s(\lambda)\big) = \frac{1}{2}\begin{pmatrix} B_s + \frac{1}{B_s} & B_s - \frac{1}{B_s} \\ B_s - \frac{1}{B_s} & B_s + \frac{1}{B_s} \end{pmatrix}, \quad B_s > 0$$

from Lemma 2.3. Thus $\psi(\rho_s(\lambda))$ corresponds to $(\gamma_\lambda, 0)$, where $\gamma_\lambda = \frac{B_s^2-1}{B_s^2+1}$.

4 Proof of Theorem

As the knot exterior M satisfies $H^2(M;\mathbb{Z}) = 0$, any $\rho_s\colon G \to SL_2(\mathbb{R})$ lifts to a representation $\widetilde{\rho}\colon G \to \widetilde{SL_2(\mathbb{R})}$ [7]. Moreover, any two lifts $\widetilde{\rho}$ and $\widetilde{\rho}'$ are related as follows:

$$\widetilde{\rho}'(g) = h(g)\widetilde{\rho}(g),$$

where $h\colon G \to \ker\Phi \subset \widetilde{SL_2(\mathbb{R})}$. Since $\ker\Phi = \{(0, 2m\pi) \mid m \in \mathbb{Z}\}$ is isomorphic to $\mathbb{Z}$, the homomorphism h factors through $H_1(M)$, so it is determined only by the value $h(x)$ of a meridian x (see [9]).

The following result, which was originally claimed in [9], is the key in [3] for the figure-eight knot. Our proof follows that of [3] for the most part, but it is much simpler, because of the values of $\psi(\rho_s(x))$ and $\psi(\rho_s(\lambda))$, which were calculated in Section 3.

Lemma 4.1 *Let $\widetilde{\rho}\colon G \to \widetilde{SL_2(\mathbb{R})}$ be a lift of ρ_s. Then, replacing $\widetilde{\rho}$ by a representation $\widetilde{\rho}' = h\cdot\widetilde{\rho}$ for some $h\colon G \to \widetilde{SL_2(\mathbb{R})}$, we can suppose that $\widetilde{\rho}(\pi_1(\partial M))$ is contained in the subgroup $(-1,1)\times\{0\}$ of $\widetilde{SL_2(\mathbb{R})}$.*

Proof Since $\Phi(\widetilde{\rho}(\lambda)) = (\gamma_\lambda, 0)$, $\gamma_\lambda \in (-1,1)$ and $\widetilde{\rho}(\lambda) = (\gamma_\lambda, 2j\pi)$ for some j. On the other hand, λ is a commutator, because our knot is genus one. Therefore [17, (5.5)] implies $-3\pi/2 < 2j\pi < 3\pi/2$. Thus we have $\widetilde{\rho}(\lambda) = (\gamma_\lambda, 0)$.

Similarly, $\widetilde{\rho}(x) = (\gamma_x, 2\ell\pi)$ for some ℓ, where $\gamma_x \in (-1,1)$. Let us choose $h\colon G \to \widetilde{SL_2(\mathbb{R})}$ so that $h(x) = (0, -2\ell\pi)$. Set $\widetilde{\rho}' = h\cdot\widetilde{\rho}$. Then a direct calculation shows that $\widetilde{\rho}'(x) = (\gamma_x, 0)$ and $\widetilde{\rho}'(\lambda) = (\gamma_\lambda, 0)$. Since x and λ generate the peripheral subgroup $\pi_1(\partial M)$, the conclusion follows from these. ■

Proof of Theorem 1.1 Let $r = p/q \in (0,4)$. By Lemma 2.4, we can fix s so that $g(s) = r$. Choose a lift $\widetilde{\rho}$ of ρ_s so that $\widetilde{\rho}(\pi_1(\partial M)) \subset (-1,1)\times\{0\}$. Then $\rho_s(x^p\lambda^q) = I$, so $\Phi(\widetilde{\rho}(x^p\lambda^q)) = I$. This means that $\widetilde{\rho}(x^p\lambda^q)$ lies in

$$\ker\Phi = \left\{(0, 2m\pi) \mid m \in \mathbb{Z}\right\}.$$

Hence $\widetilde{\rho}(x^p\lambda^q) = (0,0)$. Then $\widetilde{\rho}$ can induce a homomorphism $\pi_1(M(r)) \to \widetilde{SL_2(\mathbb{R})}$ with non-abelian image. Recall that $\widetilde{SL_2(\mathbb{R})}$ is left-orderable [2]. Since $M(r)$ is irreducible [8], $\pi_1(M(r))$ is left-orderable by [4, Theorem 1.1]. This completes the proof. ■

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