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Photon polarizability and its effect on the dispersion of plasma waves

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High-frequency photons travelling in plasma exhibit a linear polarizability that can influence the dispersion of linear plasma waves. We present a detailed calculation of this effect for Langmuir waves as a characteristic example. Two alternative formulations are given. In the first formulation, we calculate the modified dispersion of Langmuir waves by solving the governing equations for the electron fluid, where the photon contribution enters as a ponderomotive force. In the second formulation, we provide a derivation based on the photon polarizability. Then, the calculation of ponderomotive forces is not needed, and the result is more general.

Key words: plasma nonlinear phenomena, plasma waves

1. Introduction

As we showed recently (Ruiz & Dodin 2016), high-frequency photons travelling in plasma exhibit a well-defined linear polarizability. Hence, they contribute to the linear dielectric tensor just like any other plasma particles, such as electrons and ions. This implies that high-frequency photons can influence the dispersion of linear plasma waves. Here, we present a detailed calculation of this effect for Langmuir waves (LW).

Specifically, we develop a theory linear with respect to the LW amplitude ε. The photon density is assumed $O(\varepsilon^0)$, and perturbations to the photon density are assumed $O(\varepsilon^1)$. Hence, the LW can be understood as the linear modulational dynamics of the electromagnetic (EM) radiation. Two alternative formulations of this dynamics are given. In the first formulation (§ 2), we calculate the LW dispersion by solving the governing equations for the electron fluid, where the photon contribution enters as a ponderomotive force. A related calculation was also reported previously (Bingham, Mendonça & Dawson 1997; Mendonça 2000), but it contains omissions that warrant a reconsideration. In the second formulation (§ 3), we invoke the photon-polarizability concept. Then, the theory becomes linear, ponderomotive forces do not need to be considered, and consequently, more general results are obtained.

Although we focus on LW, the calculation presented here is only a characteristic example. The concept of the photon (plasmon, phonon, etc.) polarizability can be useful also in more general settings. For example, effects related to those considered

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here also apply to waves in solid-state media (Dylov & Fleischer 2008). We leave the consideration of such specific examples (other than LW) to future publications. The present paper aims only to illustrate the basic idea.

2. Approach based on partial-differential equations

In this section, we present a formulation that is based on the partial-differential equations (PDE) governing the plasma motion. We assume that, to the zeroth order in ε, the plasma (including the photon content) is stationary and homogeneous. We also assume that the ions are motionless, that the electrons can be modelled as a fluid, and that the $O(\varepsilon^0)$ velocity of the electron fluid is zero. We also assume that the electrons are collisionless and non-magnetized. Hence, the wave dynamics is described as follows.

Consider the electron continuity equation

$$\partial_t n + \nabla \cdot (n\boldsymbol{v}) = 0, \tag{2.1}$$

where n is the electron density, and $\boldsymbol{v}$ is the electron flow velocity. After the linearization, equation (2.1) becomes

$$\partial_t \tilde{n} + n_0 \nabla \cdot \tilde{\boldsymbol{v}} = 0. \tag{2.2}$$

(We use subscript 0 to denote $O(\varepsilon^0)$ quantities and tilde to denote $O(\varepsilon^1)$ quantities.) The velocity $\tilde{\boldsymbol{v}}$ is found from the electron momentum equation

$$mn(\partial_t + \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = ne\boldsymbol{E} - \nabla P + \boldsymbol{\eta}, \tag{2.3}$$

where m and $e < 0$ are the electron mass and charge, P is the pressure, and $\boldsymbol{\eta}$ is the ponderomotive force density (averaged over the EM-field oscillations yet not over the LW oscillations). After the linearization, equation (2.3) becomes

$$\partial_t \tilde{\boldsymbol{v}} = \frac{e}{m}\tilde{\boldsymbol{E}} - \frac{\nabla \tilde{P}}{mn_0} + \frac{\tilde{\boldsymbol{\eta}}}{mn_0}. \tag{2.4}$$

The electron gas is considered adiabatic, so $\tilde{P} = 3mv_T^2\tilde{n}$, where the constant v_T is the unperturbed thermal speed of electrons. (See, e.g. appendix B.2 in Dodin, Geyko & Fisch (2009), particularly (B21) and the references cited therein.) Hence, we obtain

$$\partial_t \tilde{\boldsymbol{v}} = \frac{e}{m}\tilde{\boldsymbol{E}} - 3v_T^2 \frac{\nabla \tilde{n}}{n_0} + \frac{\tilde{\boldsymbol{\eta}}}{mn_0}. \tag{2.5}$$

Substituting this into (2.2) gives $\partial_t^2 \tilde{n} = -n_0 \nabla \cdot \partial_t \tilde{\boldsymbol{v}}$, or

$$\partial_t^2 \tilde{n} = -\frac{en_0}{m}\nabla \cdot \tilde{\boldsymbol{E}} + 3v_T^2 \nabla^2 \tilde{n} - \frac{\nabla \cdot \tilde{\boldsymbol{\eta}}}{m}. \tag{2.6}$$

Using Gauss's law, $\nabla \cdot \tilde{\boldsymbol{E}} = 4\pi e\tilde{n}$, we then obtain

$$\partial_t^2 \tilde{n} + \omega_{p,0}^2 \tilde{n} - 3v_T^2 \nabla^2 \tilde{n} = -\frac{\nabla \cdot \tilde{\boldsymbol{\eta}}}{m}, \tag{2.7}$$

where $\omega_p^2 \doteq (4\pi n e^2/m)^{1/2}$. (We use the symbol $\doteq$ to denote definitions.) Assume $\tilde{n} = \mathrm{Re}\,(n_c \mathrm{e}^{-\mathrm{i}\Omega t + \mathrm{i}\boldsymbol{K}\cdot\boldsymbol{x}})$. A similar notation will also be assumed for other 'tilded' quantities; e.g. $\tilde{\boldsymbol{\eta}}$ is real and $\tilde{\boldsymbol{\eta}}_c$ is complex. Then,

$$[\Omega^2 - \Omega_0^2(\boldsymbol{K})]\tilde{n}_c = \mathrm{i}\boldsymbol{K}\cdot\tilde{\boldsymbol{\eta}}_c/m, \tag{2.8}$$

where $\Omega_0(\boldsymbol{K}) = (\omega_{p,0}^2 + 3K^2 v_T^2)^{1/2}$ is the LW frequency absent photons. Note that $K v_T \ll \Omega$ is implied because otherwise electrons cannot be considered adiabatic but rather must be described kinetically (Stix 1992); hence,

$$\Omega_0(\boldsymbol{K}) \approx \omega_{p,0}(1 + 3K^2 v_T^2/2\omega_{p,0}^2). \tag{2.9}$$

In the case of broad-band EM radiation, the average $\boldsymbol{\eta}$ equals the sum of the ponderomotive forces produced by its individual quasimonochromatic constituents, i.e. travelling geometrical-optics (GO) waves with well-defined wave vectors $\boldsymbol{k}$ and the corresponding frequencies

$$\omega = (\omega_p^2 + c^2 k^2)^{1/2}. \tag{2.10}$$

(In order to distinguish frequencies and wave vectors of EM waves from those of LW, we denote the former as ω and $\boldsymbol{k}$. As a reminder, the frequency and wave vector of the LW are denoted as Ω and $\boldsymbol{K}$.) Each such EM wave produces a per-electron average force $-\nabla\Phi_{\boldsymbol{k}}$, where $\Phi_{\boldsymbol{k}} = e^2|\boldsymbol{E}_c|^2/(4m\omega^2)$ is the ponderomotive potential, and $\boldsymbol{E}_c$ is the complex amplitude of the EM-wave electric field $\boldsymbol{E}$, which may have any polarization (Gaponov & Miller 1958). (More specifically, we adopt $\boldsymbol{E} = \mathrm{Re}\,(\boldsymbol{E}_c \mathrm{e}^{\mathrm{i}\theta})$, where θ is the wave rapid phase. Accordingly, $\omega \doteq -\partial_t\theta$ and $\boldsymbol{k} \doteq \nabla\theta$.) Hence, the average force density is $\boldsymbol{\eta}_{\boldsymbol{k}} = -n\nabla\Phi_{\boldsymbol{k}}$. Let us also express this in terms of the wave action density $\mathcal{I} = \mathcal{E}/\omega$ (i.e. the photon density times $\hbar$), where (Dodin & Fisch 2012)

$$\mathcal{E} = \frac{1}{16\pi\omega}\boldsymbol{E}_c^* \cdot \partial_\omega[\omega^2 \boldsymbol{\epsilon}(\omega, \boldsymbol{k})] \cdot \boldsymbol{E}_c \tag{2.11}$$

is the wave energy density, and $\boldsymbol{\epsilon}$ is the dielectric tensor. Using $\boldsymbol{\epsilon}(\omega, \boldsymbol{k}) = 1 - \omega_p^2/\omega^2$, we obtain $\mathcal{E} = |\boldsymbol{E}_c|^2/(8\pi)$. Then, irrespective of polarization, the average force density is given by the following expression:

$$\boldsymbol{\eta}_{\boldsymbol{k}} = -\frac{\omega_p^2}{2}\nabla\left(\frac{\mathcal{I}}{\omega}\right). \tag{2.12}$$

As a side remark, note that the force $\boldsymbol{\eta}_{\boldsymbol{k}}$ induced by a modulation of an EM wave on a plasma is qualitatively different from the force induced by a modulation of a matter wave on a plasma. In the electrostatic limit considered here, a matter wave causes an electrostatic force that is entirely determined by the electron density modulation through Gauss's law. In contrast, $\boldsymbol{\eta}_{\boldsymbol{k}}$ is determined by the modulation of both the photon density ($\mathcal{I}/\hbar$) and photon energy ($\hbar\omega$), so it can be non-zero even when the photon density is homogeneous. Considering that the photon Hamiltonian, or frequency (2.10), is very similar to the Hamiltonian of a relativistic electron, this difference may seem surprising; one could expect a one-to-one correspondence between electrostatic and ponderomotive forces. However, the correspondence is limited to free particles only. An electron has a constant rest mass and is coupled to the environment through the electrostatic potential. In contrast, a photon has a

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variable rest mass determined by ω_p and is coupled to the environment through modulations of this mass. Even in the limit $\omega_p \ll kc$, the interaction Hamiltonian of a photon remains k-dependent, so it cannot be represented as a potential energy in principle. This explains the difference between the corresponding forces. (As another side remark, this also explains the non-zero term (A 3) in the photon ponderomotive potential (A 2), which has no analogue in the electron ponderomotive potential, at least in the electrostatic gauge.)

The force density $\boldsymbol{\eta}$ produced by broad-band radiation can be written as $\boldsymbol{\eta} = \int \boldsymbol{\eta}_{\boldsymbol{k}}\, \mathrm{d}^3 k$, where $\mathcal{I}$ is replaced with the phase-space action density F summed over all polarization states. (Accordingly, the result holds for polarized and depolarized radiation equally. Also note that $F/\hbar$ can be understood as the phase-space photon probability distribution (Dodin 2014*a*).) This gives

$$\boldsymbol{\eta}(t, \boldsymbol{x}) = -\frac{\omega_p^2}{2} \nabla \int \frac{F(t, \boldsymbol{x}, \boldsymbol{k})}{\omega(t, \boldsymbol{x}, \boldsymbol{k})}\, \mathrm{d}^3 k. \tag{2.13}$$

Equation (2.13) is in agreement with the formula reported previously (Bingham *et al.* 1997; Mendonça 2000), at least up to a factor of two. However, in this work, we report a different linearization, which is as follows:

$$\tilde{\boldsymbol{\eta}}_c = -\frac{\mathrm{i}}{2} \boldsymbol{K} \omega_{p,0}^2 \int \left(\frac{\tilde{F}_c}{\omega_0} - \frac{\tilde{\omega}_c F_0}{\omega_0^2} \right) \mathrm{d}^3 k. \tag{2.14}$$

Here, $\omega_0 \doteq (\omega_{p,0}^2 + c^2 k^2)^{1/2}$ is the unperturbed frequency of the EM wave, and

$$\tilde{\omega}_c = \frac{\omega_{p,0}^2}{2\omega_0} \frac{\tilde{n}_c}{n_0} \tag{2.15}$$

is the perturbation on the EM-wave frequency due to the plasma density variations caused by the LW. The second term under the integral in (2.14) is specific to the photon–plasma interaction and has no direct analogue in the electron–plasma interaction for reasons discussed in the previous paragraph.

The perturbation of the photon distribution $\tilde{F}_c$ is obtained from the wave kinetic equation

$$\partial_t F + \partial_{\boldsymbol{k}} \omega \boldsymbol{\cdot} \nabla F - \partial_{\boldsymbol{x}} \omega \boldsymbol{\cdot} \nabla_{\boldsymbol{k}} F = 0, \tag{2.16}$$

where $\omega = \omega(t, \boldsymbol{x}, \boldsymbol{k})$ and $\partial_{\boldsymbol{k}} \omega$ is understood as the group velocity. Since the unperturbed plasma is considered homogeneous, linearizing equation (2.16) gives

$$\tilde{F}_c = -\frac{\tilde{\omega}_c \boldsymbol{K} \boldsymbol{\cdot} \nabla_{\boldsymbol{k}} F_0}{\Omega - \boldsymbol{K} \boldsymbol{\cdot} \boldsymbol{v}_*}, \tag{2.17}$$

where $\boldsymbol{v}_* \doteq c\boldsymbol{k}/\omega_0$ is the unperturbed group velocity. Inserting equations (2.15) and (2.17) into (2.14), we obtain

$$\tilde{\boldsymbol{\eta}}_c = \frac{\mathrm{i}}{4} \left(\frac{\tilde{n}_c}{n_0} \right) \boldsymbol{K} \omega_{p,0}^4 Q(\Omega, \boldsymbol{K}), \tag{2.18}$$

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where we introduced the following function:

$$Q(\Omega, \boldsymbol{K}) \doteq \int \left[\frac{\boldsymbol{K} \cdot \nabla_k F_0(\boldsymbol{k})}{(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_*)\omega_0^2(k)} + \frac{F_0(\boldsymbol{k})}{\omega_0^3(k)} \right] \mathrm{d}^3 k. \tag{2.19}$$

Note that the second term in the brackets is due to the second term in (2.14).

By substituting (2.18) into (2.8), we obtain

$$\Omega^2 - \Omega_0^2(\boldsymbol{K}) = -\frac{\omega_{p,0}^4 K^2}{4mn_0} Q(\Omega, \boldsymbol{K}). \tag{2.20}$$

Assuming that Ω is close to $\Omega_0 \approx \omega_{p,0}$, we can also simplify the left-hand side here as follows:

$$\Omega^2 - \Omega_0^2(\boldsymbol{K}) \approx 2\omega_{p,0}[\Omega - \Omega_0(\boldsymbol{K})]. \tag{2.21}$$

Then, the dispersion relation becomes

$$\Omega \approx \Omega_0(\boldsymbol{K}) - \frac{\omega_{p,0}^3 K^2}{8mn_0} Q(\Omega, \boldsymbol{K}). \tag{2.22}$$

The second term on the right-hand side is the modification of the LW dispersion relation caused by the photon gas. Equation (2.22) differs from the corresponding relation reported previously (Bingham *et al.* 1997; Mendonça 2000) in the following aspects: (a) the order-one numerical coefficient in front of the integral is different, and most importantly, the second term in the integrand in our expression (2.19) for Q is new. Notably, such terms are missed also in the general method reported by Tsytovich (1970) for calculating the nonlinear plasma dispersion. The wave-scattering paradigm that underlies this method assumes that the interaction between each pair of waves can be modelled as instantaneous, so the effect of the adiabatic frequency shift $\omega - \omega_0$ cannot be captured.

In order to estimate the photon contribution, suppose for clarity that $\omega \sim kc$ and $k \sim K$. (Under the assumptions adopted in the present section, this regime is accessible only marginally, but a more general theory given in § 3 leads to similar estimates.) Then, the ratio of the two terms in the right-hand side of (2.22) is roughly

$$\frac{\omega_{p,0}^3 K^2}{mn_0 \Omega_0} Q \sim \left(\frac{eE_c}{mc\omega} \right)^2 \equiv a^2, \tag{2.23}$$

where we assumed $Q \sim \omega^{-3} \int F_0 \, \mathrm{d}^3 k \sim \mathcal{I}/\omega^3 \sim |E_c|^2/(8\pi\omega^4)$. Note that a is the amplitude of the EM-driven momentum oscillations in units mc. Thus, in the non-relativistic limit assumed here, one has $a \ll 1$, so the photon contribution is small. Nevertheless, photons can have an important effect on the LW stability. For example, when resonant photons are present, the integrand in (2.19) has a pole on the real axis, and the integral must be taken along the Landau contour (Stix 1992). Then, Ω is complex, which signifies dissipation (positive or negative) of LW on photons. This effect is known as photon Landau damping (Bingham *et al.* 1997; Mendonça 2000) and has been demonstrated experimentally, albeit in a solid-state medium rather than plasma (Dylov & Fleischer 2008). Also note that LW are considered here only as a simple example, and the effect of photons on the dispersion of other waves can be more substantial.

3. Polarizability-based approach

Although the calculations of the ponderomotive forces are relatively straightforward in the situation considered in this paper, the problem can be much harder when Φ_k is velocity-dependent, i.e. when kinetic effects are essential (Dodin & Fisch 2008*a*,*b*; Dodin 2014*b*). Thus, it would be advantageous to develop a formulation of the modulational dynamics that avoids this step altogether. Below, we propose such formulation that utilizes the photon-polarizability concept (Ruiz & Dodin 2016). Within this approach, ponderomotive forces do not need to be considered, and a more general dispersion relation is obtained.

3.1. *General theory*

The dispersion relation of electrostatic oscillations in plasma with dielectric tensor $\boldsymbol{\epsilon}$ is given by

$$\boldsymbol{e}_K \boldsymbol{\cdot} \boldsymbol{\epsilon}(\varOmega, \boldsymbol{K}) \boldsymbol{\cdot} \boldsymbol{e}_K = 0, \tag{3.1}$$

where $\boldsymbol{e}_K \doteq \boldsymbol{K}/K$ is the unit vector along the LW wave vector $\boldsymbol{K}$ and $\varOmega$ is the LW frequency. Consider a plasma whose dielectric tensor is some $\boldsymbol{\epsilon}_0$ plus the contribution from the photon gas. The latter contribution is the photon susceptibility, which can be written as follows:

$$\boldsymbol{\chi}_{ph}(\varOmega, \boldsymbol{K}) = 4\pi \int \boldsymbol{\alpha}_{ph}(\varOmega, \boldsymbol{K}, \boldsymbol{k}) f_{ph}(\boldsymbol{k})\, \mathrm{d}^3 k. \tag{3.2}$$

Here, f_{ph} is the unperturbed photon distribution, and $\boldsymbol{\alpha}_{ph}$ is the polarizability of a single photon. As shown previously (Ruiz & Dodin 2016),

$$\boldsymbol{\alpha}_{ph} = \frac{\hbar e^2 K^2 \varXi}{4m^2\omega_0^3}\, \boldsymbol{e}_K \boldsymbol{e}_K, \tag{3.3}$$

where $\varXi$ is a dimensionless coefficient given by

$$\varXi \doteq \frac{\varOmega^2 - c^2K^2}{(\varOmega - \boldsymbol{K}\boldsymbol{\cdot}\boldsymbol{v}_*)^2 - (\varOmega^2 - c^2K^2)^2/4\omega_0^2(k)}, \tag{3.4}$$

or, equivalently,

$$\varXi = -\sum_{\sigma=\pm 1} \frac{\sigma\omega_0(k)}{R_\sigma(\varOmega, \boldsymbol{K}, \boldsymbol{k})}, \tag{3.5}$$

$$R_\sigma(\varOmega, \boldsymbol{K}, \boldsymbol{k}) \doteq \varOmega - \boldsymbol{K}\boldsymbol{\cdot}\boldsymbol{v}_*(\boldsymbol{k}) + \sigma\frac{\varOmega^2 - c^2K^2}{2\omega_0(k)}. \tag{3.6}$$

Thus, equation (3.1) can be expressed as follows:

$$\epsilon_0(\varOmega, \boldsymbol{K}) + \chi_{ph}(\varOmega, \boldsymbol{K}) = 0. \tag{3.7}$$

Here, $\epsilon_0 \doteq \boldsymbol{e}_K \boldsymbol{\cdot} \boldsymbol{\epsilon}_0(\varOmega, \boldsymbol{K}) \boldsymbol{\cdot} \boldsymbol{e}_K$ and $\chi_{ph}(\varOmega, \boldsymbol{K}) \doteq \boldsymbol{e}_K \boldsymbol{\cdot} \boldsymbol{\chi}_{ph}(\varOmega, \boldsymbol{K}) \boldsymbol{\cdot} \boldsymbol{e}_K$ or, explicitly,

$$\chi_{ph}(\varOmega, \boldsymbol{K}) = -\frac{\omega_{p,0}^2 K^2}{4mn_0} \sum_{\sigma=\pm 1} \int \frac{\sigma F_0(\boldsymbol{k})}{\omega_0^2(k) R_\sigma(\varOmega, \boldsymbol{K}, \boldsymbol{k})}\, \mathrm{d}^3 k, \tag{3.8}$$

where $F_0 \doteq \hbar f_{ph}$ is the photon action density, which is a classical quantity. Accordingly, equation (3.7) becomes

$$\epsilon_0(\Omega, \boldsymbol{K}) - \frac{\omega_{p,0}^2 K^2}{4mn_0} \sum_{\sigma=\pm 1} \int \frac{\sigma F_0(\boldsymbol{k})}{\omega_0^2(k) R_\sigma(\Omega, \boldsymbol{K}, \boldsymbol{k})} \, \mathrm{d}^3 k = 0. \tag{3.9}$$

Compared to (2.22) that was obtained within a PDE-based approach, equation (3.9) is more general. It allows for $k \sim K$ and does not assume any specific model of the background plasma; i.e. no assumptions regarding $\boldsymbol{\epsilon}_0$ are made. Moreover, the derivation applies as is to the case where the background plasma is weakly non-stationary and (or) weakly inhomogeneous. (For more details, see Ruiz & Dodin (2016).) Also importantly, the same approach is readily extended to describe modulational dynamics of other waves too, as will be reported separately.

3.2. *Small-K limit*

Finally, let us show how (3.9) reduces to (2.22) under the additional assumptions adopted in § 2. First, assume the GO approximation for photons, namely, $K \ll k$. This implies (Dodin & Fisch 2014; Ruiz & Dodin 2016)

$$\Xi(\Omega, \boldsymbol{K}, \boldsymbol{k}) \approx \frac{\Omega^2 - c^2 K^2}{(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_*)^2}. \tag{3.10}$$

Hence, the photon susceptibility becomes

$$\chi_{ph}(\Omega, \boldsymbol{K}) = \frac{\omega_{p,0}^2 K^2}{4mn_0} \int \frac{\Omega^2 - c^2 K^2}{(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_*)^2 \omega_0^3(k)} F_0(\boldsymbol{k}) \, \mathrm{d}^3 k. \tag{3.11}$$

As can be checked by a direct calculation,

$$\boldsymbol{K} \cdot \frac{\partial}{\partial \boldsymbol{k}} \left[\frac{1}{(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_*)\omega_0^2} \right] = \frac{c^2 K^2 - \Omega^2}{(\Omega - \boldsymbol{K} \cdot \boldsymbol{v}_*)^2 \omega_0^3} + \frac{1}{\omega_0^3}. \tag{3.12}$$

(This non-intuitive step can be avoided as explained in appendix.) Thus, one can also express χ_{ph} equivalently as follows:

$$\chi_{ph}(\Omega, \boldsymbol{K}) = \frac{\omega_{p,0}^2 K^2}{4mn_0} Q(\Omega, \boldsymbol{K}), \tag{3.13}$$

where Q is given by (2.19). Second, assume that ions are motionless and electrons have a non-zero yet small enough thermal speed $v_T \ll \Omega / K$; then (Stix 1992),

$$\epsilon_0(\Omega, \boldsymbol{K}) = 1 - \frac{\omega_{p,0}^2}{\Omega^2} \left(1 + \frac{3K^2 v_T^2}{\omega_{p,0}^2} \right), \tag{3.14}$$

and Ω is close to the unperturbed linear frequency Ω_0 (2.9). This can be simplified to $\epsilon_0 \approx 2[\Omega - \Omega_0(\boldsymbol{K})]/\omega_{p,0}$. Hence, equation (3.7) immediately leads to (2.22).

4. Conclusions

In summary, we calculated the influence of the photon polarizability on the dispersion of linear Langmuir waves in a collisionless non-magnetized electron plasma. Two alternative formulations are given here. In the first formulation, we calculate the Langmuir wave dispersion by solving the equations of motion for the electron fluid, where the photon contribution enters as a ponderomotive force. A related calculation was reported previously (Bingham *et al.* 1997; Mendonça 2000), but it contains omissions, which are corrected in the present work. In the second formulation, we explicitly invoke the photon-polarizability concept (Ruiz & Dodin 2016). Then, the theory becomes linear, ponderomotive forces do not need to be considered, and consequently, more general results are obtained.

Although we focus on LW, the calculation presented here is only a characteristic example. The concept of the photon (plasmon, phonon, etc.) polarizability is of broader generality and can help calculate the modification of the dispersion relation also of other waves, including waves in media other than plasma. We leave the consideration of specific examples to future publications.

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Appendix. Variational approach

Here, we present an even more straightforward derivation of the GO dispersion relation (2.22) that does not involve the non-intuitive step of using (3.12). Instead of invoking the photon-polarizability concept, we start directly with the LW Lagrangian density (Ruiz & Dodin 2016):

$$\mathfrak{L} = \epsilon_0(\varOmega, \boldsymbol{K})\frac{|\tilde{\boldsymbol{E}}_c|^2}{16\pi} - \int \varPhi_{ph} f_{ph}\, \mathrm{d}^3k. \tag{A 1}$$

Here, the first term is the Lagrangian density of LW absent photons (Dodin 2014*b*), the second one is the photon contribution, and $\varPhi_{ph}$ is the LW-produced ponderomotive potential of a photon. Let us use the GO approximation of $\varPhi_{ph}$ that was derived in Ruiz & Dodin (2016), Dodin & Fisch (2014):

$$\hbar^{-1}\varPhi_{ph} = \langle \omega - \omega_0 \rangle + \frac{\boldsymbol{K}}{4} \boldsymbol{\cdot} \frac{\partial}{\partial \boldsymbol{k}} \left(\frac{|\tilde{\omega}_c|^2}{\varOmega - \boldsymbol{K}\boldsymbol{\cdot}\boldsymbol{v}_*} \right), \tag{A 2}$$

where the angular brackets denote averaging over time. As seen from (2.10),

$$\langle \omega - \omega_0 \rangle = -\frac{\omega_{p,0}^4}{16\omega_0^3}\frac{|\tilde{n}_c|^2}{n_0^2}. \tag{A 3}$$

Also, from Gauss's law, $\tilde{n}_c = \mathrm{i}\boldsymbol{K}\boldsymbol{\cdot}\tilde{\boldsymbol{E}}_c/(4\pi e)$, so

$$\frac{|\tilde{n}_c|^2}{n_0^2} = \frac{K^2|\tilde{\boldsymbol{E}}_c|^2}{16\pi^2 n_0^2 e^2} = \frac{|\tilde{\boldsymbol{E}}_c|^2}{16\pi}\frac{4K^2}{mn_0\omega_{p,0}^2}, \tag{A 4}$$

where we used that the electrostatic field $\tilde{\boldsymbol{E}}_c$ is parallel to $\boldsymbol{K}$. This gives

$$\hbar^{-1}\Phi_{ph} = \frac{\omega_{p,0}^2 K^2}{4mn_0}\left\{-\frac{1}{\omega_0^3} + \boldsymbol{K}\boldsymbol{\cdot}\frac{\partial}{\partial\boldsymbol{k}}\left[\frac{1}{(\Omega - \boldsymbol{K}\boldsymbol{\cdot}\boldsymbol{v}_*)\omega_0^2}\right]\right\}, \tag{A 5}$$

where we used (2.15). By substituting this result into (A 1) and integrating by parts, one obtains

$$\mathfrak{L} = \epsilon(\Omega, \boldsymbol{K})\frac{|\tilde{\boldsymbol{E}}_c|^2}{16\pi}, \tag{A 6}$$

where ϵ is given by

$$\epsilon(\Omega, \boldsymbol{K}) \doteq \epsilon_0(\Omega, \boldsymbol{K}) + \frac{\omega_{p,0}^2 K^2}{4mn_0}Q(\Omega, \boldsymbol{K}), \tag{A 7}$$

and Q is given by (2.19). The action integral corresponding to the Lagrangian density (A 6) can be considered as a functional of $\mathcal{A} \doteq |\tilde{\boldsymbol{E}}_c|^2$ and the LW phase Θ:

$$S = \int \mathfrak{L}(\mathcal{A}, \underbrace{-\partial_t\Theta}_{\Omega}, \underbrace{\nabla\Theta}_{\boldsymbol{K}})\,\mathrm{d}t\,\mathrm{d}^3x. \tag{A 8}$$

The dispersion relation is obtained from $\delta S/\delta\mathcal{A} = 0$. This gives $\epsilon(\Omega, \boldsymbol{K}) = 0$, so one is again led to (2.22). Also notably, if ϵ depends on $(t, \boldsymbol{x})$, the amplitude equation (action conservation theorem) is obtained from $\delta S/\delta\Theta = 0$; namely,

$$\partial_t\mathcal{I} + \nabla\boldsymbol{\cdot}(\boldsymbol{V}_g\mathcal{I}) = 0. \tag{A 9}$$

Here, $\mathcal{I} = (\partial_\Omega\epsilon)|\tilde{\boldsymbol{E}}_c|^2/(16\pi)$ is the LW wave action density, $\boldsymbol{V}_g \doteq -(\partial_{\boldsymbol{K}}\epsilon)/(\partial_\Omega\epsilon)$ is the LW group velocity, Ω is found from the dispersion relation, and the wave vector is treated as a field; i.e. $\boldsymbol{K} = \boldsymbol{K}(t, \boldsymbol{x})$ (Dodin 2014*a*; Tracy *et al.* 2014).

Although this variational formalism does not capture dissipation (which is reflected in the conservative form of (A 9)), it can be extended to dissipative processes as described in Dodin, Zhmoginov & Ruiz (2016). In particular, the dispersion relation derived from the above variational formalism is valid for complex frequencies too provided that the integral in the expression (2.19) for Q is taken using the Landau rule.

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