A FINITE ELEMENT APPROACH TO THE DESIGN OF THE SUPPORT SYSTEM FOR THE ESO 1 M. ACTIVE OPTICS EXPERIMENT

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1. Introduction

A way to follow in order to reduce the weight of the primary mirror is to accept an higher ratio diameter-thickness and to find a remedy to the consequent greater deformability of the mirror by an active control system of it. A technique using closed loop active optics control has been proposed in references |1, 2|.
In these papers, a coherent scheme of active optics control for the ESO New Technology Telescope (NTT) was presented, based on analysis of the image errors in terms of an appropriate polynomial (the ESO off-line telescope test polynomial) and the production of equivalent correction terms by force modulation of the primary axial support.
The "calibrations" of the force changes required to generate these terms were performed using analytical theory by Schwesinger |1|.
It is well known that the mirror behaves as a high-sensitivity structure: minor variations of the forces applied by the actuators may lead to major consequences in terms of deformability.
It is thus necessary to perform a quite accurate static analysis in order to solve the following problems:

a - Designing the support system and thus choosing the actuator position and the values of their effects in order to keep the mirror, loaded by its own weight as near as possible to the wished position.

b - Controlling the mirror and thus choosing the values of the variations of actuators effects that must be given in order to compensate possible distorsions of the mirror surface and to return it as near as possible to the original configuration.

Both the above problems may be reduced to the optimization of quadratic functions |3|. Nevertheless some difficulties may arise in working out the solution, mainly due to the following reasons:

(i) The mirror has an axisymmetric geometric shape but the actuators are not axisymmetrically placed. Even if their distance is small, it is not possible

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to regard their effects as continous but it is necessary to consider the applied forces as concentrated.

(ii) The mirror has a curvature. Even if the meniscus is small, the mirror behaves more as a shell than a plane plate.

As a consequence of the above statements one must consider that the analytical approach based on the mathematical theory of plates and shells |4| may be not sufficient to solve all the problems with the wished precision.

Aim of this paper is to examine a well defined problem with finite elements technique in order to point out the advantages of its application to the design of the support system together with the difficulties that may arise by examining high sensitivity structures with numerical methods.

The mirror of fig. 1 will be considered. It is a mirror of 1.05 m in diameter with a curvature radius of 6.5 m. It is provided by 75 actuators able to vary the applied forces and it is supposed to work with a fixed vertical axis.

This mirror is a model test bench for the NTT active optics system and has been scaled to have the same gravity flexure as the NTT 3,5 m primary |5, 6|.

2. The Theoretical Approach

The approach is based on the mathematical considerations made in |3| that are briefly summed up in the following.

The design problem may be formulated as:

$$\min \sum_{1}^{n}{}_{i} \; \gamma_i \; (-w_i^o + \sum_{1}^{m}{}_{j} \; F_j \; a_{ij})^2 \qquad (1)$$

where:

n, m	are the number of control points and of actuators respectively
w_i^o (x_i, y_i)	is the displacement component at the point P_i $(i=1,2...n)$ in the direction corresponding to the control signal, due to the weight of the mirror
F_j	is the modulus of the force exerted by the actuator applied at point Q_j $(j=1,2...m)$
a_{ij}	is the influence coefficient, that is the analogous displacement components due to the unit effect of the actuator applied at point Q_j
γ_i	is the weight of point P_i, that is the influence of the portion of surface sorrounding the point P_i on the total surface distortion

The unknowns of the problem are the forces F_j and the position of actuators Q_j

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They may be worked out by an iterative procedure that implies at each step the knowledge of the influence coefficients a_{ij} and of displacement components w_i^o. For this reason the design problem can be solved by using an efficient numerical procedure as the mathematical theory of plane plates.

Once the positions of actuators has been decided, the values of forces F_j can be determined by solving the problem (1). This can be effectively carried out by calculating the influence coefficients a_{ij} and the displacement components w_i^o through the finite element method.

The control problem may be formulated as two different optimization problems or it can be solved by means of a direct combination formula. In a general way one can:

(i) find the values ΔF_j of the forces that must be provided by the actuators in order to return the mirror as near as possible to the design position;

(ii) find the values ΔF_j of the forces that must be provided by the actuators in order to minimize the energy supplied by the actuators and to limit the maximum gap between the distorted and the design configuration.

Both statements (i) and (ii) lead to quadratic programming problems subjected to linear constraints.

More simply, one may choose to correct the distorsions of the surface by moving the points at which the actuators are placed of the quantity $\Delta\bar{w}_h$, in order to return them only in their original design position. Of course, this procedure may be not optimal but it achieve the goal of the greatest simplicity.

Therefore, one has to:

- work out the influence matrix $|c_{hk}|$ calculating the displacement components of the points at which the actuators are applied;
- determine the inverse matrix $|e_{kh}|=|c_{hk}|^{-1}$ and then;
- compute the correcting forces ΔF_k, as

$$\Delta F_k = \sum_{1}^{m}{}_h \quad e_{kh}\,\Delta\bar{w}_h \tag{2}$$

The main difficulty is to compute the influence matrix $|c_{hk}|$. The entry c_{hk} represents the displacement component at the point P_h, where the h-th actuator is placed due to a unit-force brought by the k-th actuator. For this purpose a Finite Element analysis can be carried out considering as many load conditions as the number of actuators is.

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3. The Design of the Support System

The mirror shown in Fig. 1 has the following characteristics

internal radius	R_i	= 100 mm
external radius	R_e	= 525 mm
thickness	t	= 18 mm
specific mass	δ	= 0.0252 kg/cm^3
elasticity modulus	E	= 90252 N/mm^2
Poisson coefficient	ν	= 0.245
total weight	G	= 371.33 N
radius of curvature	r	= 6500 mm

Four sets of 8, 16, 24 and 30 points corresponding to 75 actuators and 3 fixed points are placed along four concentrical circumferences. The 3 fixed points are included in the third ring. This support geometry is a scaled-down version of the NTT primary support system |2|.

The radii of the 4 actuator rings were already established by ESO, according to the Schwesinger calculations |1|, at the following position:

R_1 = 143.1 mm; R_2 = 253.9 mm; R_3 = 367.1 mm; R_4 = 481.6 mm

If the coefficients a_{ij} and w_i^o are determined by using the mathematical theory of thin circular plates loaded by ring-distributed loads (thus disregarding the concentration effects of the loads but spreading out the actuator forces along circumferences) the problem (1) with γ_i=1 give the following values:

F_1 = 38.075 N; F_2 = 81.307 N; F_3 = 118.042 N; F_4 = 133.905 N;

The quality of the solution was checked analysing the mirror with the Finite Element technique. The mesh of Fig. 2 was choosen and the mirror was studied for the loading condition given by the mirror selfweight and the previously calculated actuator forces, assumed to be concentrated in their actual position. SAP V computer program was used considering 1536 thin plate finite elements. This leads to 4896 linear equations if in-plane displacement components are disregarded, to 9792 equations if they are taken into account.

Two computational models were appointed:

(i) the curvature of the mirror was disregarded in order to have a comparison between the analytical solution obtained with the theory of axisymmetric thin circular plate and the Finite Element solution, taking into account the load concentration. The results obtained are presented in Fig. 3;

(ii) the curved mirror was examined taking into account also the in-plane displacements. This is the most realistic even if more onerous model. The results are given in Fig. 4

The displacement cicumferential patterns are plotted in Fig. 5 and 6 for plane and curved mirror respectively. The differencies of the periodicity of the ripples may be attributed to the evaluation of the nodal loads in the irregular elements sorrounding the outer actuators. From radial patterns it may be noted that the curvature may have some influence in the evaluation of optimal values of forces.

4. The Control of the Mirror

The control of the mirror was achieved by the procedure expressed by formula (2). The matrix $|c_{hk}|$ is a 75x75 matrix. The entries of the k-th column have to be evaluated by applying a unit force where the k-th actuator is located, (Fig. 7), and then computing the displacements at the point at which the 75 actuators are placed.

The polynomial chosen for the wave front analysis had the following terms.

k_1	rigid translations
$k_2\ r \cos\ (\theta+\Phi)$	wave front tilt
$k_3\ r^2$	longitudinal defocusing
$k_4\ r^4$	3rd order spherical aberration
$k_5\ r^6$	5th " " "
$k_6\ r^2 \cos\ 2\theta$	3rd " astigmatism
$k_7\ r^3 \cos\ 3\theta$	triangular astigmatism
$k_8\ r^4 \cos\ 4\theta$	quadratic astigmatism
$k_9\ r^5 \cos\ \theta$	higher order coma

with:

$r,\ \theta$	polar coordinates
$k_1 \ldots k_9$	coefficients
Φ	shift of angular origin

At the three fixed points, the surface displacement must be zero.

Thus the following polynomial terms where assumed in order to simulate the displacements $\bar{S}$ of distorted surface. This is essentially the ESO telescope test polinomial, |7| modified and extended for the NTT:

$$\bar{S} = 0{,}5\ \lambda\ (\rho^2 - k_1^2)$$
$$\bar{S} = 0{,}5\ \lambda\ (\rho^4 - k_1^4)$$
$$\bar{S} = 0{,}5\ \lambda\ (\rho^6 - k_1^6)$$
$$\bar{S} = 0{,}5\ \lambda\ (\rho^2 \cos\ 2\theta + k_2\ \rho\ \mathrm{sen}\theta)$$
$$\bar{S} = 0{,}5\ \lambda\ \rho^3 \cos\ 3\theta$$

$\bar{S} = 0{,}5\ \lambda\ (\rho^4 \cos 4\theta - k_2^3\ \rho\ \mathrm{sen}\theta)$

$\bar{S} = 0{,}5\ \lambda\ (\rho^5 \cos \theta - k_2^4\ \rho\ \cos\theta)$

with:

$\rho = R/R_e$

$\lambda = 0.550\ \mu$

$R_e = 525$ mm; $R_i = 100$ mm; $R_3 = 367.1$ mm; $k_1 = k_2 = R_3/R_e$

Figg. 8 - 12 show the values of the forces (in N) that must be applied. Also in this case the two computations presented at point 3 were performed in order to underline the influence of the models. From the analysis of the results one may conclude that the effect of curvature has not a significant influence on the computation of correcting forces if the mirror is considered in horizontal position.

Acknowledgements

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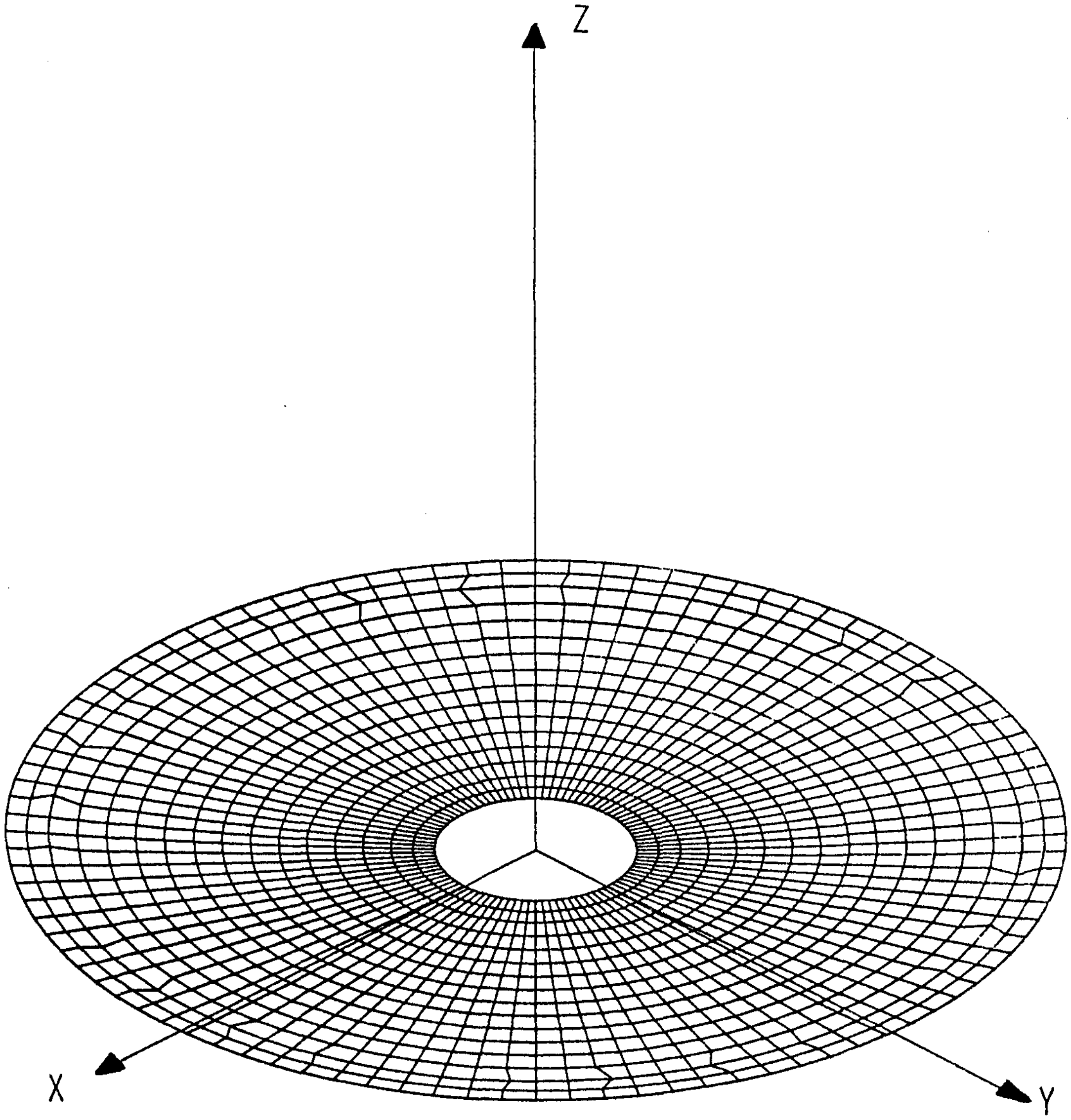

Fig. 1

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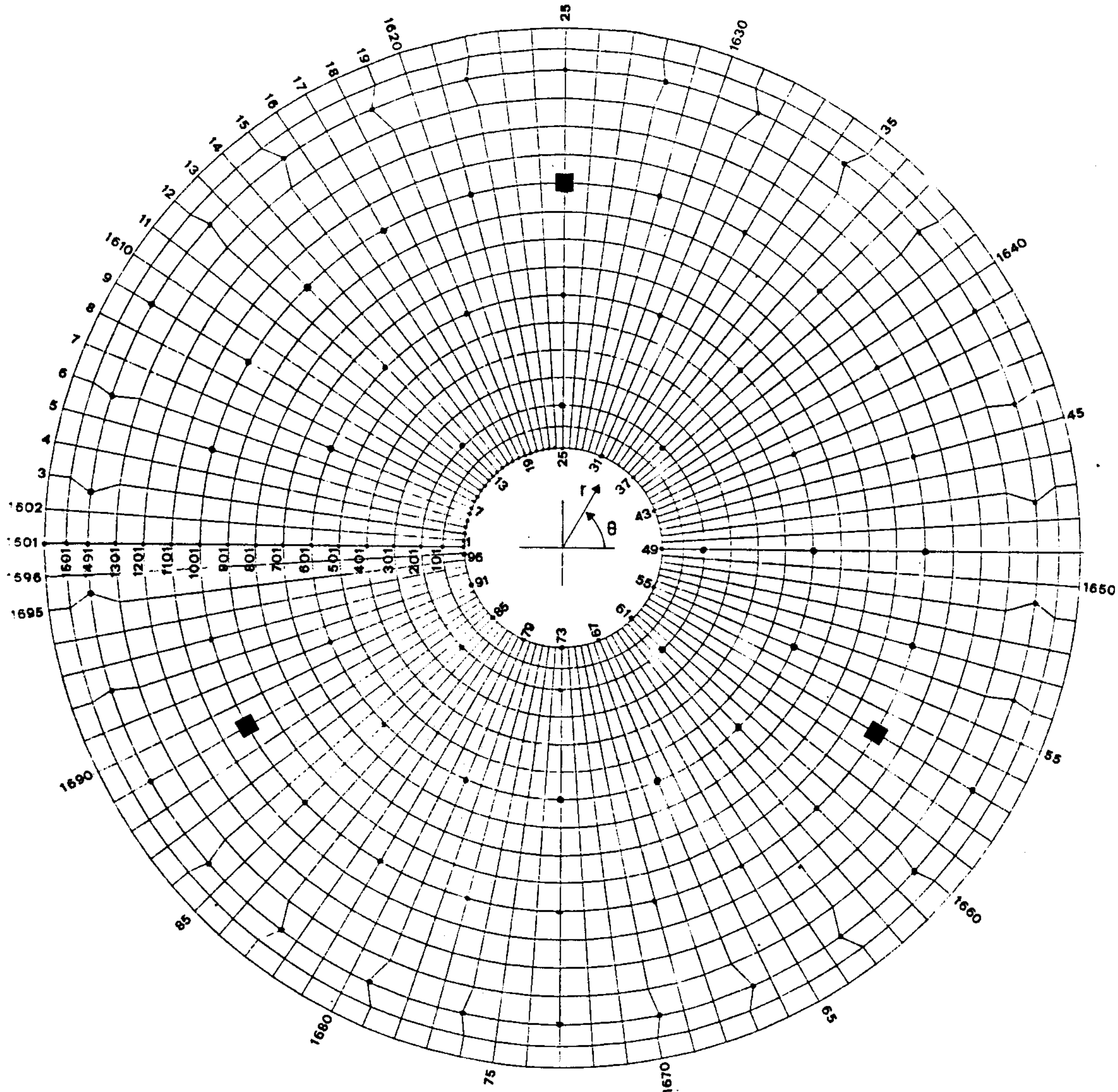

Fig. 2

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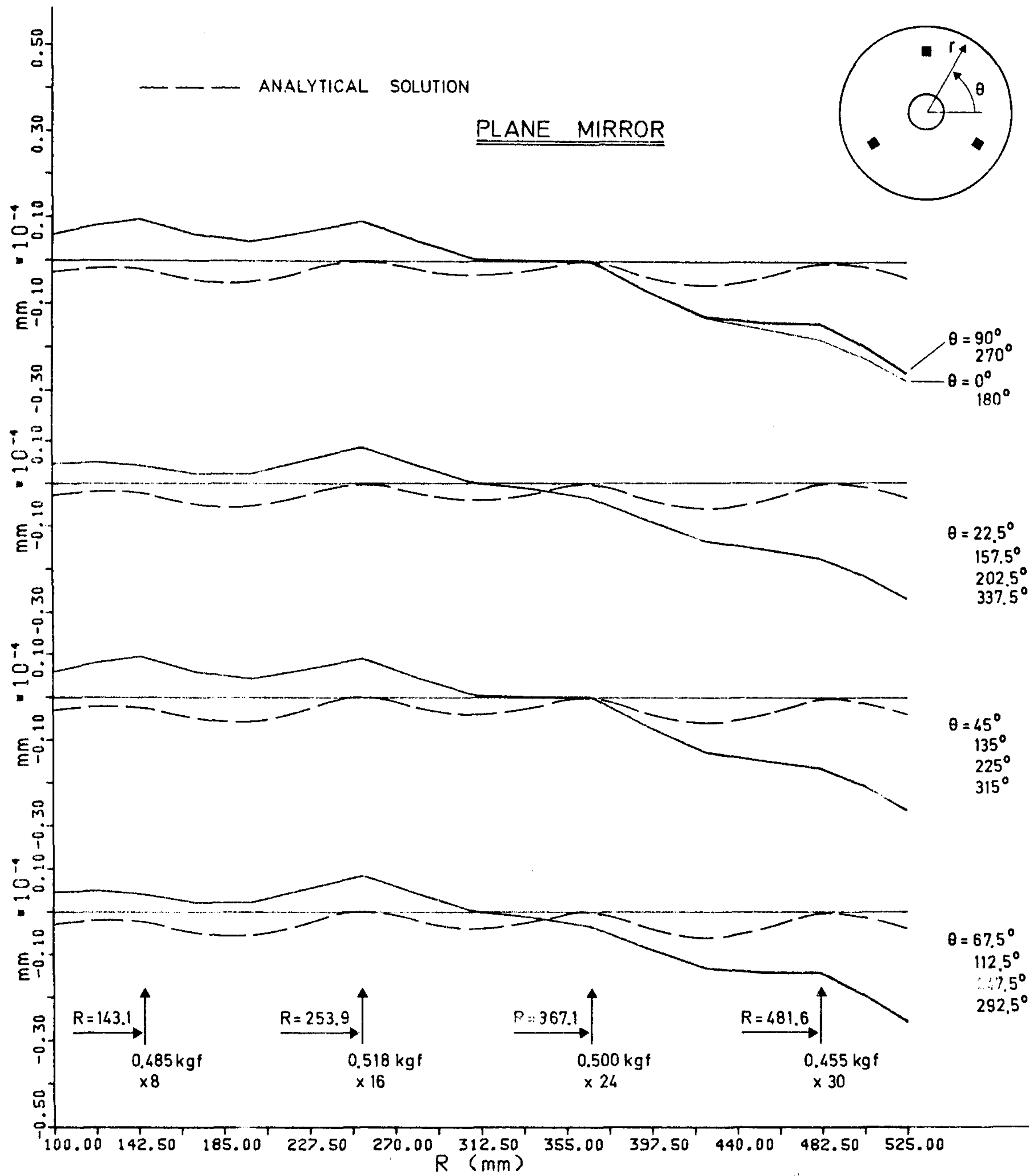

Fig. 3

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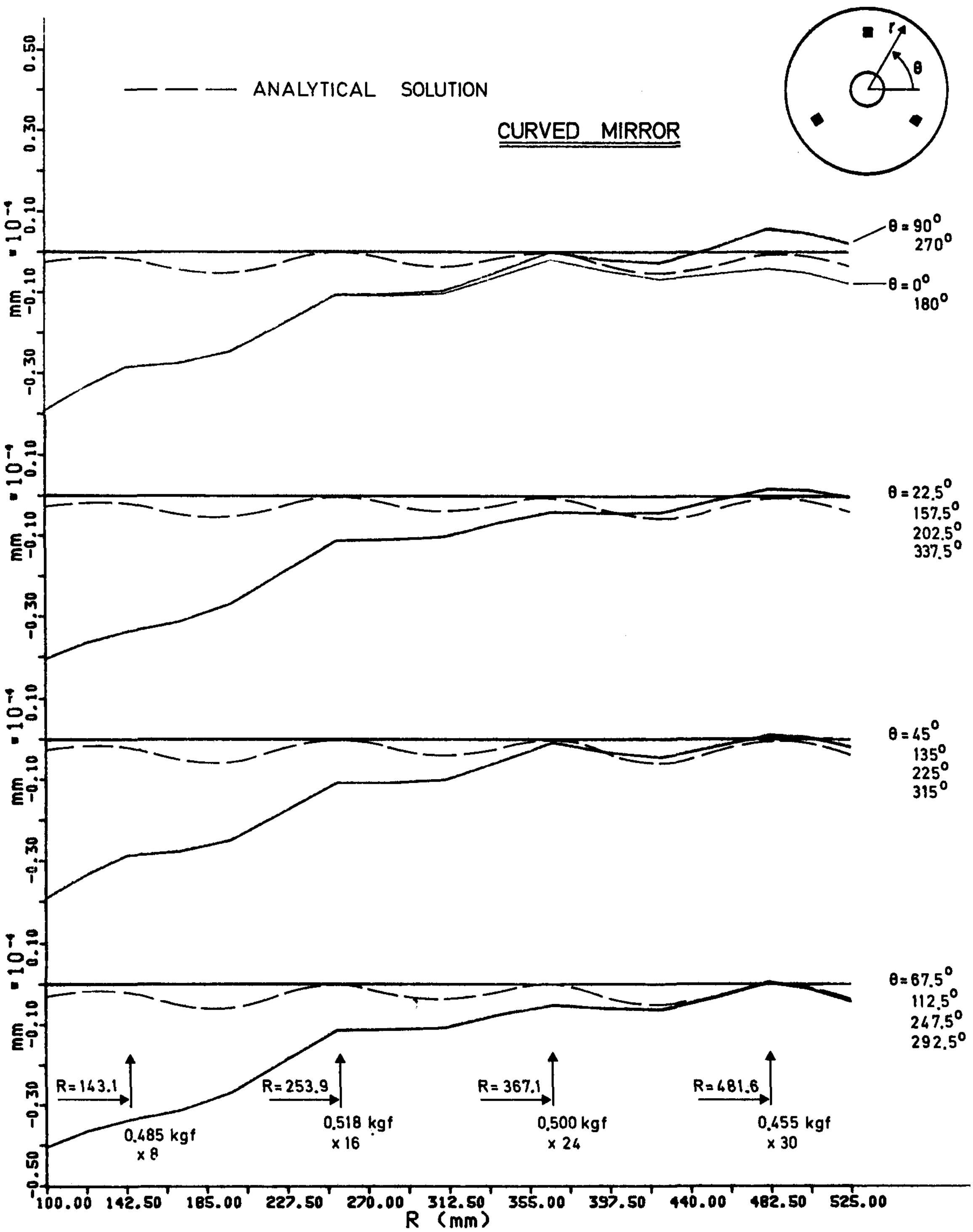

Fig. 4

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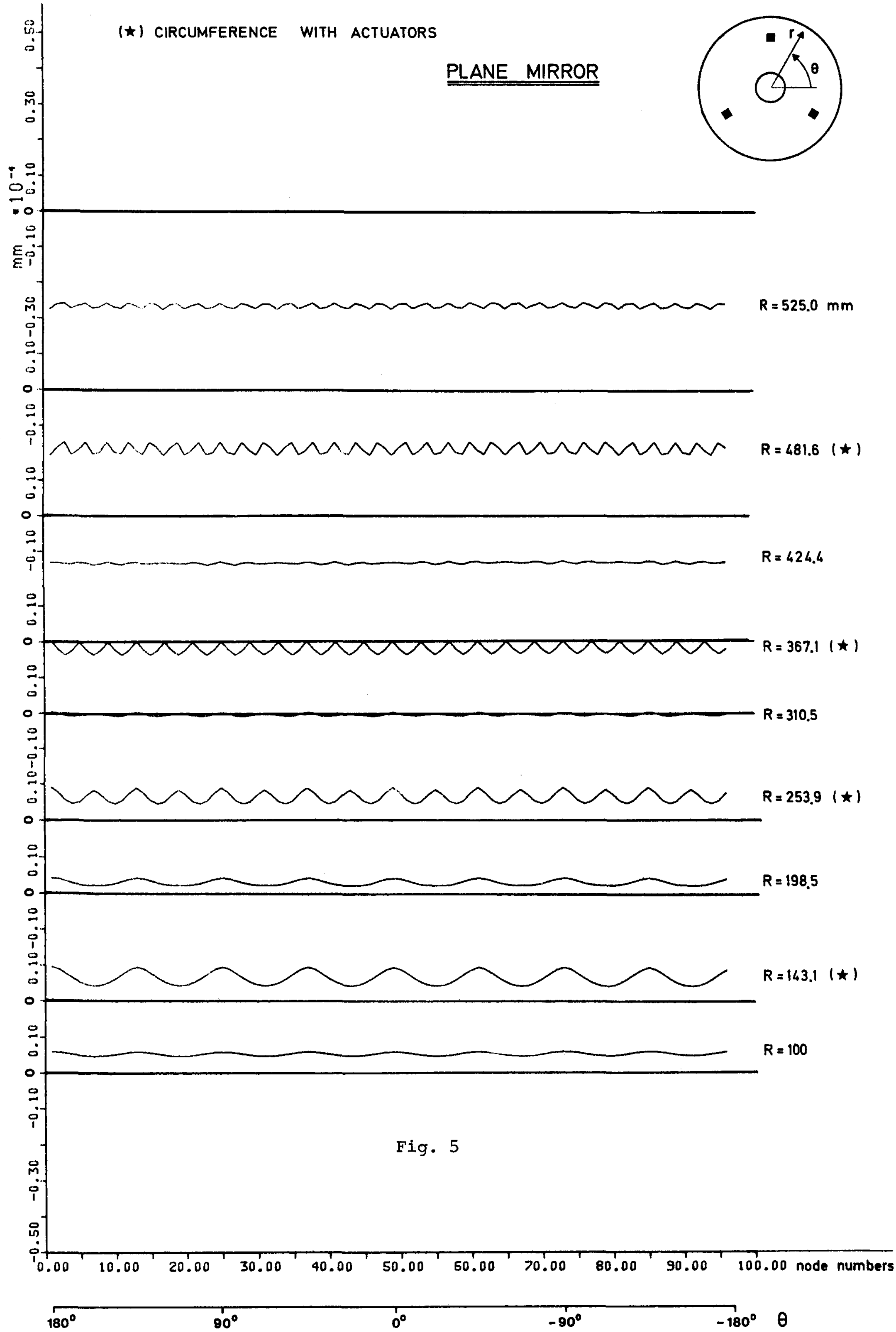

Fig. 5

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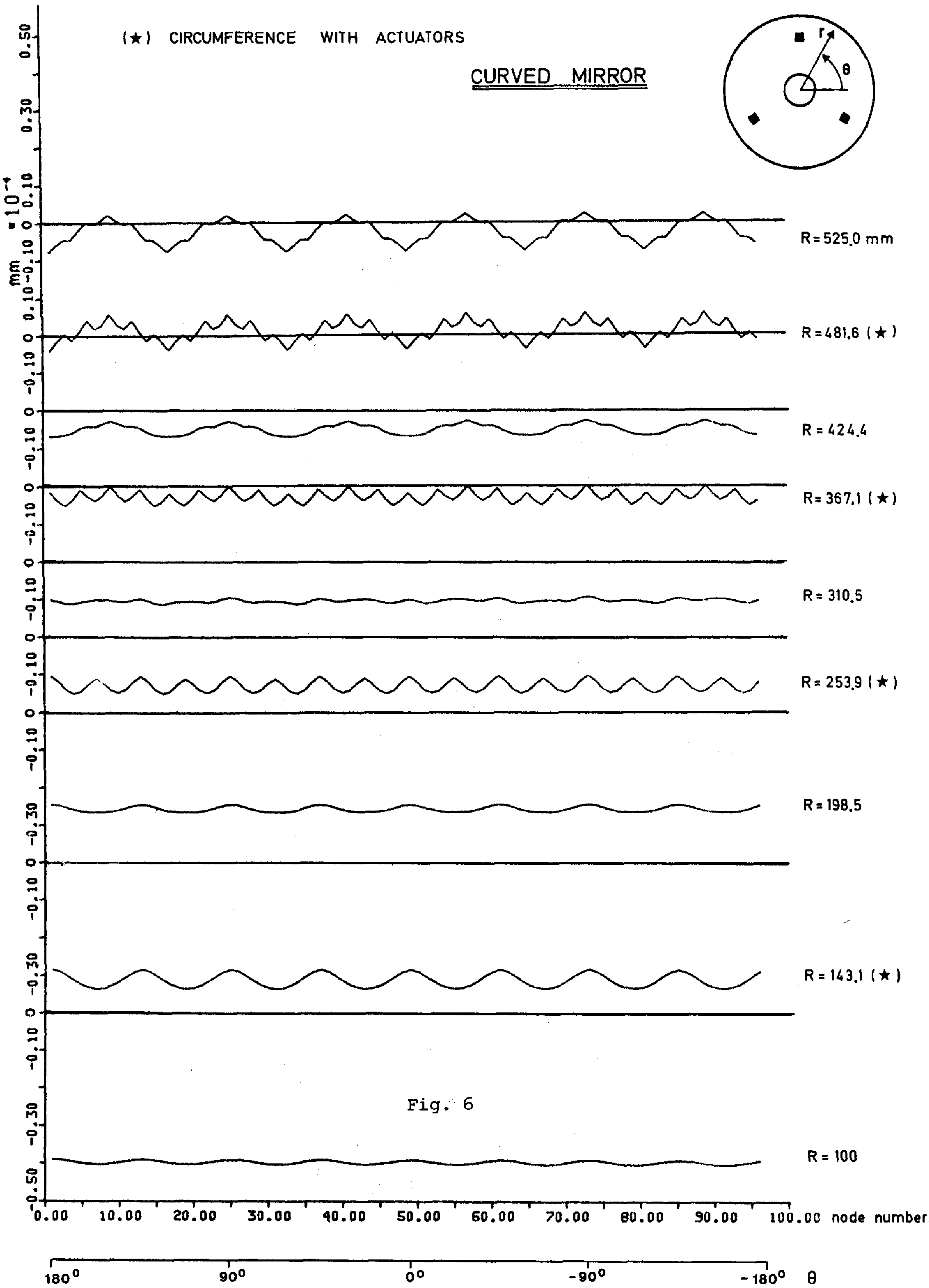

Fig. 6

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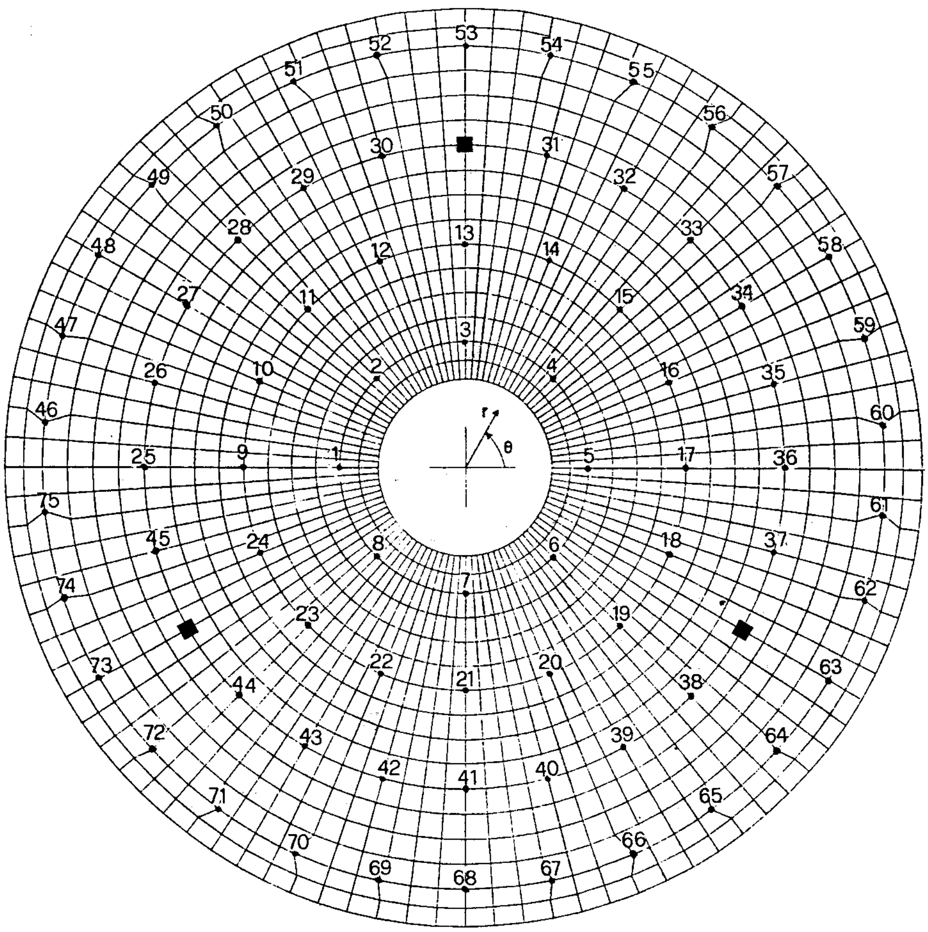

Fig. 7

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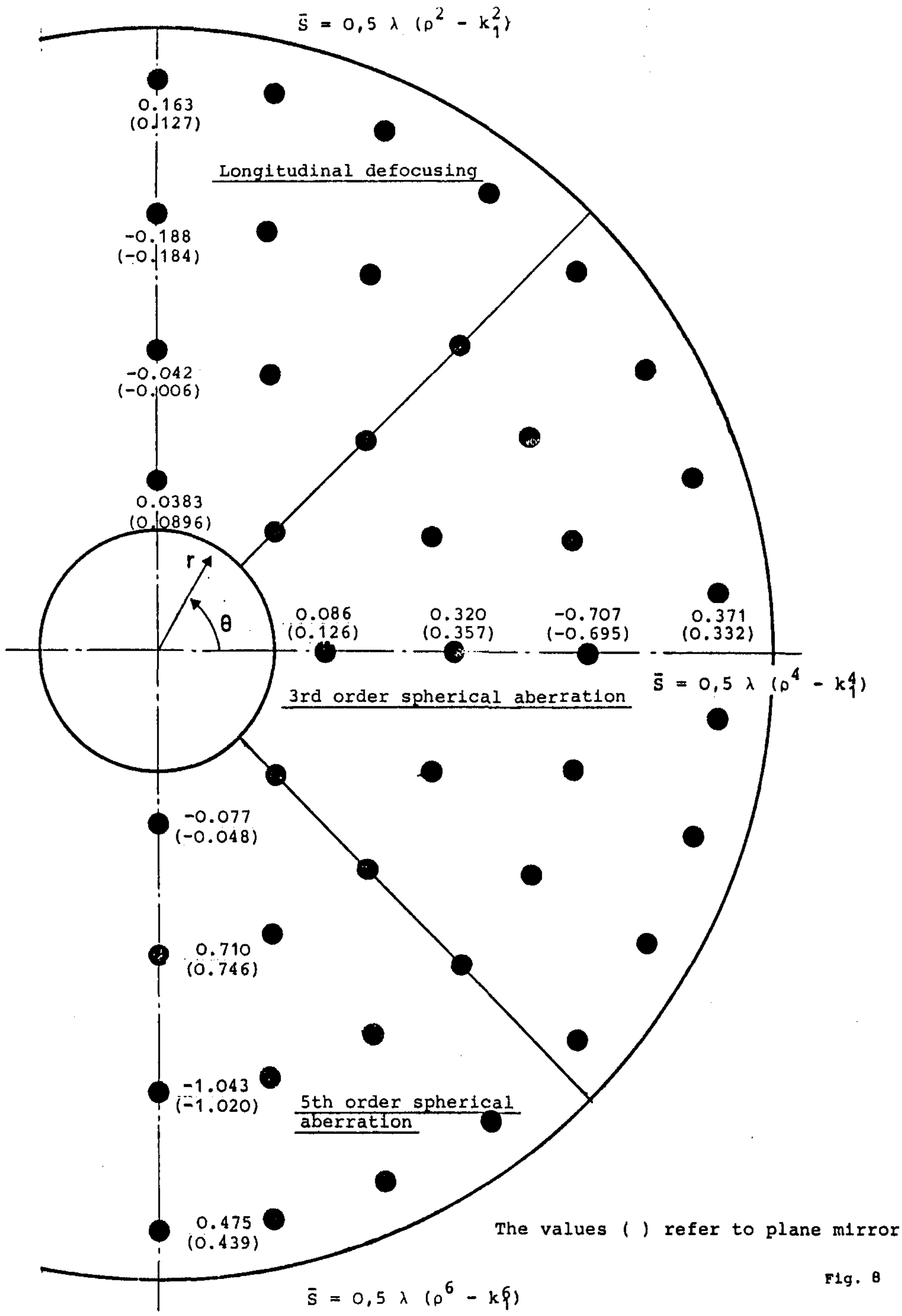
$\bar{S} = 0,5\ \lambda\ (\rho^2 - k_1^2)$
0.163
(0.127)
Longitudinal defocusing
-0.188
(-0.184)
-0.042
(-0.006)
0.0383
(0.0896)
r
θ
0.086
(0.126)
0.320
(0.357)
-0.707
(-0.695)
0.371
(0.332)
3rd order spherical aberration
$\bar{S} = 0,5\ \lambda\ (\rho^4 - k_1^4)$
-0.077
(-0.048)
0.710
(0.746)
-1.043
(-1.020)
5th order spherical aberration
0.475
(0.439)
The values ( ) refer to plane mirror
$\bar{S} = 0,5\ \lambda\ (\rho^6 - k_1^6)$

Fig. 8

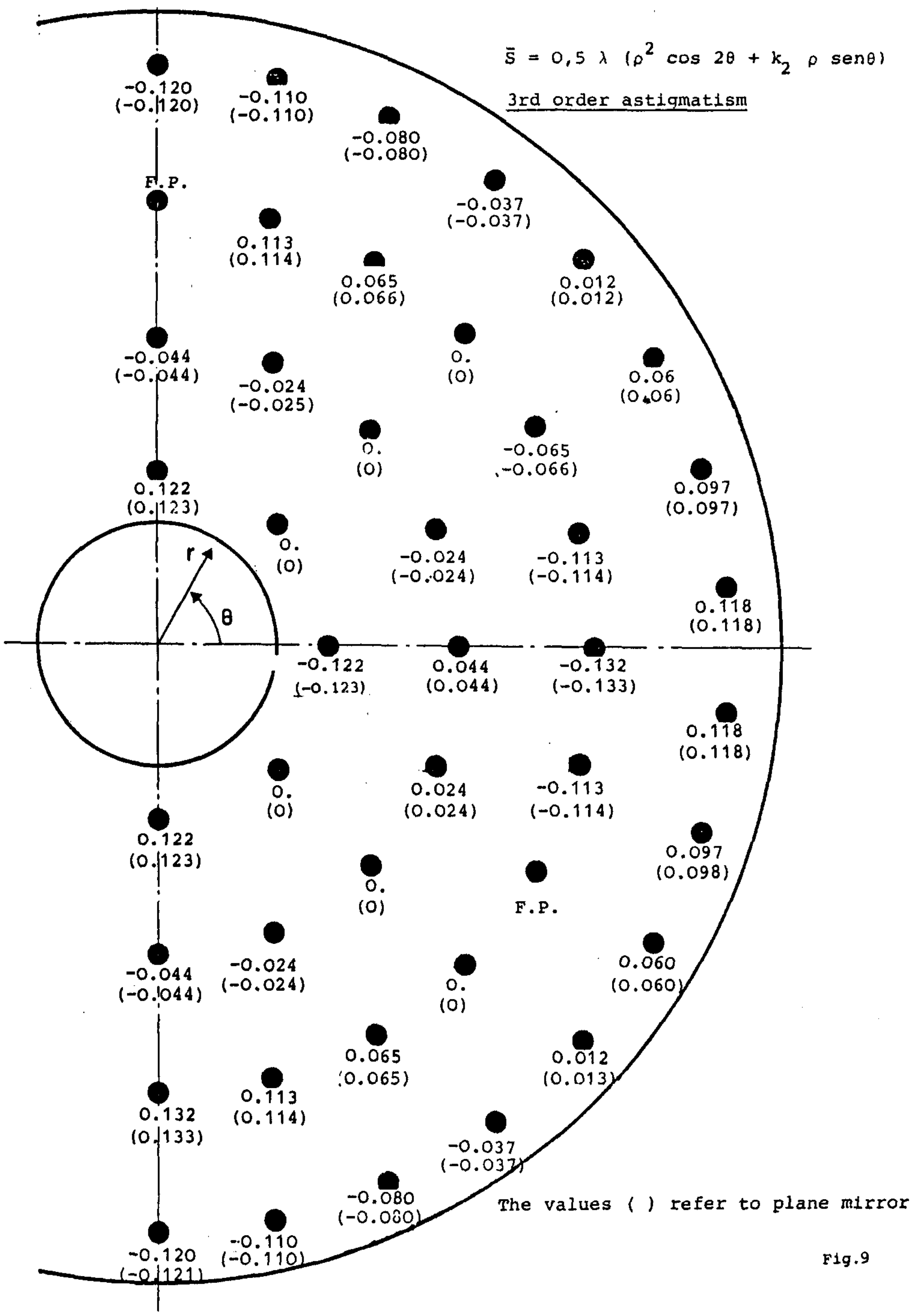

Fig.9

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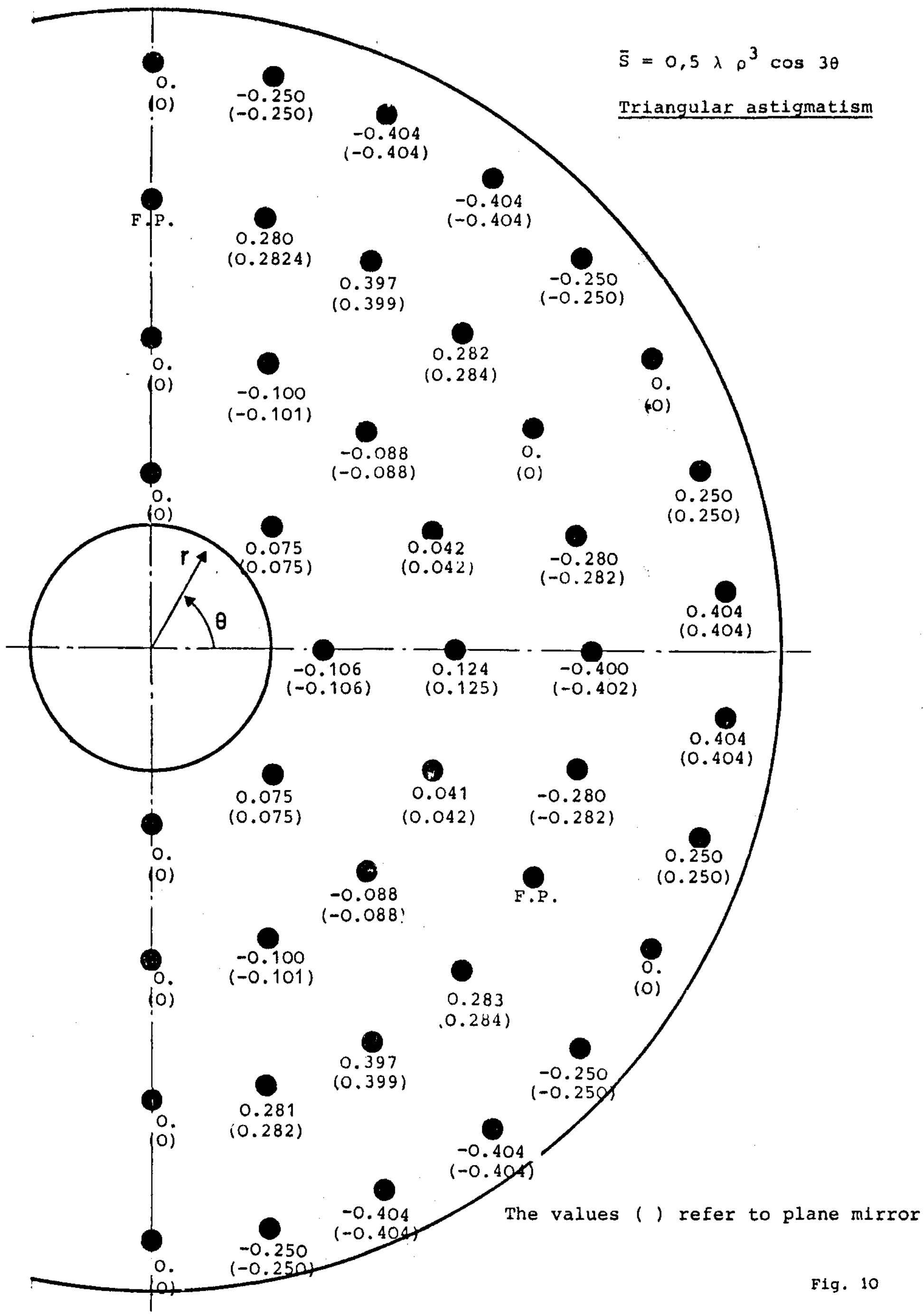
$\bar{S}$ = 0,5 λ ρ^3 cos 3θ
Triangular astigmatism
The values ( ) refer to plane mirror
F.P.
r
θ
0.
(0)
-0.250
(-0.250)
-0.404
(-0.404)
0.280
(0.2824)
0.397
(0.399)
-0.250
(-0.250)
0.282
(0.284)
-0.100
(-0.101)
-0.088
(-0.088)
0.250
(0.250)
0.075
(0.075)
0.042
(0.042)
-0.280
(-0.282)
0.404
(0.404)
-0.106
(-0.106)
0.124
(0.125)
-0.400
(-0.402)
0.041
(0.042)
0.283
(0.284)
0.281
(0.282)

Fig. 10

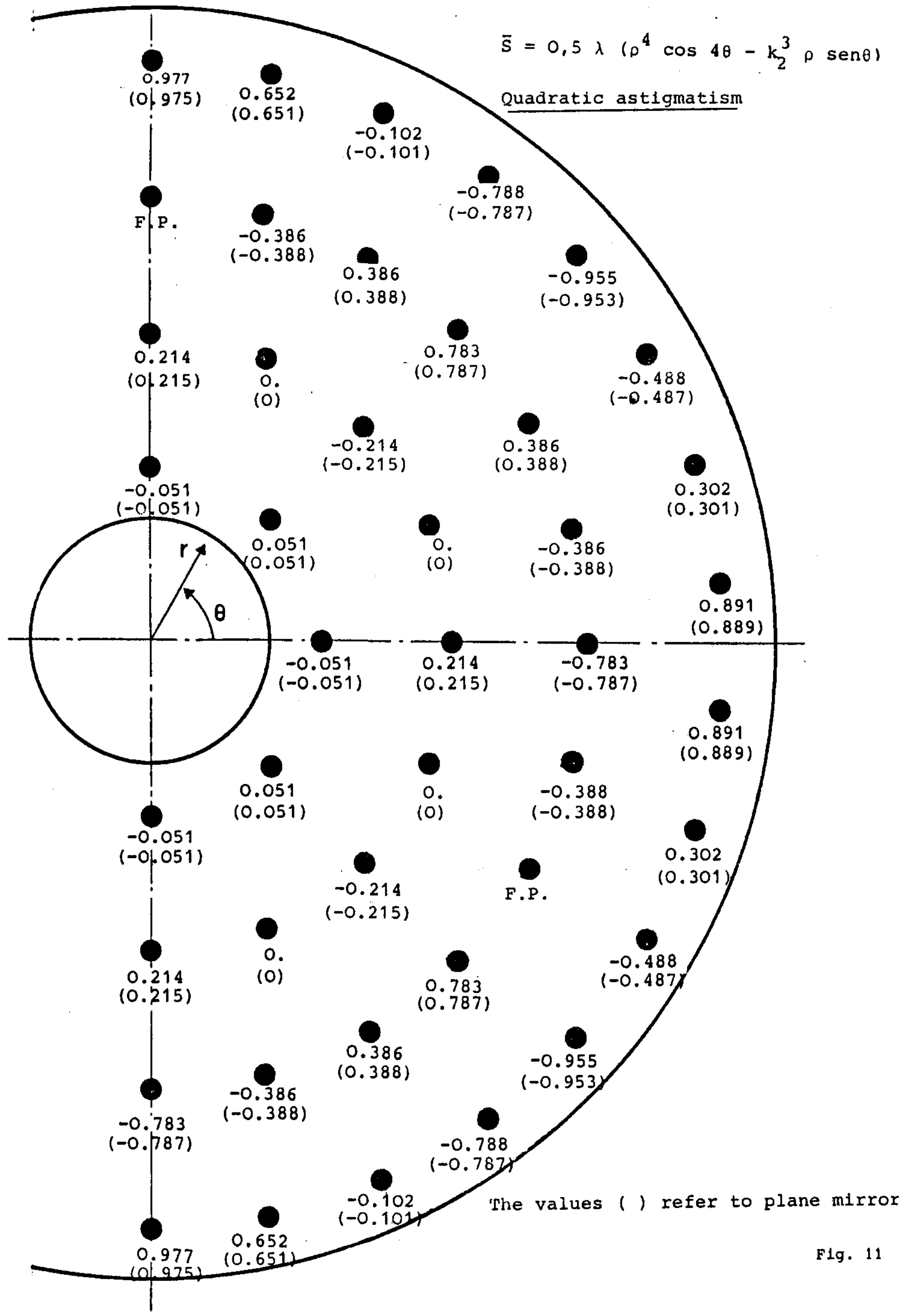

Fig. 11

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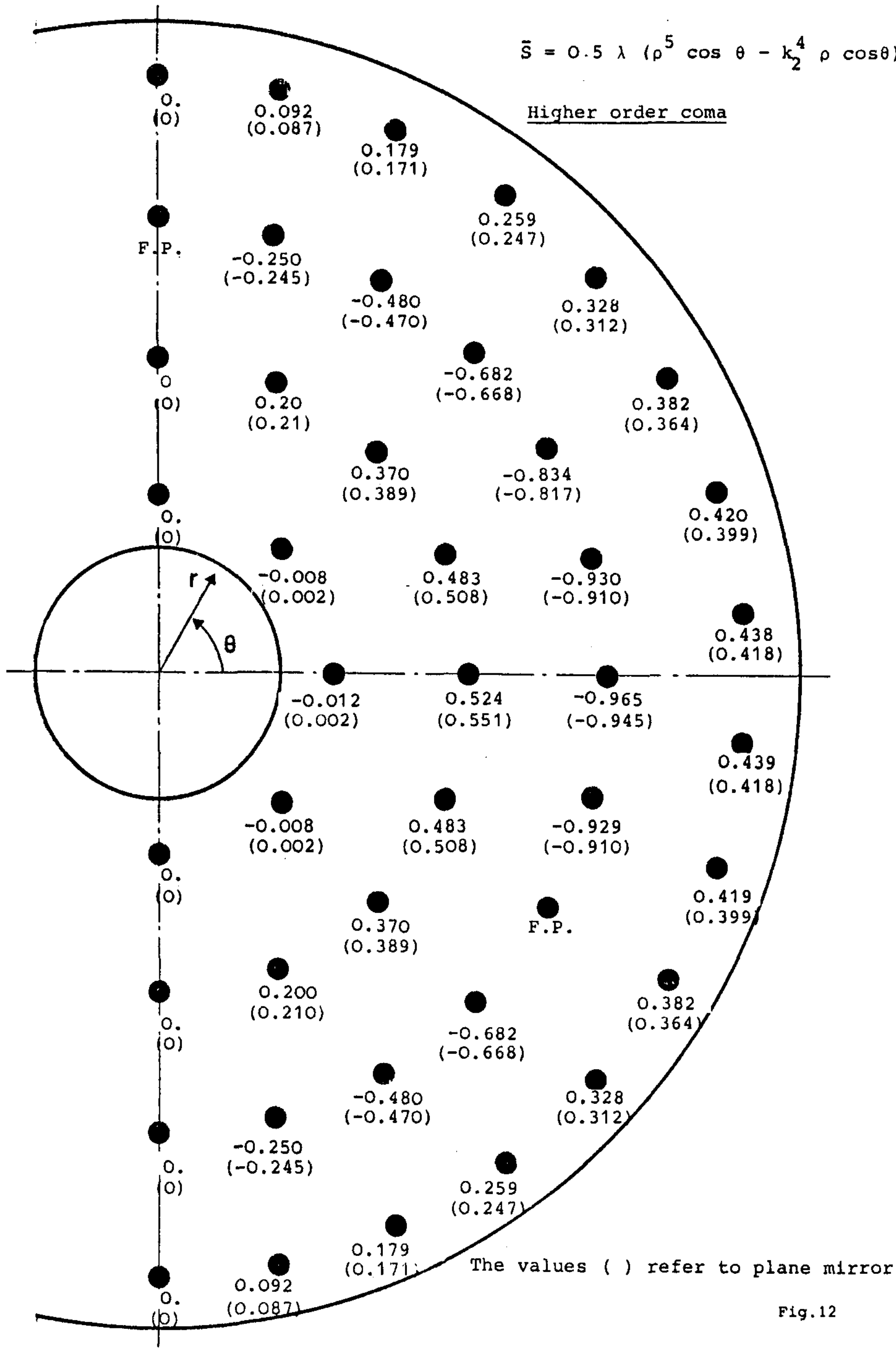

Fig.12

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DISCUSSION

J. Nelson: I enjoyed your analysis very much. The question of what perturbing forces should be applied through your actuators to remove an optical aberration is an interesting and subtle one. In the UCTMT project M. Budionsky has been exploring this, and he finds that one can often limit the maximum point load without appreciably altering the desired deformation. This can be critical, since otherwise one may need to apply enormous forces to remove an aberration.

G. Ballio: The removal of an optical aberration may be formulated as two different optimum problems, both leading to quadratic programming problems with linear constraints.
1st formulation. Find the values of the effects provided by the actuators able to return the mirror as near as possible to the design position.
2nd formulation. Find the values of the effects provided by the actuators in order to minimize the energy supplied to the actuators and to limit the maximum gap between the distorted and the wished configuration under an assigned value ε^2 . When the difference between the distorted and the wished configuration are measured on the actuators points and $\varepsilon^2=0$ the two formulations lead to solving a linear system of equations but obviously you can find unrealistic values for forces to be applied.

J. Nelson: (to G. Ballio concerning high forces resulting from calibration procedure.)

R. Wilson: It should be mentioned that these "calibrations" by finite element calculations of the force changes needed to produce a 1λ coefficient of a given aberration are a follow-up of analytical calibrations done by Schwesinger for our ESO NTT primary mirror.

Concerning the question of Jerry Nelson whether such finite element calibration might not lead to excessively high force changes, there is indeed a real danger of this happening. If the sampling of the desired function is such that a constraint is applied with regard to higher order flexure terms, then this confirmation may produce excessive force requirements. This is the equivalent of certain optical design optimisation algorithms operating with too many zero points in the function and constraining small higher order aberration residuals in an unnatural and unnecessary way, the result being excessive paths in parameter space associated with ill-conditioned matrices with very high eigenvalue ratios.

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