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CORRIGENDUM

Relativistic quantum kinetic theory of stimulated bremsstrahlung of an axial vacuum mode by an electron beam travelling in a uniform magnetic field

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In an electron-cyclotron maser (ECM) using an axial uniform magnetic field whose magnetic induction is $\mathbf{B} = B\hat{\mathbf{z}}$, all electrons rotate around the magnetic field lines in the counter-clockwise direction. Hence the time-reversed process of emission (absorption) by an electron implies absorption (emission) by an electron rotating around a magnetic field line in the clockwise direction. However, the electron cannot rotate around in the clockwise direction, and so, for any process taking place in a uniform axial magnetic field, there can be no corresponding time-reversed process in the same field. Mathematically,

$$(E-cp_z)\,k+mc\omega_c \neq 0 \quad \text{for} \quad k, \omega_c > 0.$$

This was not taken into account in the above-mentioned paper, and thus the gain formula (3.3) obtained was incorrect. In addition, the wrong conclusions were drawn regarding the validity of the classical theory (Davydovsky 1962; Kolomensky & Lebedev 1962). We shall correct (3.3) here.

We shall derive (3.3), starting from spontaneous emission. Some additional important results will also be derived. With no loss of generality, we can assume that the wave vector of the emission is $\mathbf{k} = k_z\,\hat{\mathbf{z}}+k_y\,\hat{\mathbf{y}}$. Then the perturbing Hamiltonian representing the emission of a photon of polarization vector $\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\nu}$ is written as (Kim 1993)

$$\mathscr{H}'_e(z,t) = -\frac{e\boldsymbol{\alpha}\,.\,\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\nu}}{\mathscr{V}^{\frac{1}{2}}}a^{\dagger}_{\mathbf{k}\nu}\exp\left[-i(k_z z+k_y y-\omega t)\right]. \tag{1}$$

The transition probability per unit time from state 1 with z momentum p_{1z} and Landau quantum number l_1 to state 2 with z momentum p_{2z} and Landau quantum number l_2 by the spontaneous emission of a photon of wave vector $\mathbf{k}$ polarized in the $\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\nu}$ direction is

$$\mathscr{T}_{\text{spon. em.}}(1\rightarrow 2\,|\,k_z\,\hat{\mathbf{z}}+k_y\,\hat{\mathbf{y}},\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\nu}) = \frac{\pi^2c^2e^2p_\perp^2}{2\hbar kE^2\mathscr{V}}\delta_{p_{2z},\,p_{1z}-\hbar k_z}\delta_{k_y,\,0}$$
$$\times\left[\delta\left(\frac{c(p_z k_z+m\omega_c)}{E}-\frac{\omega}{c}\right)\delta_{l_2,\,l_{1-1}}+\delta\left(\frac{c(p_z k_z-m\omega_c)}{E}-\frac{\omega}{c}\right)\delta_{l_2,\,l_1+1}\right]. \tag{2}$$

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The $\delta_{k_y,0}$ prescribes that the emission takes place only in the z direction. Accordingly, we assume from now on that $\mathbf{k} = k\hat{\mathbf{z}}$. Since the emission is only in the z direction, we should assume that the z dimension L_z of the interaction (emission) system is unspecified, while its x and y dimensions L_x and L_y respectively should be taken as equal to the emission wavelength λ, so that $\mathscr{V} = L_z\lambda^2$. This is because, for an arbitrary function $A(k, \theta, \psi)$,

$$\iiint A(\mathbf{k})\,\delta_{k\sin\theta,0}\frac{\mathscr{V}k^2\,dk\,d\Theta}{(2\pi)^3} = \iiint A(k,\theta,\psi)\frac{\delta(\theta)\,\delta(\psi)}{\sin\theta}\frac{\mathscr{V}k^2\,dk\sin\theta\,d\theta\,d\psi}{(2\pi)^3}$$

$$= \int A(k,0,0)\frac{L_z\,dk}{2\pi}$$

is satisfied only when $L_x L_y = \lambda^2$.

From the above, we find that

$$\mathscr{T}_{\text{spon. em.}}(1\to 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu) = \frac{\pi^2 e^2 p_\perp^2}{\hbar k^2\mathscr{V}}\delta_{p_{2z},\,p_{1z}-\hbar k}\,\delta_{l_2,\,l_1-1}\,\delta(p_\perp^2-p_0^2)$$

$$= \frac{\pi^2 e^2 p_\perp^2\,k_0}{2\hbar m^2c^2\mathscr{V}k_c(k_c+k_0)}\delta_{p_{2z},\,p_{1z}-\hbar k}\,\delta_{l_2,\,l_1-1}\,\delta(k-k_0), \qquad (3)$$

where $\hat{\boldsymbol{\epsilon}}_1 = \hat{\mathbf{x}}$, $\hat{\boldsymbol{\epsilon}}_2 = \hat{\mathbf{y}}$, $p_0^2 = \dfrac{m^2c^2}{k^2}(k_c^2-k^2+2k_c\,kp_z/mc)$,

with $k_c = \dfrac{\omega_c}{c}$, $k_0 = \dfrac{(\gamma+p_z/mc)\,k_c}{1+p_\perp^2/(mc)^2}$,

and $\delta[c^2(p_z k - m\omega_c)/E-\omega] = 0$ for $p_z > 0$ has been used ($S(-\omega_c)$ in (3.2) of the original paper should be zero, and all terms arising from this term in the subsequent equations should be ignored).

For a given polarization, the number of states of the photon whose wavenumber is between k and $k+dk$ is $L_z\,dk/2\pi$. Unlike stimulated emission, whose final photon state is determined by the incident photon, nothing determines the final state of the photon emitted by spontaneous emission. Therefore the probability per unit time that an electron initially in state 1 will emit a photon with wavenumber between k and $k+dk$ and polarization $\hat{\boldsymbol{\epsilon}}_\nu$ is

$$\sum_2 \mathscr{T}_{\text{spon. em.}}(1\to 2\,|\,\mathbf{k},\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\nu})\frac{L_z\,dk}{2\pi}.$$

Accordingly, the total axial spontaneous emission power for an average electron is given by

$$\mathscr{P}_{\text{spon. em.}} = \iint f(\mathbf{p})\sum_{\nu=1}^{2}\sum_2 \mathscr{T}_{\text{spon. em.}}(1\to 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\frac{L_z\hbar ck\,dk\,d^3p}{2\pi}$$

$$= \frac{e^2\langle p_\perp^2\rangle\,\bar{k}_0^4}{8\pi m^2ck_c(\bar{k}_0+k_c)}\int\delta(k-\bar{k}_0)\,dk, \qquad (4)$$

where $\bar{k}_0 = \dfrac{(\gamma+\langle p_z\rangle/mc)\,k_c}{1+\langle p_\perp^2\rangle/(mc)^2}$,

and $f(\mathbf{p})$ is the normalized momentum distribution function, taken as $f(\mathbf{p}) = \delta(p_z-\langle p_z\rangle)\,\delta(p_\perp^2-\langle p_\perp^2\rangle)$.

From (4), the frequency and axial spontaneous emission power from an electron in the ECM are given by

$$\omega^{ECM} = \begin{cases} \dfrac{2\gamma\omega_c}{1+\langle p_\perp^2\rangle/m^2c^2} & (\gamma \gg 1), \\ \omega_c & (\gamma \approx 1), \end{cases} \tag{5}$$

$$P^{ECM}_{\text{spon. em.}}(\hat{\mathbf{z}}) = \begin{cases} \dfrac{e^4B^2\gamma^3\langle p_\perp^2\rangle/m^2c^2}{\pi m^2c^3(1+\langle p_\perp^3\rangle/m^2c^2)^3} & (\gamma \gg 1), \\ \dfrac{e^2\langle p_\perp^2\rangle\,\omega_c^2}{16\pi m^2c^3} & (\gamma \approx 1). \end{cases} \tag{6}$$

The wave vector of the photon produced by stimulated emission is exactly the same as that of the incident photon inducing the emission. Let the transition probabilities per unit time from state 1 to 2 by stimulated emission (or stimulated bremsstrahlung) and stimulated absorption (or inverse bremsstrahlung) of photons with wavenumber between k and $k+dk$ and polarization $\hat{\boldsymbol{\epsilon}}_{k\nu}$ be $\mathscr{T}_{\text{st. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\,dk$ and $\mathscr{T}_{\text{st. ab.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\,dk$ respectively. We readily find that

$$\begin{aligned}\mathscr{T}_{\text{st. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\,dk &= \mathscr{T}_{\text{st. ab.}}(2\rightarrow 1\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\,dk \\ &= \frac{\mathscr{T}_{\text{spon. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\,\mathscr{V}\,i(k)\,dk}{\hbar c^2k},\end{aligned} \tag{7}$$

where $i(k)\,dk$ is the energy passing through unit area normal to the z direction per unit time carried by photons with wavenumbers between k and $k+dk$.

The intensity (or the energy passing through unit area normal to the z direction per unit time) at z is given by

$$I(z) = \int \sum_\nu i(z,k,\hat{\boldsymbol{\epsilon}}_\nu)\,dk.$$

Consider a cylinder with axis parallel to the z direction. Let z and $z+dz$ be the z co-ordinates of the bottom and top of the cylinder respectively, and let S be the cross-sectional area of the cylinder. Since $[I(z+dz)-I(z)]S$ is the energy radiated in the cylinder that will pass through the top per unit time (Kim 1986), we have

$$\begin{aligned} S\,dz\frac{dI(z)}{dz} &= S\,dz\int\sum_{\nu=1}^{2}\sum_1\sum_2\Big\{N_1\,\mathscr{T}_{\text{spon. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\frac{\hbar ckL_z}{2\pi} \\ &\quad+[N_1\,\mathscr{T}_{\text{st. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)-N_2\,\mathscr{T}_{\text{st. ab.}}(2\rightarrow 1\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)]\,\hbar ck\Big\}\,dk \\ &= NS\,dz\int dk\int d^3p\sum_{\nu=1}^{2}\sum_2\mathscr{T}_{\text{spon. em.}}(1\rightarrow 2\,|\,k\hat{\mathbf{z}},\hat{\boldsymbol{\epsilon}}_\nu)\Big\{f(\mathbf{p}_1)\frac{\hbar ckL_z}{(2\pi)} \\ &\quad+[f(\mathbf{p}_1)-f(\mathbf{p}_2)]\frac{i(z,k)\,\mathscr{V}}{c}\Big\},\end{aligned} \tag{8}$$

where N_i is the number of photons in state i per unit volume.

The gain is defined by

$$\mu(k,z) = \lim_{i(z,k)\rightarrow\infty}\frac{1}{i(z,k)}\frac{di(z,k)}{dz}.$$

Then, from (3) and (8), we find that (3.3) should be written as

$$\mu = N\int \sum_{\nu=1}^{2} \sum_{2} \mathscr{T}_{\text{spon. em.}}(1 \rightarrow 2 \,|\, k\hat{\mathbf{z}}, \hat{\boldsymbol{\epsilon}}_\nu)\, \hbar \left[k\frac{\partial f(\mathbf{p})}{\partial p_z} + 2m\omega_c \frac{\partial f(\mathbf{p})}{\partial p_\perp^2}\right]\frac{\mathscr{V}\, d^3p}{c}$$

$$= -\frac{4\pi^2 cme^2\omega_c N}{\omega^2}\int f(\mathbf{p})\,\delta(p_\perp^2 - p_0^2)\, d^3p. \qquad (3.3)$$

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