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# Small divisor problems via a general theorem

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The "small divisor" difficulty arises in the study of certain stability problems, particularly in celestial mechanics. Sternberg [5] approached these problems by using Taylor's formula generalized to normed vector spaces. His work seems to be the first real attempt to provide a conceptual framework for these various small divisor problems and the resulting theory, while complicated, is significantly simpler than previous work.

In this thesis the theory of Sternberg is refined and the main result is the development of a single theorem which will deal, in a routine way, with many important small divisor problems. This theorem is referred to as the "stage 3" accelerated convergence theorem.

Application of this accelerated convergence theorem is made possible by the use of a new general formula for finding the Fréchet derivative of the type of composite functions such as occur in conjugacy problems [1].

In order to motivate the rather involved "stage 3" accelerated convergence theorem two simplified versions of this theorem are stated and proved. The first is an extremely simple "prototype" which has very limited applications. It is used, by way of illustration, to prove an elementary stability theorem.

The "stage 2" accelerated convergence theorem is used to prove, for the first time in its full generality in higher dimensions, the Siegel centre theorem. Proofs have been given for the one dimensional case but the higher dimensional version previously involved the introduction of an

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important restrictive condition [5, page 96].

This same "stage 2" theorem is applied to give a proof of a reducibility theorem of Mitropol'skiĭ and Samoĭlenko [4] for quasiperiodic linear systems of differential equations. This application differs fundamentally from the Siegel centre theorem, the Moser twist theorem and the simple stability theorem in that it does not seem possible to write it as a conjugacy problem.

The "stage 3" accelerated convergence theorem which implies both the "stage 2" and "prototype" theorems is used to prove a version of the well-known Moser twist theorem. All of the applications of the method of accelerated convergence which are treated in the thesis are to holomorphic systems but the same theory can be employed to prove corresponding results for differentiable systems. This, however, involves the use of "smoothing operators" which is outside the scope of the thesis.

The main results of the thesis are contained in the papers [1], [2], [3].

## References

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