

Abstracts of Australasian PhD theses

Finite difference methods with application to the cavity problem

Gregory Woodford

The choice of suitable graded meshes and the use of extrapolation techniques as applied to finite difference methods of solution for firstly, linear two-point boundary value problems and secondly, the prototype cavity problem in fluid dynamics are examined.

Consider two-point boundary value problems of the form

$$\frac{d^2y}{dx^2} + a(x) \frac{dy}{dx} + b(x)y = 0$$

subject to the conditions

$$y(0) = y(1) = 1 .$$

It is shown how to construct systematically graded meshes that ensure that the appropriate finite difference method has optimal numerical properties. This scheme for choosing graded meshes is designed to allow the use of extrapolation processes as well. This is an attribute not usually available when using graded meshes. Both first order and second order formulations of the above two-point boundary value are examined.

The prototype cavity problem of fluid dynamics is used as an example of the application of some of the above-mentioned ideas. The cavity problem consists of finding the flow pattern for an inviscid fluid in a square two-dimensional box with a sliding roof. A similar method to the above is not known as yet for choosing an appropriate graded mesh for the non-linear Navier-Stokes equation but the principle for choosing a graded mesh is applicable and does still allow extrapolation to be used with that

Received 5 August 1975. Thesis submitted to the Australian National University, February 1975. Degree approved, September 1975. Supervisor: Dr M.R. Osborne.

graded mesh.

The cavity problem is examined using regular meshes and a number of graded meshes for a Reynolds number of 50 . Extrapolation is applied in all cases. Both the convective and the divergence forms of the vorticity transport equation are studied. A graded mesh is demonstrated that is too severe for the problem in hand to emphasize the dangers involved in the choice of a graded mesh for problems where a specific algorithm for choosing the optimal mesh is not known.