LETTERS TO THE EDITOR

Dear Editor,

On honest Bernoulli excursions

Recently, in an elegant paper, Smith and Diaconis (1988) derived a limiting approximation for the chance that the simple symmetric random walk has travelled distance k from its start, given that its first return is at time $2n$. We show that their result holds for all simple (not necessarily symmetric) random walk.

Smith and Diaconis (1988) considered the simple random walk on the integers. Starting at zero, a particle moves one step up with probability $P = 1/2$ or one step down with probability $1 - P$. They derived a limiting aproximation for the distribution of the maximum distance from zero reached by the walk up to time $2n$, given that its first return is at time $T = 2n$. The aim of this note is to generalize their result for all $0 < P < 1$.

From the classical ballot problem we have:

$$P(T = 2j) = \frac{1}{2j-1}\binom{2j}{j} P^j(1-P)^j$$

(1)

$$= \frac{1}{2j-1}\binom{2j-1}{j} 2P^j(1-P)^j,$$

(1) is the generalization of (1.3) in Smith and Diaconis (1988) for all $0 < P < 1$. Conditioning on the outcome of the first step we get the generalization of (2.1). That is $P(M_T < K \text{ and } T = 2n)$ equals

(2)
$$2P^n(1-P)^n \sum_j \left\{ \binom{2n-2}{n-1+jk} - \binom{2n-2}{n-2+(j+1)k} \right\},$$

where M_T is the maximum distance from zero reached by the walk up to time T. Our generalization follows from (1) and (2) since the distribution of M_T, given that $T = 2n$, does not depend on P.

The idea used to extend the results from $P = 1/2$ to general P works for several other problems connected to random fluctuation given the value of this sum at time n (see, for example, Csáki and Mohanty (1981), (1986) and Vervaat (1979)). Another example is the number of changes of sign given $S_{2n} = 0$. For an example where the results depend on P, see, for example, Takács (1979).

Acknowledgement

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Yours sincerely,
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Dear Editor,

A note on the stability of interior ESSs in a diploid population

An outstanding problem in evolutionarily stable strategy (ESS) theory has been the consistency of the underlying genetic structure of a population with the method of finding stable equilibria by the ESS criteria of comparing phenotypic (i.e. strategy) fitnesses. Even the most basic problem of analyzing the stability properties of an interior ESS, S^*, in a randomly-mating diploid population where strategy is determined at a single multi-allelic locus has produced misleading and/or incorrect statements in the literature.

For instance, Cressman and Hines (1984) asserted in their Theorem 4.1(d) that, for semi-dominant inheritance patterns (i.e. incompletely dominant in the terminology of Cressman (1988)), the mean strategy evolves either towards the boundary or towards S^*. The proof relied on the exponentiation of a matrix product and, to be correct, needed the matrices to commute — an unwarranted assumption in the circumstances. The inappropriateness of this approach was pointed out to one of us in private communications with Josef Hofbauer. As a consequence, the corollary at the end of this paper (asserting that S^* is globally stable if there are three or less alleles) is incorrect as can be seen by a counter-example in the last exercise of Section 28.4 of Hofbauer and Sigmund (1988).

On the other hand, the other main conclusions of Cressman and Hines (1984), that is Theorems 4.1 and 4.2, remain valid. In particular, S^* is locally stable for semi-dominant