Notes on Factoring.

By J. W. Butters.

§ 1. When it is necessary to find the factors of such an expression as $x^2 - 7x - 120$ the difficulty for beginners lies in finding the pair of factors of 120 which satisfy the middle term: in this case so that their difference is 7. They may write down the pairs in order (1×120; 2×60; etc.) and then choose the suitable pair, but this method is often long; or they may guess until they chance to find the pair required. This is often done in so haphazard a fashion that much time is wasted, especially if the given expression have no *rational* factors.

§ 2. A combination of these methods may be used as follows: Take any pair of factors as a first trial—say 10×12; here the difference is too small; hence the required factors must *differ more* than 10 and 12, and therefore the smaller factor must be made still smaller. Try now *in order the natural numbers* less than 10. We find that 9 is not a factor of 120, but that 8 is (for $120 = 8 \times 15$) and that this gives the required difference. Hence

$$x^2 - 7x - 120 = (x + 8)(x - 15).$$

§ 3. It sometimes happens that the first trial gives a sum or difference which *differs greatly* from the one required, and then the above method is rather long. For example, if for $x^2 + 59x + 168$ we take 8×21 as a first trial, we get in succession 7×24; 6×28; 4×42; and 3×56 which is the pair required. In such a case we may begin by removing a factor from one of the pair and multiplying it into the other. We should then have (say) 8×21; 4×42; 3×56. It ought to be noticed, however, that by this method we often *pass* the required pair. Thus we might have (by transferring 4 instead of 2) 8×21; 2×84 where the first pair has too small a sum and the second too great. In most cases the method of § 2 is sufficient, and if in using it we find two consecutive pairs with their (algebraic) sums too great and too small respectively, we see that the given expression cannot have *rational* factors.

§ 4. Just as from the identity

$$(x + a)(x + b) \equiv x^2 + (a + b)x + ab$$

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we know that an expression of the form x^2+px+q may be resolved into factors provided we can find two factors of q whose sum is p, so we may extend this method* to the form Ax^2+Bx+C by means of the identity

$$(ax+m)(bx+n) \equiv abx^2+(an+bm)x+mn.$$

Here we see that it is necessary to find two factors of AC whose sum is B (viz., an and bm); also that the required factors are obtained by removing a and b from the expressions $(abx+an)$ and $(abx+bm)$, viz., $(bx+n)(ax+m)$.

§ 5. One or two numerical examples will show how this works out in practice.

Find the factors of $18x^2-111x+80$.

Begin by writing down the product 18×80. [It is convenient to keep throughout the smaller number first.] This gives a sum of 98 instead of 111. Hence 18 must be reduced. Applying now the method of *natural numbers* as in § 2 we find 16×90 (sum $=106$, too small) and 15×96 (sum 111, as required). Affix the proper signs : -15×-96. Now divide $(18x-15)$, $(18x-96)$ by 3 and 6 (the G C.M.'s of 18 and 15 and 18 and 96 respectively) and we get $(6x-5)(3x-16)$ as the required factors.

§ 6. In the following the necessary work alone is given:

$12x^2-5x-72$	12×72 (A)
$\equiv(4x+9)(3x-8)$	24×36
	$+27 \times -32$

[EXPLANATION: (A) The difference between 12 and 72 being *very much* greater than 5, the factor 2 is transferred from 72 to 12 before applying the method of the *natural numbers* as in § 2.]

§ 7. This example may be used to present the theory in another form.

$$12x^2-5x-72 \equiv \frac{1}{12}\{(12x)^2-5(12x)-12 \times 72\}$$

$$\equiv \frac{1}{12}(12x+27)(12x-32) \equiv \frac{1}{3}(12x+27) \cdot \frac{1}{4}(12x-32)$$

$$\equiv(4x+9)(3x-8)$$

* The extension is due to Mr Jas. M'Kean, F E.I.S., Lecturer on Mathematics in the Heriot-Watt College, Edinburgh.

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Since "factoring" is eminently a *practical* part of Algebra it is perhaps better not to burden the working of each case with the exemplification of the theory but to treat all as in § 6.

§ 8. This form of the theory suggests that instead of transforming quadratic equations from $ax^2+bx+c=0$ to $x^2+\frac{b}{a}x+\frac{c}{a}=0$, it is better to change to $(ax)^2+b(ax)+ac=0$. We thus avoid fractions and introduce early the important notion of "change of variable."

Against a Current Pseudo-Definition of Varying Velocity.

By Mr R. F. Muirhead

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