

# Design tool for topology and particle damping optimization of additively manufactured parts

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**ABSTRACT** Thanks to its design freedom, additive manufacturing (AM) offers the possibility of directly integrating functions and effects into parts. One of these effects is particle damping, which can significantly increase the damping of parts due to the friction of loose particles in closed cavities. However, the design of these cavities is challenging due to a large number of influencing factors. This article presents a tool that optimizes the component topology and creates particle damping cavities. Using the bidirectional evolutionary structure optimization method, an optimization of mass, stiffness, and damping is achieved. The verification of the tool shows that, in addition to reducing the part mass, the integration of the particle dampers is successfully implemented in compliance with the design principles from the literature. Furthermore, restrictions of the AM process were implemented.

**KEYWORDS:** Additive manufacturing, topology optimization, particle dampers, powder bed fusion by laser beam, effect engineering

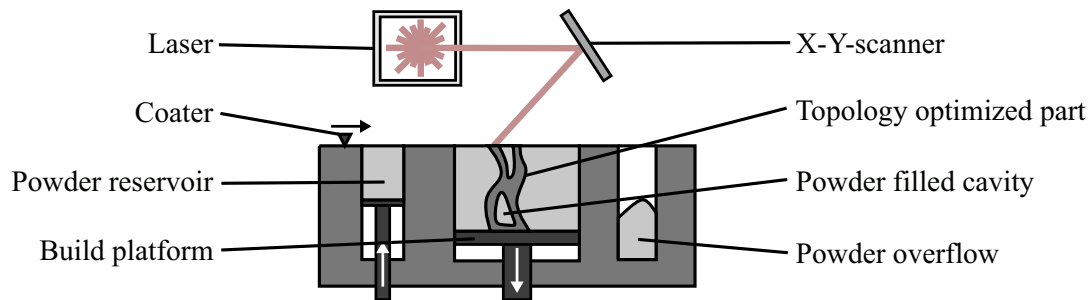
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## 1. Introduction

The process of additive manufacturing offers significant advantages in terms of the available degrees of freedom compared to conventional manufacturing processes due to the layer-by-layer material application (Lachmayer et al., 2024). While the possibility of manufacturing free-form surfaces and filigree structures has so far been used in particular for the additive manufacturing of load-adapted structures to reduce part mass, the extended design freedoms of the manufacturing process are increasingly being used for the integration of application-specific effects and functions. Examples of such functional integration include internal channels for cooling or lubrication (Lachmayer et al., 2022). The further development of the additive manufacturing process is continuously expanding the degrees of freedom. For example, multi-material additive manufacturing enables the resolving of material-related conflicts so that ideal material properties, such as high thermal or electrical conductivity, can be used voxel by voxel (Meyer et al., 2023; Oel et al., 2023). The aim of using additive manufacturing is to maximize part performance by integrating as many functions and effects as possible into one part (Ehlers et al., 2023).

One effect made possible by powder bed fusion of metals using a laser beam (PBF-LB/M, also known as LPBF or SLM) is the direct integration of particle dampers into parts during additive manufacturing (Scott-Emuakpor et al., 2018; Kunneke & Zimmer, 2017). A particle damper is a closed cavity filled with particles that can be used to reduce vibrations. In the event of part vibration, friction, and impacts occur between the individual particles and between the particles and the cavity walls (Lu et al., 2018). The resulting energy dissipation reduces the vibrations of the part. Although the principle of particle dampers has been known since 1985 (Araki et al., 1985), it is the subject of current research due to the development of the powder bed based additive manufacturing processes. The PBF-LB/M process can be

used to create these closed cavities inside a part directly in the manufacturing process. The cavities are filled with the powder, which serves as the feedstock for PBF-LB/M, turning them into particle dampers, which is shown in Figure 1. This direct integration of the particle dampers during the manufacturing process eliminates the need for additional manufacturing steps and is particularly promising in combination with topology-optimized parts, which have filigree structures and are therefore prone to large part vibrations (Ehlers et al., 2023). In previous studies, the damping of additively manufactured parts and test specimens was increased by a factor of up to 20 through particle damping (Ehlers et al., 2021). Since the effect of particle damping is based on partially non-linear friction and impact interactions, the characterization of the effect and the use of this knowledge for product development is challenging. In the following, the basics and influencing factors of additively manufactured particle dampers are explained based on current literature.



**Figure 1. Manufacturing of a topology-optimized part with integrated particle dampers in the PBF-LB/M process**

### 1.1. Influencing factors of particle dampers

The effect of particle damping can be influenced by the application and the associated excitation. The design and the particle material also have an influence on the damping behavior. A significant factor influencing system damping is the predominant excitation. For each particle damper, there is an optimal excitation level dependent on the dimensions of the cavity, which results in a maximum system damping (Fowler et al., 2001). Due to frictional forces between individual particles as well as between particles and the cavity wall, the excitation must exceed a threshold to ensure particle movement within the cavity and thus affect the damping (Papalou & Masri, 1996). Xu et al. (2004) provide a recommendation for a threshold across parts in terms of acceleration; the product of displacement and the square of the excitation frequency should not be lower than  $3.5\text{ms}^{-2}$ . The positioning and sizing of the cavities are additional design parameters for particle damper configuration. Essentially, particle dampers should be placed at positions of maximum kinetic energy, hence maximum displacement (Hollkamp & Gordon, 1998; Oel et al., 2024). Regarding the dimensioning of the cavity, damping can be increased with larger cavity volumes by integrating additional particles into the damper (Ehlers et al., 2023). However, this is limited in the direction of gravity since particles at the bottom are compressed by the layers above, restricting their movement and thereby reducing the efficiency of the particle damper (Hollkamp & Gordon, 1998; Ehlers et al., 2023). Furthermore, the characteristics of the particles themselves affect the damping behavior. The mass of the particles is a design parameter that significantly influences the system damping of particle-damped structures. Changes in total particle mass can be achieved by adjusting either the particle density or the particle volume (Hollkamp & Gordon, 1998; Masmoudi et al., 2016). Generally, an increase in particle mass leads to an increase in damping. However, beyond a specific threshold, no significant improvement in damping is observed due to movement constraints of the lower particle layers, which is similar to the effective cavity height (Hollkamp & Gordon, 1998). Modifying the particle mass shifts the natural frequencies, which must be considered in the design (Fowler et al., 2001). Moreover, the system damping can be influenced by the particle diameters. An optimum for particle diameter depends on the excitation frequency, with the recommended diameter increasing with higher frequencies (Xu et al., 2004). Fundamentally, particles should have a small diameter to increase the number of interactions between individual particles and between particles and the cavity wall, resulting in higher energy dissipation (Sathishkumar et al., 2014). The hardness of the particles also serves as a design parameter for particle dampers. Sathishkumar et al. observed improved damping behavior with increased particle hardness (Sathishkumar et al., 2014). Additionally, the packing density impacts the

damping behavior. This describes the ratio of the particle volume to the volume of the corresponding cavity (Ehlers et al., 2021). Similar to the particle size, an optimal packing density depends on the prevailing excitation frequency. Generally, lower packing densities are suitable for low frequencies, and higher packing densities suit high frequencies (Xu et al., 2004). Ideally, the packing density should range from 40-80% to achieve efficient damping (Schmitt et al., 2017). If the packing density is too low, insufficient interactions occur within the system. Conversely, if it is too high, the potential movements of individual particles are restricted. Both scenarios result in inadequate energy dissipation (Ehlers et al., 2023). It should be noted that a variation in packing density at constant particle density implies a change in particle mass or cavity dimensions (Hollkamp & Gordon, 1998). With regard to additive manufacturing, it can be concluded that the damping behavior can be influenced by the placement, dimensioning, and design of the cavities in the product design. Other influencing factors relating to the particle material or the filling volume are determined by the additive manufacturing process and cannot be influenced.

For this reason, this article focuses on the design of particle-damped parts, in particular, the integration of cavities at the ideal location according to the influencing factors. Since the potential for the particle damping effect is particularly high for weight-optimized and topology-optimized parts, the development of a tool that combines conventional topology optimization with the integration of closed cavities to increase the damping properties is presented below.

## 2. Methods

To generate topology-optimized parts with integrated particle dampers, existing methods of topology optimization need to be further developed. For this purpose, a decision structure for generating internal closed cavities is to be developed in FE-based optimization algorithms. With the help of Evolutionary Structure Optimization (ESO), the solution space can be divided into discrete element states and the part volume can be minimized by gradually removing inefficient material (Wang et al., 2023). Compared to the Solid Isotropic Material with Penalisation (SIMP) method, the discrete description of the elements makes it possible to describe the powder-filled cavities. Bidirectional Evolutionary Structure Optimization (BESO) can be used to change the state of the elements in both directions (Querini et al., 1998). There are powerful commercially available software solutions that apply the BESO algorithm for the topology optimization of ordinary parts. However, to enable the algorithm to be adapted for the optimization of integrated particle dampers, the Abaqus FE software is combined with Python-based scripts. This combination enables the user-friendly definition of load cases and support conditions as well as simple meshing within the graphical user interface of Abaqus CAE. With the help of a Python script, all field and history variables can still be accessed and used for the decision structures in the optimization algorithm. The BESO algorithm according to Zuo & Xie (2015), which serves as the basis, is explained below. The optimization algorithm tailored to the integration of particle dampers is then explained before both are combined to form an optimization tool.

### 2.1. Topology optimization

The method for topology optimization is based on the procedure of Zuo & Xie (2015), which is available as a simple Python code for the optimization of three-dimensional parts. The method uses a BESO algorithm, where the goal is to minimize the objective function  $C$ , also known as compliance. The sensitivity  $\alpha_e$  is used for the optimization, which is determined by differentiating the objective function  $C$  in Equation 1.

$$\min_x : C(\mathbf{X}) = \mathbf{F}^T \mathbf{U} = \mathbf{U}^T \mathbf{K} \mathbf{U} \quad (1)$$

$$\alpha_e = \frac{\partial C}{\partial x_e}$$

Where  $\mathbf{F}$  is the force vector,  $\mathbf{U}$  is the displacement vector, and  $\mathbf{K}$  is the stiffness matrix.  $x_e$  is the design variable of the element  $e$ . A detailed explanation of the calculation of the objective function as well as for the filtering and averaging of the sensitivity, can be found in Huang & Xie (2007) and Zuo & Xie (2015).

The procedure of the topology optimization algorithm used here is shown in Figure 2. The model initialization includes the placement of the part and the definition of boundary conditions and forces. The volume target and the evolutionary ratio  $ert$ , which specifies the change per iteration step, are also defined. After initializing the model, a static-mechanical FEM simulation is carried out. The sensitivities of the individual elements are then calculated, filtered, and averaged from the results. A threshold is determined that decides whether an element is assigned to the solid set ( $x_e = 1$ ) or the void set ( $x_e = 0.001$ ). In each iteration, the part volume is reduced by the evolutionary rate  $ert$ . Once the elements have been adjusted, the model is updated and then checked to ensure that convergence and the defined volume target  $v_{Fr}$  have been achieved. The volume target  $v_{Fr}$  is defined as the fraction of the volume of the optimized part to the initial part volume. If this is the case, the optimization is terminated, otherwise a further iteration step is carried out.

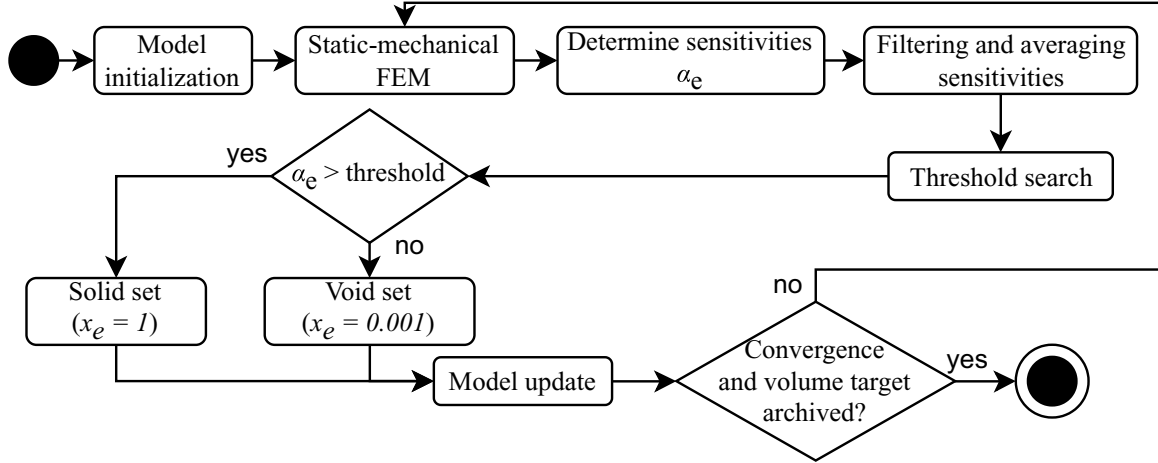


Figure 2. Method for topology optimization according to Zuo & Xie (2015)

## 2.2. Integrating optimized cavity position and geometry

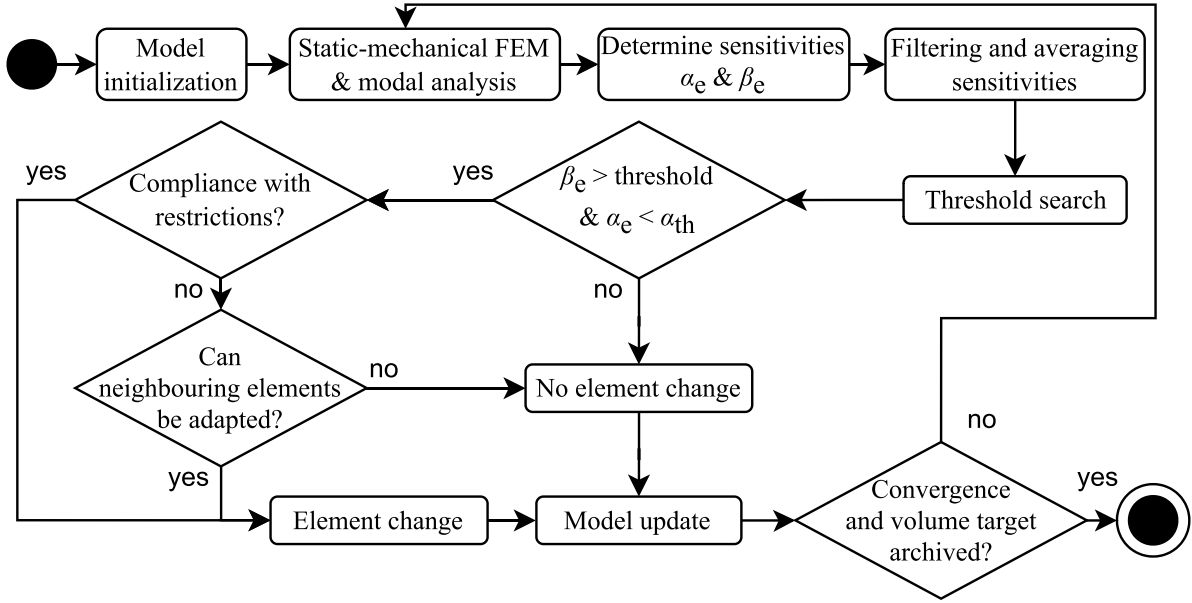
In order to integrate the particle damping in an iterative optimization process, an objective function must be defined that ensures that the part damping is maximized. According to Oel et al. (2024), an increase in the displacement and volume of the cavities leads to an increase in the damping  $D$ . Therefore, the objective function  $Z_{PD}$  is defined in Equation 2.

$$\max D(Y) \sim \max \sum_{e=1}^N (y_e v_e q_e) = \max Z_{PD}(Y) \quad Y = \{y_e\} \quad \forall e = 1, \dots, N \quad (2)$$

Here,  $q_e$  indicates the normalized displacement amplitude and  $v_e$  the volume of an element  $e$ .  $y_e$  defines the affiliation of an element  $e$ , where  $y_e = 1$  for a particle damper (PD) element and  $y_e = 0$  for solid or void elements. The sensitivity  $\beta_e$  is defined on the basis of the objective function (see Equation 3).

$$\beta_e = q_e \quad (3)$$

The complete procedure for the integration of internal closed cavities for particle damping is shown in Figure 3. In addition to the model initialization and the static-mechanical FEM as explained in section 2.1, a FE modal analysis is carried out using Abaqus to read out the displacement of the individual elements. The sensitivities  $\alpha_e$  and  $\beta_e$  are then calculated, filtered, and averaged from the element-by-element results. In addition to comparing the sensitivity  $\beta_e$  with the threshold value, it is checked whether the sensitivity  $\alpha_e$  of an element is lower than the threshold value  $\alpha_{th}$ . This is to prevent highly loaded elements from being assigned to the PD set and converted into a cavity. The mechanical properties of the PD elements are identical to those of the void elements, but the mass of the particles is taken into account when calculating the weight reduction. If the potential to generate a PD element is given, it must



**Figure 3. Method for integrating internal cavities for particle damping**

be checked whether the resulting change to the part complies with the restrictions. In this context, it is checked whether the inner cavities are closed and whether the minimum wall thickness  $w_t$  is maintained. These restrictions, which are shown in Figure 4 (a), are checked by querying neighboring elements and elements within a certain radius. If possible, other elements are adjusted to comply with the restrictions, as shown in Figure 4 (b) as an example. To increase the optimization speed, an assignment of neighboring elements is carried out during model initialization so that the neighboring elements only need to be checked and not searched for when inspecting the wall thickness.

Once the element assignment has been adjusted, the model is updated and the convergence as well as whether the volume target  $v_{PD,Fr}$  has been reached is checked in the same way as for topology optimization. The volume target of the particle damping integration  $v_{PD,Fr}$  specifies the fraction of the volume of the cavities to the volume of the optimized part. If necessary, a new iteration is started, otherwise the optimization is terminated.

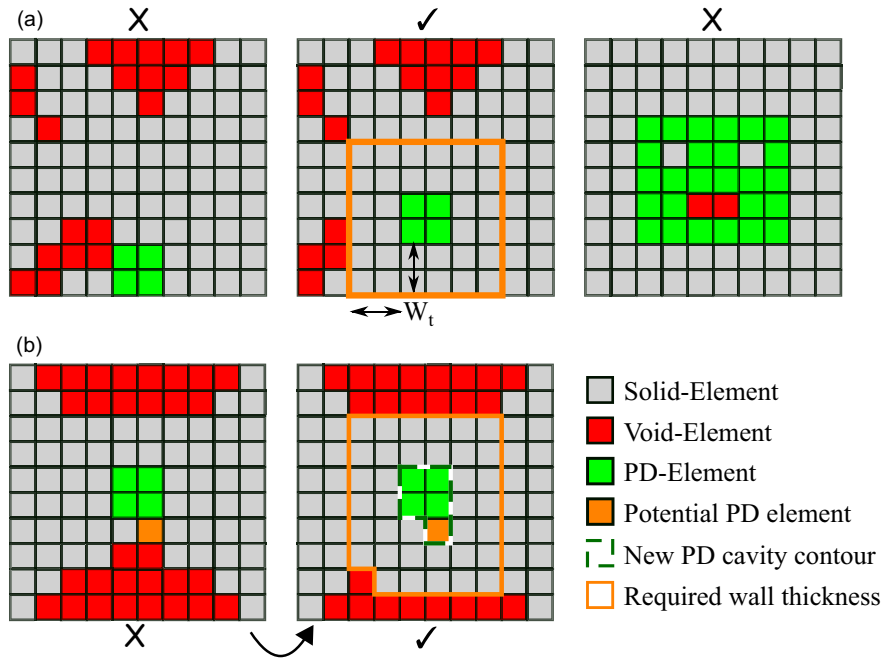
### 2.3. Optimization tool for mass, stiffness, and damping

In order to combine the two objectives of weight reduction and integration of cavities for particle damping, the two optimization algorithms presented above are combined in one tool. The entire optimization process consists of individual iterations that either change the topology for weight reduction or generate the internal cavities.

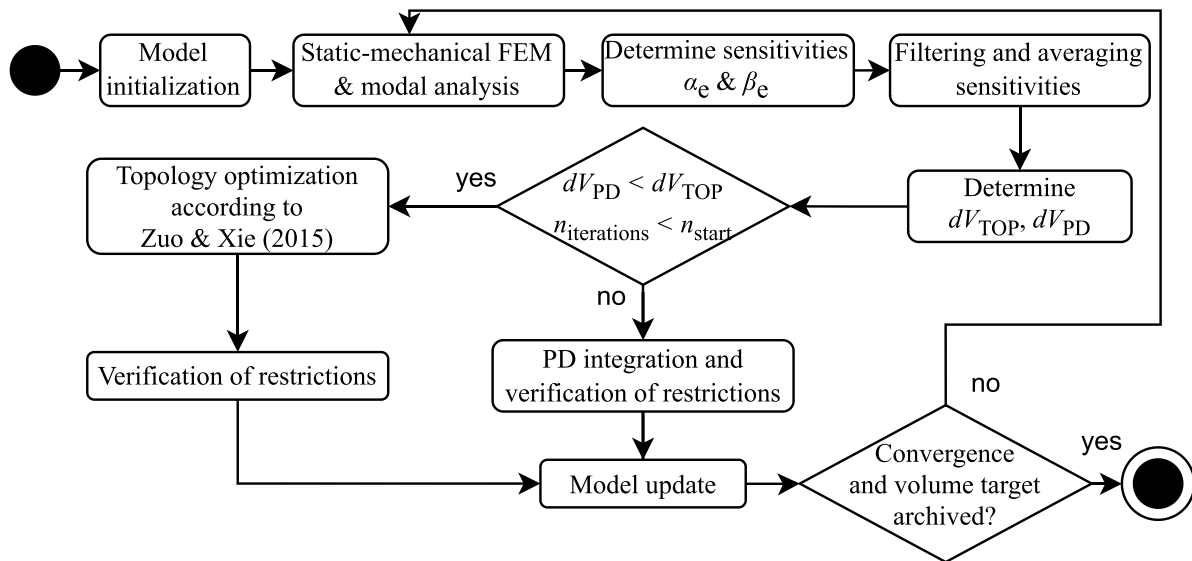
The decision as to which of the two optimization processes is carried out is based on the distance  $dV_{TOP}$  and  $dV_{PD}$  to the volume targets  $v_{Fr}$  respectively  $v_{PD,Fr}$ . The distances to the volume targets  $dV_{TOP}$  and  $dV_{PD}$  are calculated as follows:

$$dV_{TOP} = \frac{v_{Fr}^{k-1} - v_{Fr}}{1 - v_{Fr}} \quad dV_{PD} = \frac{v_{PD,Fr} - v_{PD,Fr}^{k-1}}{v_{PD,Fr}} \quad (4)$$

Where  $v^{k-1}$  describes the volume fraction of the total volume of the previous iteration and  $v_{pd}^{k-1}$  is the volume fraction of the cavity in the target volume of the previous iteration. As there is a significant change in the topology, particularly in the first iterations, the modal system also changes significantly. To prevent cavities from being created in vibration maxima of the initial topology, which would prevent effective optimization of the part in the further course, topology-only optimizations are carried out at the beginning. Once these start iterations  $n_{Start}$  have been completed, the modes to be used for the integration of the cavities can be selected. The procedure of the entire optimization tool is shown in Figure 5. The model initialization, FEM analysis and determination, filtering, and averaging of the sensitivities is carried out in the same way as in the previous procedure. The distances to the respective defined overall



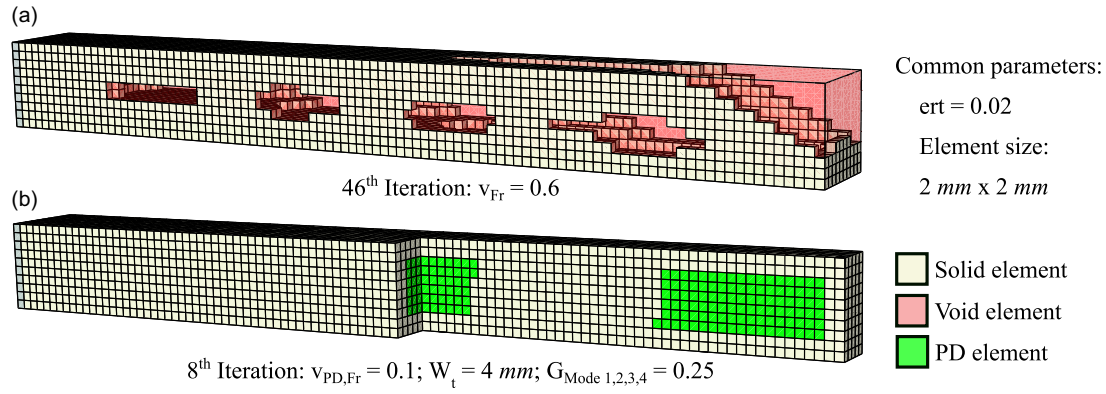
**Figure 4. Visualization of the restrictions. (a) Integrating the cavities, (b) adaptation to neighboring elements**



**Figure 5. Method for iterative optimization of part topology and integration of internal cavities for particle damping**

targets are then calculated. Together with checking whether the initial topology-only optimization iterations  $n_{start}$  have already been completed, a decision is made regarding which of the two optimization algorithms will be applied in this iteration. If the topology is to be optimized, the procedure from section 2.1 is followed. In addition, compliance with the defined restrictions is checked. For the integration of particle damping, the procedure from section 2.2 is performed. After updating the model, the accomplishment of convergence and volume targets is checked. If necessary, a further iteration step is started, otherwise the optimization is completed.





**Figure 6. Results of the topology optimization (a) and the integration of the cavities (b) using a bending beam**

### 3. Verification and results

In order to verify that the developed tool for topology and damping optimization works according to the presented design criterion, it is tested using examples. First, the two algorithms presented in sections 2.1 and 2.2 are tested individually using a bending beam. The entire system is then tested using the GE Aircraft Engine Bracket presented by Carter et al. (2014).

The bending beam has the dimensions  $20\text{ mm} \times 20\text{ mm} \times 200\text{ mm}$  and is clamped on one side. For the static-mechanical simulation, a force of  $F = 1000\text{ N}$  is applied at the free end. The meshing is performed with an element size of  $2\text{ mm}$  and the evolutionary ratio is set to  $ert = 0.02$ . A target volume fraction of  $v_{Fr} = 0.6$  is defined for the topology optimization. For the integration of the cavities, a volume fraction of  $v_{PD,Fr} = 0.1$ , a minimum wall thickness  $w_t = 4\text{ mm}$  and an equal weighting of  $G_{Mode} = 0.25$  of the first four modes are specified. The modes are the first two bending modes in both spatial directions. The results of the two optimizations are shown in Figure 6.

The results clearly show that the generated part structure matches the defined design criteria. The implementation of the topology optimization algorithm of Zuo & Xie (2015) works without any problems, the desired target volume is reached after 46 iterations. The cut-outs and cavities in the beam increase towards the tip, which corresponds to the decreasing bending stress towards the tip. The results of the damping optimization show that the powder-filled cavities are integrated in the area of the maxima of the vibration modes. These are expanded to the maximum over the entire cross-section so that the minimum wall thickness is still maintained.

The overall system is verified using the GE Aircraft Jet Engine Bracket, which serves as a widely used example of topology optimization (Carter et al., 2014). To simplify matters, the example has been reduced to one load case, which is shown in Figure 7. The damping is to be optimized for the first two modes, whose natural frequencies are  $4292\text{ Hz}$  and  $4770\text{ Hz}$ . The target volume fraction is defined as  $v_{Fr} = 0.6$  for the total volume and  $v_{PD,Fr} = 0.16$  for the cavities. The evolutionary ratio is  $ert = 0.02$  and the initial iterations with topology-only optimization  $n_{start} = 4$ . The result after achieving convergence and volume targets within 48 iterations is shown in Figure 7. Over these 48 iterations, the part volume decreased and the volume of the particle-filled cavities increased.

As can be seen in Figure 8 a, the objective function  $Z_{PD}$  increases, but in a zig-zag path. This can also be seen in the volume fraction  $v_{PD,Fr}$  in Figure 8 b.

After the optimization, it was also found that the resonant frequencies of the modes observed have shifted due to the change in topology. The first mode has decreased by  $363\text{ Hz}$ , while the frequency of the second mode has increased by  $540\text{ Hz}$ .

### 4. Discussion

Based on the results presented for the topology optimization and particle damping integration alone, it can be seen that these algorithms generate the expected result and therefore the part optimization works according to the defined target function. The defined restrictions successfully ensure that the cavities created are closed and that the minimum wall thickness is maintained in accordance with the design guidelines. The discrete element statuses given by the BESO algorithm are elementary for the

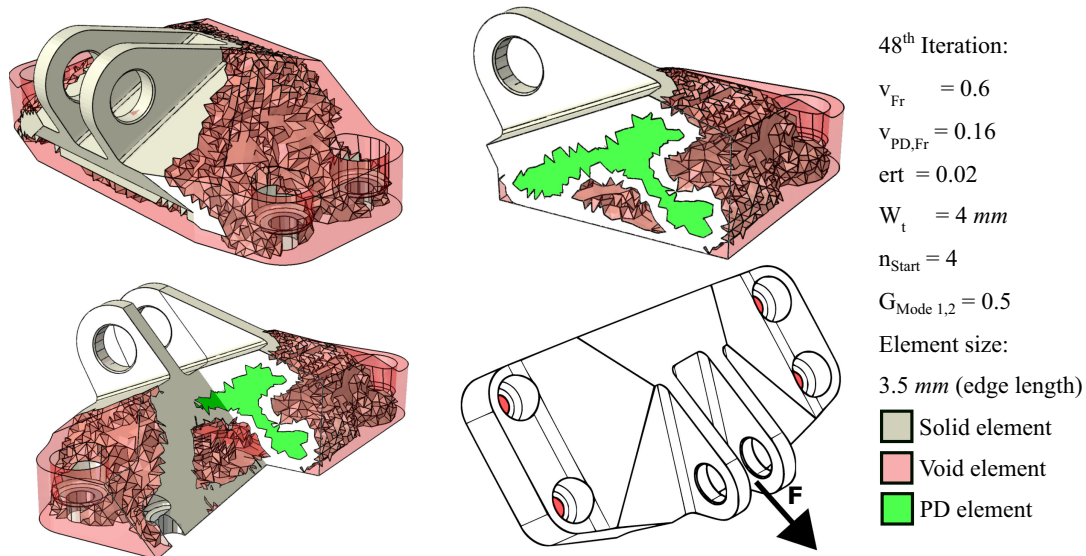


Figure 7. Results of the optimization tool for the GE Bracket example after 48 iterations

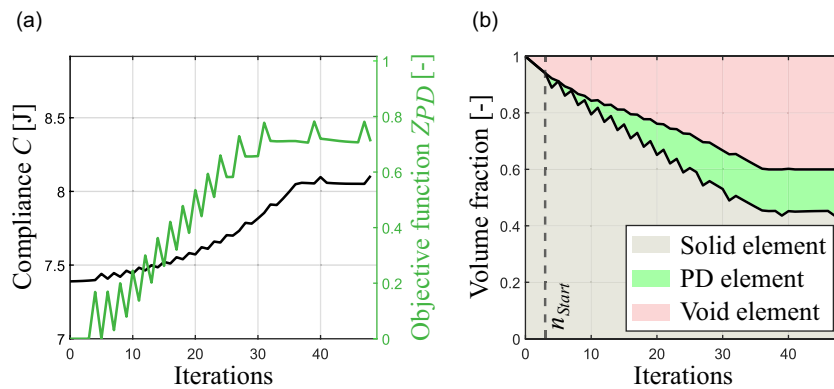


Figure 8. Iteration progression for the optimization of the GE Bracket. (a) Compliance and objective function, (b) volume fraction

implementation of these restrictions. This would not be possible when using the SIMP approach with continuous element densities. For the combination of the two algorithms, the distance to the set target was also introduced to prioritize one of the two optimizations. In addition to the sensitivity  $\beta_e$ , the threshold  $\alpha_{th}$  for highly loaded elements and the value  $n_{start}$  for the topology-only optimization iterations were also implemented.

It can be concluded that the decision criteria that determine whether the individual elements belong to the solid, void, or PD set are decisive for the quality of the result obtained. These decisions are based both on the criteria used and on the thresholds selected. While it is known from the literature that the sensitivity  $\alpha_e$  is suitable for weight and topology optimization, the results presented here show that the sensitivity  $\beta_e$  is suitable for generating particle dampers in the ideal area. The additionally introduced threshold  $\alpha_{th}$ , which is intended to prevent the change of highly loaded elements from the solid set to the PD set, has also proven to be useful for generating an efficient part structure. The alternating element assignment that can be clearly seen in the example of the GE-bracket can be explained by the fact that the threshold  $\alpha_{th}$  in combination with the selected evolutionary ratio leads to an overly large change in the topology. This makes it necessary to add solid elements again in the next step. In addition to adjusting the selected parameters, a hysteresis loop could also be used to set the threshold values.

The change in the natural frequencies is an expected behavior, as the topology of the part changes due to the optimization. The shift must be critically questioned insofar as the point of interest may shift from an operating point of the application. At the same time, other resonances that were not previously of interest may also shift into a critical range. The implementation of the value  $n_{start}$  for the definition of initial



topology-only optimizations has ensured that the definition of the considered eigenmodes only takes place later when the natural frequencies have already shifted.

## 5. Conclusions and Outlook

The development tool presented here can be used to successfully optimize additively manufactured parts in terms of their mass, stiffness, and damping. For this purpose, the methods for topology optimization available in the literature were extended with particle damping integration. This integration uses the results of an FE modal analysis to increase the part damping. Correlations that have been proven in the literature by experimental tests are used as the design criterion. The optimization tool was demonstrated using the GE Bracket as an example and the results were critically discussed. The integration of damping structures while at the same time optimizing the weight of the component is a novelty. The most important results can be summarized as follows:

- An optimization tool was developed to optimize the topology as well as the position and size of the cavities for particle damping.
- Based on a topology optimization algorithm from the literature, the tool was implemented using Abaqus and Python. Decision criteria were developed to ensure that the target was achieved.
- The design criterion of the algorithm for integrating the cavities is based on the relationship between damping and cavity displacement proven in the literature, from which the sensitivity  $\beta_e$  was derived.
- Restrictions to comply with the design guidelines were implemented.
- Using the GE Bracket as an application example, damping structures could be integrated automatically at the same time as topology optimization.

Various points can be considered for the further development of the tool: As the criteria and limit values that determine the assignment of an element to a set are decisive for the optimization result, these should be developed further. These include hysteresis loops to prevent jumps in element assignment behavior. Furthermore, the modes observed should be defined on the basis of a frequency range that corresponds to the critical operating point of the application. This ensures that the focus of the optimization remains on the relevant frequency range despite a frequency shift. In addition, design guidelines for additive manufacturing, such as critical downskin angles and overhangs, could be incorporated into the algorithm through restrictions. For further use in the product development process, part reconstruction of the mesh-based optimization result is necessary to carry out the detailed design and prepare it for additive manufacturing. Furthermore, the results of the optimization are to be validated using experimental investigations of additively manufactured components. A possible change in the damping effect over time must also be investigated.

## Acknowledgments

The project “Development methodology for laser powder bed fused lightweight structures with integrated particle dampers for vibration reduction” was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - Project number 495193504.

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