

CORRESPONDENCE.

APPROXIMATE CALCULATION OF JOINT-LIFE
ANNUITY-VALUES.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—In his interesting Note “On obtaining values of Life Annuities at isolated rates of interest”, by means of annuities-certain (*J.I.A.*, vol. liii, p. 171), Mr. E. H. Lever takes as his example No. 7 the calculation of the values of $a_{[40]:[28]}^{\text{F.M}}$ and $a_{[50]:[23]}^{\text{F.F}}$ at 5 per-cent, $O^{[6]}$ Table, having given the corresponding values at 3 per-cent and $3\frac{1}{2}$ per-cent; and he states that the examples are chosen so that the method may be compared with that given in the memorandum at the end of the British Offices Life Tables (p. 229 *et seq.*). The values found by his method are closer to the values given by a Hardy summation formula than those found by the official method of interpolated equal ages—see the British Offices Life Tables, pp. 231–2; and on the assumption that the values by Hardy’s formula are unlikely to differ from the true values by more than a small third difference error, Mr. Lever regards his results as showing that his own new method gives results more accurate on the whole than the official method. In making the assumption in question he has, I think, overlooked the remarks made on this subject by the late T. G. Ackland (*T.F.A.*, vol. iii, pp. 309–10), where he works out the same and other annuity-values by a method, due to the late G. F. Hardy, which is rigidly accurate except for very small errors in the last place due to first difference interpolations. Mr. Ackland remarks that the last mentioned method “gives results “which are practically identical with those deduced by Mr. Lidstone’s “approximate method” (*i.e.*, the method described in the memorandum at the end of the British Offices Life Tables); “and it “will, I think, also be found that the results by both methods are “practically exact.” Mr. Ackland points out that “the formula of “approximate summation employed in this comparison (probably “Mr. G. F. Hardy’s formula 39*a*) is not suitable for the calculation “of *select* annuity-values, because none of the terms of the formula

“(excepting u_0) falls within the period of selection.” Mr. Ackland tested the matter by calculating the annuity-values in question by G. F. Hardy’s extended formula 38, employing 15 and 17 ordinates respectively, and obtained results very close to those found by the method he was describing and to those found by the official method. In order to test the matter still more closely I have had the annuity-values calculated at length by the formula $\sum v^t \cdot t p_{xy}$, taking the calculations to 5 decimal places; and the following is a summary of the various results obtained :

Value	$a_{[40]:[28]}^{\text{F M}}$	$a_{[50]:[23]}^{\text{F F}}$
True	12·605	11·562
By official method (interpolated equal ages)	12·603	11·565
„ Mr. Lever’s method	12·590	11·554
„ Hardy’s method as discussed by Ackland	12·606	11·562
„ Hardy’s extended summation formula 38, as given by Ackland	12·608	11·561

These results show that the official method, in the cases here tested, give results virtually correct in the 3rd decimal place, and do not support Mr. Lever’s remark that the new method gives on the whole more accurate results. It seems desirable to correct this misunderstanding, as the method of interpolated equal ages, for which I was responsible, has been officially adopted in the British Offices Life Tables; but these remarks are not made by way of criticism of Mr. Lever’s new method, which is ingenious, and no doubt quite sufficiently accurate for most practical purposes; while, as he points out, it has the general advantage that it does not depend for its application upon the existence of a complete table of joint life annuities at equal ages at the rate of interest at which values are required.

I am, Sirs,

Your obedient Servant,

G. J. LIDSTONE.

Edinburgh,

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