Appendix E: Two-body scattering in the LAB frame

The relations between the kinematic variables are much more complicated in the LAB frame than they are in the CM frame. The exact transformation equations depend critically on the actual masses involved as well as on the relative values of a particle's velocity and the velocity of the CM system in the LAB [1]. The type of transformation is determined by the parameter

$$g_1^* = \frac{s + m_1^2 - m_2^2}{s - m_b^2 + m_t^2} \frac{\lambda^{1/2}(s, m_b^2, m_t^2)}{\lambda^{1/2}(s, m_1^2, m_2^2)} \qquad \text{(E.1)}$$

and the analogous parameter g_2^* obtained by interchanging the subscripts 1 and 2 in this equation. Recall that b and t refer to the beam and target particles, 1 and 2 refer to the two final state particles, and s is the square of the energy in the CM frame. The function $\lambda(a, b, c)$ was defined in Eq. 1.25. Particles with $g^* < 1$ can be emitted with any polar angle $(0 < \theta < 180°)$, whereas particles with $g^* \geqslant 1$ can only be emitted in the forward hemisphere $(0 < \theta < \theta_{\max} \leqslant 90°)$.

In the case of elastic scattering with $m_b = m_1 = \mu$, $m_t = m_2 = m$, and $\mu \leqslant m$, Eq. E.1 becomes

$$g_1^* = \frac{s + \mu^2 - \mathrm{m}^2}{s - \mu^2 + \mathrm{m}^2} \leqslant 1 \qquad g_2^* = 1 \qquad \text{(E.2)}$$

Since $g_2^* = 1$, the recoil particle is confined to the forward hemisphere in the LAB. The forward particle can be emitted in any direction in the LAB unless $\mu = m$, in which case it is also confined to the forward hemisphere. It is also possible to give explicit relations between p_i and θ_i and between θ_1 and θ_2 [1].

Reference

[1] E. Byckling and K. Kanjantie, *Particle Kinematics,* New York: Wiley, 1973.