

sécantes données. Chacune de ces droites passe aussi par un des points d'intersection des parallèles aux asymptotes menées par les extrémités de chacune des cordes  $CB'$  et  $BC'$ .

On peut appliquer cette propriété à la construction des hyperboles.

La méthode de transformation des figures par polaires réciproques permet de déduire des théorèmes précédents autant de théorèmes corrélatifs sur les tangentes aux coniques.

### Note on the regular solids.

By Professor STEGGALL.

The usual methods of proving the existence of regular polyhedra, as given in Wilson and in Todhunter, appear to most students somewhat difficult. It seemed worth while trying, therefore, whether a simpler or more direct proof could not be obtained. The following note shows how this may be done.

The problem is to distribute  $p$  points uniformly on a sphere. Suppose that they are arranged in  $n$ -sided polygons, and that each angle is  $2\pi/n$ , then the number of polygons  $= pm/n$ , and their total area is  $pm(2n\pi/m + 2\pi - n\pi)/n = 4\pi$ , the radius of the sphere being unity.

$$\text{Hence} \quad \frac{2}{m} + \frac{2}{n} - 1 = \frac{4}{pm}$$

of which the only admissible solutions are  $m = n = 3$ ;  $m = 3$ ,  $n = 4$  or  $5$ ;  $m = 4$  or  $5$ ,  $n = 3$ ; giving all the five figures.

### A certain cubic connected with the triangle.

By Professor STEGGALL.

In examining some of the lines that occur in connection with the recent geometry of the triangle, the cubic whose equation in trilinear co-ordinates is

$$abc(\beta^2 - \gamma^2) + \beta ca(\gamma^2 - \alpha^2) + \gamma ab(\alpha^2 - \beta^2) = 0$$

incidentally appeared, and it seems worth noting the very large number of special points it passes through. These are the vertices, the mid-points of the sides, the inscribed and escribed centres, the circumcentre, the orthocentre, the centroid and the symmedian points, or fourteen in all.

The tangents to this cubic are also interesting: that at a vertex is a symmedian line; those at the inscribed and escribed centres pass through the centroid; that at the centroid through the symmedian point.

Two other cubics also appeared, determined thus:—Let PD, PE, PF be drawn perpendicular to BC, CA, AB; then if AD, BE, CF meet in Q, we have the following loci of P and Q respectively in trilinear co-ordinates:

$$\begin{aligned} & \alpha(\beta^2 - \gamma^2)(\cos A - \cos B \cos C) \\ & + \beta(\gamma^2 - \alpha^2)(\cos B - \cos C \cos A) \\ & + \gamma(\alpha^2 - \beta^2)(\cos C - \cos A \cos B) = 0; \end{aligned} \quad (1)$$

and

$$\begin{aligned} & a \cos A (b^2 \beta^2 - c^2 \gamma^2) \\ & + \beta \cos B (c^2 \gamma^2 - a^2 \alpha^2) \\ & + \gamma \cos C (a^2 \alpha^2 - b^2 \beta^2) = 0. \end{aligned} \quad (2)$$

These two cubics agree with the first cubic discussed when  $a = b = c$ , as is otherwise clear: the cubic (1) passes through the vertices, the inscribed and escribed centres, the circumscribing centre, the orthocentre.