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Connections Between Metric Characterizations of Superreflexivity and the Radon–Nikodým Property for Dual Banach Spaces

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Abstract. Johnson and Schechtman (2009) characterized superreflexivity in terms of finite diamond graphs. The present author characterized the Radon–Nikodým property (RNP) for dual spaces in terms of the infinite diamond. This paper is devoted to further study of relations between metric characterizations of superreflexivity and the RNP for dual spaces. The main result is that finite subsets of any set M whose embeddability characterizes the RNP for dual spaces, characterize superreflexivity. It is also observed that the converse statement does not hold and that $M = \ell_2$ is a counterexample.

1 Introduction

Results of [16, 24] indicate the existence of some parallels between metric characterizations of superreflexivity and metric characterizations of dual spaces with the Radon–Nikodým property (RNP).

To state the corresponding results we recall the definition of the infinite diamond. The *diamond graph* of level 0 is denoted D_0. It has two vertices joined by an edge of length 1. D_n is obtained from D_{n-1} as follows. Each edge of D_{n-1} is of length $2^{-(n-1)}$. Given an edge $uv \in E(D_{n-1})$, it is replaced by a quadrilateral u, a, v, b with edge lengths 2^{-n}. We endow D_n with its shortest path metric. We consider the vertex set of D_n as a subset of the vertex set of D_{n+1}; it is easy to check that this defines an isometric embedding. We introduce D_ω as the union of the vertex sets of $\{D_n\}_{n=0}^\infty$. For $u, v \in D_\omega$ we introduce $d_{D_\omega}(u, v)$ as $d_{D_n}(u, v)$, where $n \in \mathbb{N}$ is any integer for which $u, v \in V(D_n)$. Since the natural embeddings $D_n \to D_{n+1}$ are isometric, $d_{D_n}(u, v)$ does not depend on the choice of n for which $u, v \in V(D_n)$. To the best of my knowledge the first paper in which diamond graphs $\{D_n\}_{n=0}^\infty$ were used in metric geometry is [13] (a conference version was published in 1999).

Theorem 1.1 ([16]) *A Banach space X is nonsuperreflexive if and only if it admits bilipschitz embeddings with uniformly bounded distortions of diamonds $\{D_n\}_{n=1}^\infty$ of all sizes.*

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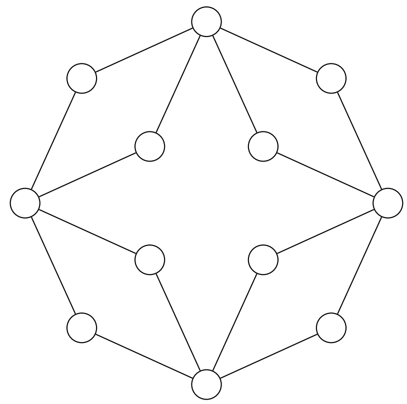

Figure 1: Diamond D_2.

Theorem 1.2 ([24]) *A dual Banach space does not have the* RNP *if and only if it admits a bilipschitz embedding of* D_ω.

Remark 1.3 It is known ([22]) that for Banach spaces that are not dual spaces, lack of the RNP does not imply embeddability of D_ω. (See [24] for more results of this type.)

Theorems 1.1 and 1.2 make it natural to try to understand whether similar results hold for other than D_ω separable metric spaces and their finite subsets. In this note we prove that in one of the directions this is true. Recall (see [30] and references therein) that a dual of a separable Banach space has the RNP if and only if it is separable. We prove the following theorem.

Theorem 1.4 *If a metric space M admits a bilipschitz embedding into any nonseparable dual of a separable Banach space, then each of its finite subsets embeds into an arbitrary non-superreflexive Banach space with uniformly bounded distortions.*

The implication in the other direction does not hold in general. We have the following result, which is proved in Section 3.

Proposition 1.5 *There exist a separable metric space M and a separable Banach space X with nonseparable dual X^* such that finite subsets of M admit embeddings into an arbitrary non-superreflexive Banach space with uniformly bounded distortions, but M does not admit a bilipschitz embedding into X^*.*

The Hilbert space ℓ_2 is an example of an M satisfying the conditions of Proposition 1.5.

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Remark 1.6 It is worth mentioning that the Hilbert space ℓ_2 is, up to an isomorphism, the only Banach space the finite subsets of which admit embeddings into an arbitrary non-superreflexive Banach space with uniformly bounded distortions. In fact, by results of James [15] and Pisier–Xu [28] there exist nonsuperreflexive spaces of type 2. It is well known that there exist nonsuperreflexive spaces of cotype 2 (for example, ℓ_1). On the other hand, Bourgain's discretization theorem [5, 12] implies that uniform bilipschitz embeddability of finite subsets implies the existence of uniformly isomorphic embeddings of finite-dimensional subspaces. Therefore, each Banach space satisfying the conditions of Proposition 1.5 has type 2 and has cotype 2; hence, by the Kwapień theorem [17], it is isomorphic to a Hilbert space.

Definition 1.7 Let X and Y be two Banach spaces. The space X is said to be *finitely representable* in Y if for any $\varepsilon > 0$ and any finite-dimensional subspace $F \subset X$ there exists a finite-dimensional subspace $G \subset Y$ such that $d(F, G) < 1 + \varepsilon$, where $d(F, G)$ is the Banach–Mazur distance.

The space X is said to be *crudely finitely representable* in Y if there exists $1 \leq C < \infty$ such that for any finite-dimensional subspace $F \subset X$ there exists a finite-dimensional subspace $G \subset Y$ such that $d(F, G) \leq C$.

Theorem 1.4 is an immediate consequence of the following result, which is proved in the next section.

Theorem 1.8 *For each non-superreflexive Banach space X there exists a non-separable dual Z^* of a separable Banach space Z, such that Z^* is crudely finitely representable in X.*

We refer to [3, 20, 21, 23, 27] for background material and presentations of some of the results used below.

2 Proof of Theorem 1.8

First we consider the case where X has no nontrivial type. In such a case, ℓ_1 is finitely representable in X (by the result of [26]), and therefore $Z = C(0, 1)$ satisfies the conditions of Theorem 1.8. In fact, it is clear that $(C(0, 1))^*$ is nonseparable. It is also known (see *e.g.*, [20, Section 5.b]) that $(C(0, 1))^*$ is finitely representable in ℓ_1.

Now we consider the case where X has nontrivial type. Replacing X, if necessary, by a nonreflexive space finitely represented in it, we can assume that X is nonreflexive. The following notion, introduced by Brunel and Sucheston, turned out to be very useful in the study of nonreflexive spaces with nontrivial type.

Definition 2.1 ([7, p. 84]) A sequence $\{e_n\}$ in a semi-normed space is called *equal signs additive* (ESA) if for any finitely non-zero sequence $\{a_i\}$ of real numbers such that $\operatorname{sign} a_k = \operatorname{sign} a_{k+1}$, the equality

$$\Big\| \sum_{i=1}^{k-1} a_i e_i + (a_k + a_{k+1}) e_k + \sum_{i=k+2}^{\infty} a_i e_i \Big\| = \Big\| \sum_{i=1}^{\infty} a_i e_i \Big\|$$

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holds.

Theorem 2.2 ([7]) *For each nonreflexive space X there is a Banach space E with an ESA basis that is finitely representable in X.*

Since this theorem is not explicitly stated in [7], we describe how to get it from the argument presented there. By [29], there is a sequence $\{x_i\}_{i=1}^{\infty}$ in B_X (the unit ball of X) satisfying the condition

$$f_n(x_k) = \begin{cases} \theta & \text{if } n \le k, \\ 0 & \text{if } n > k, \end{cases}$$

for some $0 < \theta < 1$ and some $\{f_i\}_{i=1}^{\infty} \subset B_{X^*}$. Following [6, Proposition 1] we build the spreading model $\widetilde{X}$ on the sequence $\{x_i\}$ (the term *spreading model* was not used in [6], but was introduced later; see [2, p. 359]). The natural basis $\{e_i\}_{i=1}^{\infty}$ in $\widetilde{X}$ is *invariant under spreading* (IS) in the sense that

$$\Big\| \sum_i \alpha_i e_{k_i} \Big\| = \Big\| \sum_i \alpha_i e_i \Big\|$$

for each strictly increasing sequence $\{k_i\}$ of positive integers. The space $\widetilde{X}$ is finitely representable in X; see [7, p. 83]. Now we use the procedure described in [7, p. 84], to get a Banach space E that is finitely representable in $\widetilde{X}$ and has an ESA basis. (Actually, the fact that we get a basis was not verified in [7]; this was done in [8, Proposition 1]).

Since the space E has nontrivial type, it follows from results of [8, Lemma 3, p. 290] that this basis is boundedly complete, and hence E is isomorphic to a dual space (see [21, Proposition 1.b.4]).

Remark 2.3 It would be interesting to show that E is isometric to a dual space. Then we would be able to omit the word "crudely" from the statement of Theorem 1.8.

Let R be a Banach space such that R^* is isomorphic to E. We construct the desired space Z as a transfinite dual of R. Transfinite duals were introduced in [9]. Let us recall the definition. We denote the n-th dual ($n \in \mathbb{N}$) of a Banach R by $R^{(n)}$. We say that an ordinal α is *even* if it is either a limit ordinal or an ordinal of the form $\beta + 2n$ where β is a limit ordinal and $n \in \mathbb{N}$. We define $R^{(\alpha)}$ by transfinite induction:

- $R^{(\alpha+1)} = (R^{(\alpha)})^*$.
- If α is a limit ordinal, we let $R^{(\alpha)}$ be the completion of the union

$$\bigcup_{\substack{\beta<\alpha \\ \beta \text{ is even}}} R^{(\beta)}.$$

(Observe that the union is well defined as a normed linear space, since $R^{(\beta)}$ admits a canonical isometric embedding into $R^{(\gamma)}$ if $\beta < \gamma$ and both β and γ are even.)

To complete the proof of Theorem 1.8 we prove the following two statements:

(a) The space $R^{(\omega^2+1)}$ is crudely finitely representable in E (and thus in X).

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(b) The space $R^{(\omega^2)}$ is separable and the space $R^{(\omega^2+1)}$ is nonseparable.

Statement (a) is an immediate consequence of the following lemma.

Lemma 2.4 *Let R be a Banach space and let $R^*(= R^{(1)})$ be its dual. Then $R^{(\gamma)}$ is finitely representable in R^* for every odd ordinal γ.*

Proof For finite ordinals, this result is an immediate consequence of the local reflexivity principle [18]. The same principle implies that if the statement is true for an infinite odd ordinal γ, then it is true for all ordinals of the form $\gamma + 2n$. So using the transfinite induction, it remains to show that the statement holds for ordinals of the form $\gamma = \alpha + 1$, where α is a limit ordinal, provided it holds for all smaller odd ordinals.

We have

$$R^{(\alpha)} = \mathrm{cl}\Big(\bigcup_{\substack{\beta<\alpha \\ \beta \text{ is even}}} R^{(\beta)} \Big).$$

Let F be a finite-dimensional subspace of $R^{(\alpha+1)}$, $\varepsilon > 0$, and let $\{f_i\}_{i=1}^k$ be a finite $\frac{\varepsilon}{2}$-net in S_F (the unit sphere of F). For each f_i we can find an even ordinal $\beta_i < \alpha$ and a vector $x_i \in R^{(\beta_i)}$ such that $\|x_i\| = 1$ and $f_i(x_i) \geq 1 - \frac{\varepsilon}{2}$. Let $\tau = \max_{1\leq i\leq k} \beta_i$. Then the natural restriction of F to the space $R^{(\tau)}$ is an ε-isometry, hence F is ε-isometric to a subspace in $R^{(\tau+1)}$, and the induction hypothesis implies that $R^{(\alpha+1)}$ is finitely representable in R^*. ■

To show that $R^{(\omega^2)}$ is separable, it suffices to show that $R^{(\omega n)}$ is separable for each n. This can be shown by a straightforward induction based on the following results.

Theorem 2.5 ([25, Theorem 16]) *If X is quasireflexive, then $X^{(\omega)} = X \oplus [x_i]$, where $\{x_i\}$ is an ESA basis.*

Theorem 2.6 ([8, Theorem 3]) *If a Banach space with an ESA basis has nontrivial type, then it is quasireflexive.*

The fact that $R^{(\omega^2+1)}$ is nonseparable was proved by Bellenot [4]. Since the details of the argument of Bellenot are difficult to follow, we note that this result can be derived using the argument of Davis and Lindenstrauss [10, Theorem 4]. Let us mention the modification of the argument of [10] needed to achieve this goal. To understand the discussion below the reader should be familiar with the very elegant proof in [10, pp. 194–196].[1]

We build the collections $x_{(\sigma,n)}$ and $f_{(\sigma,n)}$ in the way described in [10, pp. 194–195]. Then, for each σ in the Cantor set Δ, we pick a sequence $\{\sigma_j\}_{j=1}^{\infty}$ of end points in Δ so that $\sigma_j \to \sigma$ and let $F_\sigma \in R^{(\omega^2+1)}$ be any weak* limit point of the sequence $\{f_{(\sigma_j,1)}\}_{j=1}^{\infty}$ in $R^{(\omega^2+1)}$. We claim that $\|F_\sigma - F_\tau\| \geq \frac{1}{2}$ for each $\sigma, \tau \in \Delta$, $\sigma < \tau$, because we can find a λ that is an end point of Δ and satisfies $\sigma < \lambda < \tau$. But then,

[1] We would like to mention that there are two misprints in [10] on page 195, line 15: $f_{(\sigma,n)}$ should be $f_{(0,n)}$ and $f_{(0,n)}$ should be $f_{(1,n)}$.

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as is easy to check, for any $n \in \mathbb{N}$ we have $F_\sigma(x_{(\lambda,n)}) = 1$ and $F_\tau(x_{(\lambda,n)}) = 0$. Since $\|x_{(\lambda,n)}\| \le 2$, the conclusion follows. ■

3 Proof of Proposition 1.5

The fact that finite subsets of ℓ_2 admit embeddings into arbitrary non-superreflexive Banach spaces with uniformly bounded distortions is an immediate consequence of the Dvoretzky theorem [11].

As an example of a suitable space X, we use the James tree space (see [14, 19]), but built on ℓ_p with $p \in (2,\infty)$. More precisely, we follow the construction of [19, Section 2]. So we consider an infinite binary tree T_∞ whose vertices can be labelled with finite sequences of zeros and ones (including the empty sequence) with the norm

$$\|x\| = \sup\Big(\sum_{j=1}^{k}\Big(\sum_{\nu\in\mathcal{J}_j} x(\nu)\Big)^p\Big)^{\frac{1}{p}} < \infty,$$

where the supremum is taken over all choices of k and of pairwise disjoint finite descending paths $\mathcal{J}_1,\dots,\mathcal{J}_k$ in the tree T_∞. Denote by B the closed linear span in $(JT_p)^*$ of the biorthogonal functionals $\{e^*_\nu\}$ of the unit vector basis $\{e_\nu\}$ of JT_p.

In the same way as in [19, Section 2] one can establish the following results:

(a) JT_p is naturally isomorphic to B^*.

(b) The quotient of $(JT_p)^*$ with the kernel B is isometric to $\ell_q(\Gamma)$, where Γ is a set of cardinality continuum and $\frac{1}{q} + \frac{1}{p} = 1$.

(c) The space $(JT_p)^*$ is a nonseparable dual of a separable Banach space.

It remains to prove that $(JT_p)^*$ does not admit a bilipschitz embedding of ℓ_2. In fact, otherwise, by [3, Corollary 7.10], it would contain a linear isomorphic image of ℓ_2. Let us show that this implies that B contains a sequence $\{b_i\}_{i=1}^\infty$ equivalent to the unit vector basis of ℓ_2. Recall that $\ell_q(\Gamma)$ with $q \in (1,2)$ is totally incomparable with ℓ_2 (see [21, p. 75] for the definition and proof). Therefore the restriction of the quotient map $\varphi\colon (JT_p)^* \to \ell_q(\Gamma)$ with $\ker\varphi = B$ to a subspace H isomorphic to ℓ_2 is strictly singular. Hence (see [21, Proposition 2.c.4]), H contains a normalized sequence $\{a_i\}_{i=1}^\infty$ satisfying

$$\forall\{\alpha_i\}_{i=1}^\infty \subset \mathbb{R} \quad c\Big(\sum_i \alpha_i^2\Big)^{\frac{1}{2}} \le \Big\|\sum_i \alpha_i a_i\Big\| \le C\Big(\sum_i \alpha_i^2\Big)^{\frac{1}{2}}$$

for some $0 < c < C < \infty$, and such that $\|a_i - b_i\| \le \frac{c}{C2^{i+1}}$ for some sequence $\{b_i\}_{i=1}^\infty$ in B. It is easy to check that the sequence $\{b_i\}_{i=1}^\infty$ is equivalent to the unit vector basis of ℓ_2 (see [21, Proposition 1.a.9]).

We show that the existence of such a sequence $\{b_i\}_{i=1}^\infty$ in B leads to a contradiction. Clearly we can assume that $\{b_i\}_{i=1}^\infty$ is disjointly supported with respect to the basis $\{e^*_\nu\}$. Let $\{b^*_i\}_{i=1}^\infty \subset JT_p$ be a bounded sequence satisfying $b^*_i(b_i) = 1$. The sequence $\{b^*_i\}$ can also be assumed to be disjointly supported. By [19, Corollary 3] (see also [19, Proposition on p. 91]), we can assume that $\{b^*_i\}$ is weakly Cauchy. Then the sequence $\{b^*_{2k} - b^*_{2k-1}\}_{k=1}^\infty$ is weakly null. Using a straightforward generalization of [1, Theorem, p. 420] we get that $\{b^*_{2k} - b^*_{2k-1}\}_{k=1}^\infty$ contains a subsequence equivalent to the unit vector basis of ℓ_p. We assume that $\{b^*_{2k} - b^*_{2k-1}\}_{k=1}^\infty$ is equivalent to the

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unit vector basis of ℓ_p. Then, as is easy to see, we get that for some constant $c > 0$ and any finitely non-zero sequence $\{\alpha_k\}$ we have

$$\Big\| \sum_k \alpha_k b_{2k} \Big\| \geq c \Big(\sum_k \alpha_k^q \Big)^{\frac{1}{q}}.$$

Since $q \in (1, 2)$, this contradicts to the assumption that $\{b_n\}$ is equivalent to the unit vector basis of ℓ_2. ■

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