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ARTICLE

How to Engineer a Quantum Wavefunction

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Abstract

In a conventional experiment, scientists typically aim to learn about target systems by manipulating source systems of the same *material type*. In an analogue quantum simulation, by contrast, scientists typically aim to learn about target quantum systems of one material type via an experiment on a source quantum system of a different material type. In this article, we argue that such inferences can be justified by reference to source and target quantum systems being of the same *empirical type*. We illustrate this novel experimental practice of wavefunction engineering with reference to the example of Bose–Hubbard systems.

1. Introduction

Modern experimental practice allows for a staggering degree of control over lab-based quantum systems. This high level of control operates in terms of both the precision with which quantum systems can be probed and the range of scales of components that can be manipulated: from thousands of ultracold atoms controlled using arrays of laser beams to individual ions that can be electronically trapped. The potential implications of such quantum technology are powerful, wide ranging, and radical. In this article, our focus is on the particular context of *analogue quantum simulation* in which a well-controlled quantum system in the lab is specifically deployed by scientists to learn about features of another quantum system to which they do not have direct access (Dardashti, Thébault, and Winsberg 2017; Dardashti et al. 2019; Thébault 2019; Crowther, Linnemann, and Wüthrich 2019; Evans and Thébault 2020; Hangleiter, Carolan, and Thébault 2022; Field forthcoming; Bartha 2022).

On what basis should we categorize different physical systems as tokens of the same type? One option is to distinguish types of physical systems by their material constitution, focusing on properties like masses, atomic constitution, geometry, charges, interactions, and the like that designate the detailed physical properties of the system. Call this the *material-type* view. The second option is to distinguish types of physical systems by structural similarity in empirical behavior. In particular, we could take any two physical systems to be of the same type when, in some specified parameter regime, a set of experimental prescriptions results in appropriately similar measurement outcomes. Call this the *empirical-type* view.

The relevance of the distinction between material-type and empirical-type views arises in the context of analogue experiments in which a *source* system is manipulated in the lab with the aim of gaining understanding of a *target* system that is not directly manipulated. Significantly, the form of justification for the source–target inferences involved in analogue simulation is sensitive to how widely we draw the category of types of physical systems. When the material-type view is assumed, we find that analogue simulations by definition involve a novel form of intertype uniformity reasoning requiring justification by way of "universality" arguments.[1] However, by contrast, when the empirical-type view is assumed, a more conventional form of intratype uniformity reasoning is applied, albeit with an atypical notion of type.

The key distinction between these two ways of reconstructing the inferential practices underpinning analogue simulation is that they lead to differences in the conditions that limit the strength of inductive support for conclusions about the target system based on experimental manipulation of the source system. In particular, in the context of the material-type view, and any associated intertype uniformity reasoning requiring justification by way of universality arguments, Field (forthcoming) has convincingly argued that inferentially strong conclusions require either (1) detailed knowledge of the microstructure of the source and target or (2) empirical evidence for the applicability of relevant universality arguments via empirical access to the macrobehavior of the source and target. Correspondingly, in the context of the empirical-type view, on our account, one is licensed to draw inferentially strong conclusions regarding the target system in a context where we have (i) empirical evidence of the validity of the respective models via detailed knowledge of the microstructure of the source and target and (ii) empirical evidence for membership of the same empirical type via empirical access to the macrobehavior of the source and target.

In each case, a contrast in terms of strength of inference can be made. On one hand, in exotic examples of analogue simulation, such as analogue Hawking radiation, conditions like (1) and (2) can be expected to fail, because for the target system, we have the combination of inaccessibility and lack of reliable theories of microstructure. On the other hand, even in the case in which the conditions do obtain, we do not generically expect our inferences to meet the gold evidential standards found in systematic, direct experimentation. The empirical-type view, therefore, allows us to understand how a moderate level of evidential support can accrue for hypotheses regarding a target system in an analogue simulation in contexts in which the target system is experimentally accessible in some regime and thus in which the reliability of models of the target system, in some parameter regime, can be established. The aim of the analogue simulation is thus to probe the behavior of an accessible system in an

[1] The sense of universality here is a broad one: two systems may be of the same "universality type" in this sense without being in the same "universality class" in the Wilsonian sense (cf. Batterman 2019; Gryb, Palacios, and Thébault 2021).

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inaccessible regime based on manipulation of a further system of the same empirical type. Though such a pattern of inference is implicit in a wide range of scientific discussions, it has as yet not been subject to explicit philosophical analysis.

In this article, we argue that the scientific practice of analogue quantum simulation provides a compelling example in which the empirical-type view allows for inductive arguments toward inferences about the behavior of an accessible target system in an inaccessible regime. The fact that both systems are adequately modeled within the framework of quantum theory allows us to run a "bootstrapping" inference whereby the general empirical support for the "quantumness" of source and target is combined with direct empirical evidence of the applicability of an idealized quantum model to the target in an accessible regime, toward the inference of applicability of the target model to a broader inaccessible regime. The two ingredients in the justification of this bootstrapping thus directly correspond to specific realizations of (i) and (ii). First, we have a specific, empirical premise based on the experimental manipulation of the target system in the accessible regime, labeled (H) in what follows. Second, we have a broad, theoretical-empirical premise based on the assumed applicability of quantum theory to the target system, labeled (Q).

Through (Q), our argument pattern makes crucial use of quantum theory as a generalized framework that underlies the modeling of quantum systems. It is for this reason that we characterize the relevant experimental practice as *wavefunction engineering*. Furthermore, our argument employs a quantum uniformity principle, which can be understood as a meta-principle for this kind of modeling practice. At a high level, our argument makes use of the deidealization of a single idealized quantum model to both source and target system models and, in so doing, provides justification for reasoning based on regularity within empirical types. Although we do not claim that such empirical-type-regularity-based reasoning renders inferences about the target system in an analogue quantum simulation on a par with inferences in the context of conventional experiments, we do argue that the inferences in analogue quantum simulations command stronger inductive support than those in which the target system is inaccessible and the relevant target system model is subject to entirely theoretical support.

2. A case study of analogue quantum simulation: Bose–Hubbard physics

Successful analogue quantum simulation requires that the source and target system models be deidealizations of a single theoretical model in some appropriate regime of idealization (Hangleiter, Carolan, and Thébault 2022). The key to the simulation is that the source system can be controlled more easily than the target system, and so an experiment on the source system can probe elements of the target system that are experimentally inaccessible, given that the idealized model is appropriately verified.

One class of dynamical systems particularly ripe for modeling in analogue quantum simulation experiments comprises those that conform to the Bose–Hubbard model, which describes the dynamics of a lattice of interacting bosons. The Bose–Hubbard model was first derived by Gersch and Knollman (1963) in the context of granular superconductors—a special case of so-called type II superconductors. However, it was the discovery of *quantum phase transitions* at zero temperature between a superconducting and an insulating phase in granular superconductors that

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sparked theoretical and experimental interest in the model (Bruder, Fazio, and Schön 2005, 566). This led to the experimental investigation of other systems described by the Bose–Hubbard Hamiltonian, including thin helium films and arrays of superconductors connected by Josephson junctions.

Such implementations of the Bose–Hubbard model are engineered systems with extraordinary phase behavior that display close similarity to the behavior of natural type II superconductors. Type II superconductors are characterized by an atypical intermediate phase between their insulating and superconducting phases, an analogue of which is observed in the behavior of thin helium films, and by the formation of magnetic vortices when an external magnetic field is applied, which is observed in Josephson junction arrays. A precise understanding of superconductors is crucial for a broad range of technological applications.

Remarkably, it was found that bosonic atoms loaded into an optical lattice potential created using laser light are also described by the Bose–Hubbard model (Jaksch et al. 1998). The experimental accessibility of this system allows a great range of experimental investigations of Bose–Hubbard dynamics that is inaccessible by other means. The potential of cold atoms as an analogue simulation platform was experimentally actualized with the observation that they undergo the same phase transition at zero temperature between a superfluid and an insulator phase (Greiner et al. 2002). The phase transitions in these very different systems are underpinned by the same physical principles, that is, the "competition between the trend to global coherence, due to the hopping of bosonic particles, and the tendency towards localization induced by the strong interactions" (Bruder, Fazio, and Schön 2005, 567).

More specifically, the Bose–Hubbard model is characterized by the Hamiltonian

$$H_{\mathrm{BH}} = -J\sum_{\langle j,k\rangle}\left(b_j^\dagger b_k + b_k^\dagger b_j\right) + U\sum_j b_j^\dagger b_j^\dagger b_j b_j + \sum_j \mu_j b_j^\dagger b_j, \tag{1}$$

where the bosonic creation and annihilation operators $b_j^\dagger$ and b_j represent atoms at lattice site j, and the different terms represent the energy gain J when atoms hop between neighboring sites, the energy cost U of two atoms at the same site, and the energy offset μ_j of each lattice site. Zero-temperature or *quantum* phase transitions can be understood as the transition between regimes in which one of J or U dominates the ground state of the model. When J dominates, hopping behavior is much more likely to occur, and so the ground state consists of delocalized bosons across the lattice. This is the superfluid phase. When U dominates, a strong local repulsion between atoms occupying the same lattice site prevents global coherence. This is the Mott insulator phase (Bruder, Fazio, and Schön 2005, 567).

Cold-atom bosonic systems in an optical lattice are accessible to experimental manipulation and probing of a sort not possible for its potential target systems. A cold-atom system is typically constructed by employing counterpropagating lasers combined with a magneto-optical trap to form a space-dependent lattice potential, which is used as a location grid in which ultracold atoms, such as ^{87}Rb, can be positioned. This system is accurately described by the Bose–Hubbard Hamiltonian (Jaksch et al. 1998) in a parameter regime where (1) next-nearest-neighbor hopping and nearest-neighbor repulsion are negligible, (2) the spatial extent of the wavefunction of each oscillator ground state matches the dimensions of the lattice

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wells, and (3) the on-site interaction strength is sufficiently small for the number of particles per site. Importantly, all the model parameters can be manipulated by varying an external magnetic field and the amplitude and phase of the lasers generating the lattice potential (Hangleiter, Carolan, and Thébault 2022, 33). Thus the zero-temperature phase transition of the system can be controlled. What is more, location and momentum information of the atoms in the lattice can be measured with remarkable precision (Bruder, Fazio, and Schön 2005).

2.1. The target systems

We present here three potential target systems for analogue quantum simulation, the set of which demonstrates the variety of material physical systems that can be targeted by the cold-atom source system.

2.1.1. Superfluid ^{4}He in Vycor

Vycor is a specially manufactured high-silica glass. When manufactured as a porous structure, it is an ideal substrate for the study of confined liquids in condensed-matter physics. Helium-4 adsorbed in Vycor is observed to form a superfluid: it behaves as an interacting ideal Bose gas that typically results from the formation of a Bose–Einstein condensate (BEC) (Reppy 1984). Because the Bose–Hubbard model describes an interacting Bose gas in a lattice that behaves as a superfluid below some critical temperature, one would expect superfluid ^{4}He in Vycor to conform to Bose–Hubbard model behavior. Indeed, this system is typically modeled using the XXZ model, which is a special case of the Bose–Hubbard model in the limit of large on-site interaction strength $U \gg 1$ (van Otterlo et al. 1995), including, potentially, interactions between photons on different sites. In this "hard-core boson" limit, no two bosons are allowed to occupy the same lattice site.

At large ^{4}He densities, a conventional phase transition between a superfluid phase and a Mott insulator phase is observed at finite temperature (Fisher et al. 1989). The critical temperature, T_c, at which this phase transition occurs decreases with the density ρ of ^{4}He, reaching $T_c = 0$ at some positive density $\rho_c(T = 0)$. At zero temperature, the system then undergoes a transition from a Mott-insulating state to a superfluid as the density ρ crosses $\rho_c(T = 0)$. In addition, the phase behavior of ^{4}He adsorbed in Vycor exhibits an intermediate "Bose glass" phase, analogous to the intermediate phase of a type II superconductor. Along with subsequent observations (Weichman 2008), the quantum phase transition behavior constitutes empirical evidence that the Bose–Hubbard model with density-dependent hopping and interaction parameters is a valid characterization of the system within this parameter regime. As a result, this behavior is structurally and formally similar to the zero-temperature superfluid–insulator phase transition of the ^{87}Rb atoms in the optical lattice.

2.1.2. Triplons in quantum dimer magnets

Typical magnetic materials consist of an ordered arrangement of magnetic spin states. For "spin dimer compounds," pairs of spin states couple and, owing to the crystalline structure of the material, interact only weakly with other coupled spin states. These weakly interacting "dimers" generate a paramagnetic "spin-liquid" ground state in the material comprising local entangled spin singlet states, with an excitation gap to an excited triplet state. When a high-strength magnetic field is applied to the material, Zeeman splitting of the triplet state closes the excitation gap, and the entangled spin singlets transition to the excited triplet state and the material to a magnetically ordered state.

The dimers in such systems behave as "bosonic quasiparticles" and, when excited by a magnetic field, are known as *triplons* (Nohadani, Wessel, and Haas 2005, 1). The phase transition from the paramagnetic phase to the ordered phase can be described as the formation of a BEC. In the appropriate parameter regime, the critical temperature of the transition vanishes, and so this phase transition is analogous to the zero-temperature transition from a Mott-insulating phase to a superfluid condensate. Moreover, the phase diagram of such quantum dimer compounds contains an intermediate, partially polarized antiferromagnetic phase, making the phase behavior of the material analogous to that of a type II superconductor. This quantum phase transition behavior has been verified experimentally (Rüegg et al. 2003) and, moreover, is well modeled by a three-dimensional Heisenberg XY Hamiltonian, which can be derived from the Bose–Hubbard Hamiltonian in the hard-core boson limit $U \gg 1$ with no long-range interaction. We thus have empirical evidence that the Bose–Hubbard model describes the quantum dimer system within this parameter regime.

2.1.3. Cooper pairs in Josephson junction arrays

A Josephson junction array (JJA) is a granular superconductor given by an array of superconducting islands weakly coupled by Josephson tunnel junctions. The superconducting behavior of the system is determined by the interplay between the strength of the coupling energy between the islands and the strength of the electrostatic interaction energy of Cooper pair charges at each island. High coupling energy between the islands leads toward high superconducting coherence. High interaction energy of Cooper pairs, controlled by the island capacitance, leads toward charge localization on each island and suppression of superconducting coherence (Bruder, Fazio, and Schön 2005, 569). The behavior of Josephson tunneling and the interaction of Cooper pair charges are described by the quantum phase model Hamiltonian H_{QPM} which is formally equivalent to the Bose–Hubbard Hamiltonian in the regime of large local particle number $\langle n_i \rangle \equiv \langle b_i^\dagger b_i \rangle \gg 1$.

At high coupling energy, there is a critical temperature below which the array system is in a globally coherent superconducting state—the Cooper pairs "condense" into the same ground state. In the regime in which the electrostatic interaction energy at each island is comparable to the coupling energy between adjacent islands, lowering the temperature of the array increases the resistance between islands, and the array undergoes a transition to an insulator phase. This phase transition is experimentally well explored (van der Zant et al. 1996) to the extent that the Bose–Hubbard model is taken as a valid characterization of the behavior of the JJA in a suitable parameter regime, with the Cooper pairs behaving as the bosons. As such, this phase transition is analogous to a zero-temperature superfluid–insulator phase transition in the optical lattice system.

2.2. Summary and prospectus

Each of the four systems discussed (1) is well described by the Bose–Hubbard model within an appropriate parameter regime, (2) undergoes an analogue zero-temperature

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Table 1. Comparison of analogue Bose–Hubbard (BH) systems

System	Boson	Phase transition control	BH parameters
Cold atoms	^{87}Rb atom	Magnetic field/laser properties	
^{4}He adsorbed in Vycor	^{4}He atom	^{4}He density	$U \gg 1, U_{i,j} \neq 0$
Quantum dimer magnet	Dimer triplon	Magnetic field	$U \gg 1$
Josephson junction array	Cooper pair	Josephson energy and capacitance	$\langle n_i \rangle \gg 1$

Note. The cold-atom system tuned to a certain parameter regime can serve as the source system to study various analogue target systems. Here U denotes the on-site interaction, $U_{i,j}$ the interaction strength between distinct sites i and j, and $\langle n_i \rangle$ the expected value of the local particle numbers at site i.

phase transition from an insulating phase to a superfluid phase, (3) has a characteristic property that can be used to control the zero-temperature quantum phase transition, and yet (4) has a distinctly different material constitution. The key details are summarized in table 1.

Ultimately, the purpose of exploring such systems is to learn more about type II superconductors, with a view toward developing a better understanding of how they work and how we might be able to build, for instance, high-temperature superconductors. Some of the most promising naturally occurring candidates for such high-temperature superconductivity are so-called cuprate superconductors (materials characterized by alternating layers of copper oxides). Not only do these superconductors exhibit typical quantum phase transitions (Zhou et al. 2022) but they are in fact best known for their remarkable magnetic behavior, including the trapping of magnetic vortices in response to an external magnetic field. When these vortices are small enough (on the order of nanometers), as is the case in cuprate superconductors, the vortices exhibit ostensibly quantum behavior (Huebener 2019). However, these vortex states are difficult to observe directly, let alone probe experimentally (Berthod et al. 2017). Recent experiments employing the cold-atom source system suggest that these vortex states can potentially be probed via analogue quantum simulation (Atala et al. 2014)—whether these experiments actually do probe such states is the subject of this work.

The cold-atom system can thus act as a versatile simulator of various Bose–Hubbard systems, because it is highly tunable and more effectively probed than the target systems. However, the model is a good approximation of target system behavior only within some prescribed limit, that is, when the target systems are well described by, say, the quantum phase model, the XXZ model, or the XY model, which all reduce to the Bose–Hubbard model in a certain limit. Moreover, the cold-atom system exhibits behavior that, given the right inferential structure, could enable the investigation of phenomena we think typical of type II superconductors: quantum phase transitions between an insulator phase and a superfluid phase, an intermediate phase between insulator and superfluid phases, and the quantum behavior of magnetic vortex states generated by an external magnetic field.

In what follows, we explore the nature of the inferential structure that would lead experimenters to have confidence that probing ultracold atoms in an optical lattice can tell them something about naturally occurring superconductors. For the three target systems, we have experimental evidence for the phase transition in each

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system, which supports confidence that they are described by the Bose–Hubbard model in some limit. However, how might we gain confidence that we are successfully probing target system behavior when that behavior falls outside the limits defined by the relevant model relations, such as quantum vortex states in cuprate superconductors? Answering such questions in general and specific circumstances is the major occupation of the reminder of this article.

3. Uniformity principles in analogue quantum simulation

3.1. Tokens, types, and external validation

An *internally valid* experiment is one in which we genuinely learn about the source system we are manipulating. We will assume that all the experiments we consider here are internally validated by standard experimental means. To ensure that the outcomes of an experiment on a particular physical system are relevant to other physical systems with the same properties, we need to *externally validate* the experiment. Typically, conventional experiments are performed with systems in mind that have the same, or a similar, material constitution. Such systems are believed to behave similarly when probed in the same circumstances. External validation then amounts to ensuring that the specific lab system has the same material properties as the target systems. More abstractly speaking, in an experiment, a specific *token* physical system is probed to learn about an entire *type* of system. The inference from the token to the type is based on a *uniformity principle*, which asserts that all systems of the same material constitution behave in the same way when probed in the same circumstances.

In analogue experiments, by contrast, scientists aim for a system of one type to stand in for a system of another type, the latter of which, importantly, has a distinct material constitution. In our case study, for instance, we have the source system consisting of cold atoms and the target systems consisting of a JJA or liquid helium-4. It appears that, *by definition*, we cannot make use of an intratype uniformity principle between such systems because they are materially distinct.

To establish that a system of one type can stand in for a system of another type, we would need to perform experiments on both systems in the same setting and compare their outcomes. This would establish uniformity between tokens of different types. However, the purpose of an analogue quantum simulation is typically to probe the target system in a regime that is experimentally *inaccessible*. How can we provide a reliable means for justification of the relevant chain of inferences in such circumstances? One way to achieve this would be to establish a specific *intertype uniformity principle* between certain systems. But how could intertype uniformity be justified, and which systems would fall under it? For intratype uniformity principles, the criterion is clear: it is underpinned by the material constitution of the systems. For intertype uniformity principles (even assuming their existence), this is less clear: Are we considering a uniformity principle between two types? Should all tokens of the type, in all parameter regimes, be captured by the uniformity principle?

3.2. Material types, empirical types, and universality types

The natural implication of this discussion is that, in the context of analogue quantum simulation, we require uniformity principles that cut across the boundaries of

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different types. The corresponding notion of "type" is characterized by the material constitution of the systems. Let us therefore define the notion of a *material type* as follows:

> **Material type.** Two token systems are of the same material type if they share the same material composition as determined by the properties and spatial arrangement of the constituent particles, atoms or molecules, at the relevant physical scale.

This is a simple and intuitive notion of type in that it fleshes out the core conceptual idea of material sameness in a straightforward manner. Moreover, this idea of material type provides a simple and intuitive characterization of the kind of uniformity principle that one might naïvely take to underlie source–target inferences in a conventional experiment.

To be applicable to real scientific examples, there are of course a number of aspects of the idea of "material sameness" that need to be further specified—most obviously the *scale* at which the material composition is required to be the same. To take a famous example, two samples of carbon atoms may be of the same material type at the atomic scale but of very different material types at the level of bonded allotropes. The project of characterizing material types in a systematic and reliable manner will then crucially depend on finding an appropriate scale of structure, be this atomic, molecular, mesoscopic, or even macroscopic. Furthermore, there are good reasons to think that material similarity alone cannot be sufficient to power the types of inferences made in experimental science. Consider the example of impurities. Clearly, whether such impurities in a source system are significant enough to render an inference between source and target systems unreliable depends on the form of inference and the sensitivity of the experimental protocol. It might be perfectly valid to treat two systems as of the same material type in the context of one experimental inference despite a high level of impurities in the target, say, but entirely invalid to treat the same two systems as of the same material type in the context of another experimental inference.

The highly contextual nature of intratype reasoning in experimental science might thus prompt us to reconsider the focus on material constitution as the basis for distinguishing types in an experimental context. At the very least, there is a strong motivation to move beyond a simplistic picture of experimental inference based on source–target material similarity alone.[2]

Our focus in what follows is on the specific structure of scientific inference in the context of analogue quantum simulation. We do not take ourselves to be articulating a view on experimental science in general. However, a possible first step toward such a view, motivated by the problems with the notion of material type, would be as follows. Plausibly, what matters in the context of an experimental inference is that the source and target physical systems should behave similarly in similar situations. Let us then define a notion of *empirical type*.

[2] There are similarities here to the accounts of Bursten (2018), Roush (2018), and Norton (2021).

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Empirical type. Two token systems are of the same empirical type, in a specified parameter regime and with respect to a set of experimental prescriptions, if equivalent implementations of the prescriptions in the parameter regime result in similar measurement outcomes.

We can thus understand the intratype uniformity principles applied in conventional experimentation to be built around the assumption that tokens of a material type are also of the same empirical type. Such reasoning assumes that all tokens of the same material type can be described by a single theoretical model, which could then be validated by performing an experiment on a token system and applying the intramaterial-type uniformity principle. To justify the use of such a principle, the experiment needs to be externally validated, which requires that the concrete token system we are probing is in fact representative of the type we want to make an inference about. For a material type, this amounts to establishing similarity in material constitution of the system. A similarity in *nomic behavior* is then assumed to lead to a similarity in empirical behavior.

Whether or not this analysis is adequate in the context of conventional experimental science, it is clearly problematic in the context of analogue quantum simulation. In this context, scientists clearly are not aiming to justify an inference between source and target systems that are two tokens of the same material type and are thus not looking to establish similarities in material constitution to establish nomic and empirical uniformity.

This is clear from the analogue systems we outlined in section 2. Any putative inference from source to target in such simulations considers one physical material, say, ^{87}Rb, as a surrogate for another physical material, say, bound electron–phonon pairs in a superconductor or entangled spin states in a quantum dimer magnet. We might therefore seek to reconceptualize the schema sketched earlier and consider an *intermaterial-type* uniformity principle that would underlie the reasoning at hand in place of the *intramaterial-type* uniformity principle. To make this explicit, consider the idea of a *universality type*:

Universality type. Two systems that are of different material types are of the same universality type if, in a specified parameter regime and with respect to a set of experimental prescriptions, the behavior displayed by the systems upon equivalent implementation of the prescriptions is appropriately similar and independent of differences in their material composition.

A universality type is a particular kind of empirical type that additionally provides us with a potential route to external validation: an analogue quantum simulation might be validated on the basis of universality arguments showing the *independence* of the measurement outcomes on material constitution between source and target. Such an argument would show that the source and target systems belong to the same universality type and thus, other things being equal, would be of the same empirical type on that basis. In essence, inferential work previously performed by assumptions with regard to material types and laws of nature is now done by uniformity within the universality type. In each case, the key step is to establish target and source as members of the same empirical

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type, but in the two cases, this is achieved using a very different chain of reasoning.[3]

This suggests the question, can reasoning based on similarity as to empirical type be justified without appeal to material constitution *or* universality arguments? In other words, can we justify uniformity principles between empirical types directly?

3.3. Empirical quantum types

In this section, we consider a physical uniformity principle that cuts across material types based on independently established empirical evidence. This uniformity principle gains its inferential power by leveraging the validity of quantum theory in a well-characterized regime. The predictions of quantum theory have been confirmed to extremely high precision and at scales ranging from the size of the constituents of atoms to mechanical oscillators. In short, we are well justified to hold high confidence in the validity of quantum theory at the relevant scales and knowledge of its applicability.

As we will argue, this confidence in quantum theory can be used to justify a "quantum" notion of empirical type parallel to the idea of a universality type, the notion of an *empirical quantum type*.

> **Empirical quantum type.** Two token systems are of the same empirical quantum type, in a given parameter regime and with respect to given experimental prescriptions, if the same quantum mechanical model can be deployed in that parameter regime to provide an empirically reliable description of the systems for the experimental prescriptions.

Importantly, this definition allows for the possibility that the two systems at hand may be of different material types because, as explicitly illustrated by our case study, there clearly are cases in which the same quantum model can be employed to provide an empirically reliable description of systems with very different material constitution in the appropriate parameter regime.

More specifically, in the context of analogue quantum simulation, the relevant empirical quantum type is defined by an idealized *simulation model* M_{sim}. For the simulation systems we outlined in section 2, the simulation model is the Bose–Hubbard Hamiltonian H_{BH}. Given that both the target and the source physical systems are approximately described by the simulation model in a certain parameter regime, their empirical properties in this parameter regime will be approximately the same. Our most accurate description of the source and target systems will be specific *system models*, M_{sys}^{S} and M_{sys}^{T}, that include all known interactions and noise sources. Those are related to the simulation model by deidealization (see figure 1). We can think of all tokens of an empirical quantum type that share the same material constitution, and therefore the same system model, as a material subtype of the empirical type.

This way of thinking about an empirical quantum type in the context of analogue quantum simulation also provides a clear recipe for how to define what we called

[3] This is a point of controversy in the literature; see Dardashti, Thébault, and Winsberg (2017), Dardashti et al. (2019), Thébault (2019), and Evans and Thébault (2020) for the case in favor and Crowther, Linnemann, and Wüthrich (2019) and Field (forthcoming) for more skeptical commentary.

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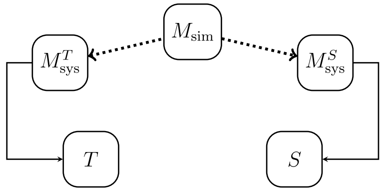

Figure 1. The inferential structure of analogue quantum simulation. The system models, M^T_{sys} and M^S_{sys}, which we take to represent (solid arrows) the target T and source S systems, respectively, are deidealizations in some controlled parameter regime (dotted arrows) of the simulation model M_{sim} that defines the quantum empirical type.

"equivalent experimental prescriptions" in section 3.2. We can specify an experimental prescription in terms of the idealized simulation model (say, $H_{\rm BH}$) that jointly and approximately describes all tokens of the empirical quantum type (say, the cold-atom optical lattice and the JJA, each constrained to the appropriate parameter regime). In other words, as long as there is a well-defined way in which an experimental prescription can be specified and translated into equivalent prescriptions for systems of different material constitutions, this prescription can figure in the definition of an empirical type. This prescription will often not be clear-cut and will incorporate our understanding of the formal model, a qualitative understanding of the physical principles underlying the behavior of the source and target systems, an understanding of how the applicability of these principles generates limitations on the parameter regime in which this behavior arises, and an understanding of any contingencies of the specific experimental apparatus employed in the simulation. For tokens of different material subtype, we can then exploit this understanding of our simulation systems to simultaneously deidealize the experimental prescriptions in accordance with the deidealization to the respective system model, giving rise to equivalent (within some operational bound) experimental prescriptions.

4. External validation of analogue quantum experiments

4.1. General inferential structure

Let us assume that we want to perform an experiment on a source quantum system S to make an inference about another target quantum system T of a distinct material constitution. Let us assume that the experiment on system S is internally valid and thus that we have established that S is accurately described by an idealized quantum simulation model M_{sim} in the parameter regime P. External validity in the context of an analogue quantum simulation experiment is equivalent to S and T being of the same empirical quantum type in the *entire parameter regime relevant to the analogue quantum experiment*.

We can *inductively* argue toward external validity; assume the following:

(Q) System T is accurately described within the framework of quantum theory in a certain parameter regime P.

(H) System T is accurately described by the idealized quantum simulation model M_{sim} for some values of its parameters $P_0 \in P$.

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(R) We have theoretical reasons to believe that M_{sim} accurately represents T in the parameter regime P.

We can then *inductively infer* the following:

(C) System T is accurately described by M_{sim} in the entire parameter regime P.

Because S and T are taken to be of the same empirical quantum type in the entire parameter regime relevant to the analogue quantum simulation, the experiment is externally valid.

Condition (Q) is supported by our confidence in the empirical reliability of quantum theory as a whole within a given parameter regime. Contemporary physics provides us with a wealth of evidence regarding the systems and regimes in which quantum behavior will be found. This evidence is wide and varied, including experimental evidence from more than a century of manipulating a broad range of quantum systems and theoretical evidence from powerful frameworks, such as effective field theory, that provide us with considerable confidence that we understand the relevant scales at which quantum theory can be applied.

Condition (H) is established by conducting a conventional experiment on the target system or a token of the same material subtype (in the conventional sense). Although the target system T may be inaccessible in some parameter regime of interest, it is typically accessible in some other regime that can be experimentally probed. Moreover, given that we are in the realm of applicability of quantum theory, in this regime we also want to be able to compare the predictions of quantum theory with experimental outcomes, so it is advantageous to perform an experiment in the computationally tractable regime.

Despite the fact that the target system T may be inaccessible in some parameter regime of interest, our expectation that the simulation model could well apply in this regime is captured by condition (R). This expectation is underpinned by our confidence that quantum theory is the right modeling framework for the relevant scale and empirical context and promotes the belief that the system will exhibit the relevant model behavior in the broader regime.

We can formulate a specific claim based on this argument pattern as follows:[4]

Claim 1. Assumptions (Q), (H), and (R) jointly provide inductive support for the conclusion (C) such that learning the conjunction $(Q) \wedge (H) \wedge (R)$ gives defeasible justification for raising one's degree of belief in (C).

The reasoning behind claim 1 is a form of "bootstrapping" argument that allows us to extend the parameter range in which we can have confidence that S and T are the same empirical type. At its core, the bootstrapping argument toward external validity works by leveraging a small piece of empirical knowledge of the target system in a narrow regime as captured by condition (H) to generalize the applicability of the

[4] We can express claim 1 in Bayesian terms as $P[C|Q + H + R] - P[C] > 0$, where C, Q, H, and R are the values of propositional variables corresponding to the truth/falsity of the relevant claims, $P[A|B]$ is the conditional probability of A given B, and we have assumed nontrivial prior probabilities.

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model M_{sim} to a broad parameter regime based on the quantum uniformity principle (Q). The condition (Q) is a key epistemic tool for the external validation of an analogue simulation because it buttresses the inferential connection between the quantum behaviors displayed by S and T. Given (Q), it is sufficient to validate the simulation model for *specific* parameters and inductively extend the applicability of the model beyond those parameters to the broader regime P, so long as we have theoretical reasons to believe that the simulation model is still applicable in the broader regime (R). Condition (R) is necessary here because the behavior of T in the parameter regime P is empirically inaccessible, and so such *theoretical* reasons are often the only evidence we have of the behavior of T in P.

Our appraisal of the inferential situation has two further significant consequences with respect to the *degree* of inferential support that the package $(Q) \wedge (H) \wedge (R)$ gives in comparison to alternative reasoning patterns that rely only on a subset of the inductive premises. We can set out these implications as follows:[5]

Claim 2. The *degree* of inductive support for the conclusion (C) provided by assumptions (Q), (H), and (R) is nontrivially greater than that provided by (H) and (R) alone.

Claim 3. The *degree* of inductive support for the conclusion (C) provided by assumptions (Q), (H), and (R) is nontrivially greater than that provided by (Q) and (R) alone.

To see that this is the case, consider the inferential weakness of reasoning based on the relevant subsets of premises.

With respect to claim 2, consider a situation in which we assume (H) and (R) but not (Q). In such circumstances, we have *experimental* evidence that T is accurately described by the idealized quantum simulation model M_{sim} in a specific parameter regime, and we have *theoretical* reasons to believe that M_{sim} accurately represents T in the wider parameter regime. However, without (Q), we have no inferential link between the behavior of T and the behavior of S and therefore no link to the (by assumption internally valid) experiment that probes S in the salient regime. Condition (Q) captures the well-justified assumption that the modeling framework of quantum theory, as provided by the core apparatus of Hilbert space representation together with some minimal interpretation given by the Born rule, does not break down (and is almost certainly applicable) at the relevant scales at which, for instance, quantum vortex states arise in type II superconductors. Put simply, based on our general empirical knowledge about the applicability of quantum mechanics, it is a

[5] In Bayesian terms, claim 2 is equivalent to $P[C|Q + H + R] \gg P[C|H + R]$ and claim 3 is equivalent to $P[C|Q + H + R] \gg P[C|Q + R]$, where the $\gg$ sign should be read qualitatively as indicating that the inductive support is nontrivially larger, but not necessarily many orders of magnitude larger. We make no claims regarding the scale of these values of inductive support, and it is perfectly plausible that they be low relative to the inductive support that may accrue in a conventional pattern of experimental inference. Plausibly, however, we take our arguments to imply that one may understand $P[C|Q + H + R] - P[C]$ to be nontrivial even if one expects trivial inductive support in cases in which the target system is entirely inaccessible and thus that $P[C|Q + R] - P[C] \approx 0$ (cf. Dardashti et al. 2019; Field, forthcoming).

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very reasonable assumption that the elementary objects at play in the source and target systems of an analogue quantum simulation are quantum objects. Without (Q), on their own, (H) and (R) provide a comparatively weak inductive base for the conclusion (C) precisely because the relevant bootstrapping argument can get no traction. This supports claim 2.

With respect to claim 3, consider a situation in which we assume (Q) and (R) but not (H). In such circumstances, there *is* an inferential connection between the behavior of S and T. However, because our assumption no longer contains any *experimentally derived* inductive evidence regarding the system T, the strength of inference we can make is greatly diminished. In particular, we are open to the possibility that beyond the features encoded in our broad quantum uniformity hypothesis, our basic theory of the target system may be completely wrong. Without (H), we have no *specific empirical evidence* that guides the selection of the most adequate model within the modeling framework of quantum theory to describe the target. All we have to constrain our reasoning with respect to the target system is theory, and although (Q) gives us a principle to connect S and T, it does not alone license strong reasoning with regard to the detailed physics underlying T. Hence claim 3 is supported.[6]

Let us now consider the specific implementation of this novel yet robust pattern of inference in the context of our case study.

4.2 External validation of Bose–Hubbard analogue simulations

Our framework for understanding the external validity of analogue quantum experiments can be applied to the context of the Bose–Hubbard analogue simulations outlined in section 2. The optical lattice system is our source system S_{CA}, and the superfluid helium, quantum dimer magnet, and JJA are our target systems T_{He}, T_{DM}, and T_{JJA}, respectively. We take each of these systems to be described by the same idealized quantum simulation model M_{sim}—the Bose–Hubbard model H_{BH}—in the right parameter regime P_0 (such as for $U \gg 1$ or $\langle n_i \rangle \gg 1$). And we take there to be high confidence that the relevant probing experiments on the optical lattice system are internally valid.

To see how this schema works in practice, let us consider an example. Although the ultimate goal of such quantum simulations is to learn about, say, the nature of natural type II superconductors, it will be instructive to consider as an example the JJA target system, T_{JJA}. The inferential structure of this simulation is depicted in figure 2. According to our framework, whether the optical lattice analogue quantum simulation counts as externally valid turns on whether for T_{JJA} each of (Q), (H), and (R) is satisfied.

Beginning with condition (H), we considered in section 2.1.3 the manner in which the superconductor–insulator transition is established by way of conventional experimentation on the JJA. It is illustrative, however, for understanding the inferential role played by the narrow parameter regime P_0 to provide some more detail. Suppose we are interested in determining the critical value of the ratio x_{cr} of the Josephson coupling energy to the capacitance at which we think a

[6] We note that claim 3 is very much in line with the analysis of Field (forthcoming) of the case of analogue simulations in which the target system is inaccessible and the relevant inferential link is built in terms of universality arguments (cf. Crowther, Linnemann, and Wüthrich 2019).

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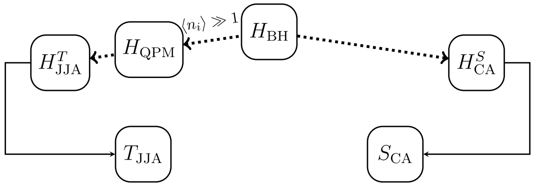

Figure 2. The inferential structure of the analogue quantum simulation of a JJA system, T_{JJA}, by a cold-atom optical lattice system, S_{CA}. As discussed in section 2.1.3, the JJA system is described by the quantum phase model Hamiltonian H_{QPM}, which can be reduced to the Bose–Hubbard model in the limit $\langle n_i \rangle \gg 1$.

superconductor–insulator phase transition occurs in a JJA and so support the claim that the Bose–Hubbard model is the appropriate model for the system. Analytic and quantitative investigations using the Bose–Hubbard model provide a zero-temperature hypothesis for where this value might lie. Observations of the array show that, for each trialed value of x_{cr}, there is a characteristic response in the resistance across the array as a function of lowering the temperature. Graphical inspection of these characteristic responses enables determination of the boundary between superconducting and insulating behavior and so also determination of the value of x_{cr}. Comparison of this value with analytic derivations lends support to the proposal of a quantum phase transition in the system (Fazio and Herre 2001). Without this direct empirical evidence of the applicability of H_{BH} to T_{JJA} in the narrow parameter regime, we would be incapable of assuming (H) and so in a relatively impoverished situation with regard to inductively supporting (C), as per claim 3.

However, a number of idealizations are required to enable the phase transition to emerge, and not only to ensure that the system is well characterized by the Bose–Hubbard model. In practice, the dynamical behavior of the array can be influenced by random offset charges at each island, which introduce an intrinsic degree of disorder to the array, especially at the phase transition boundary; dissipation due to coupling to the environment, which can dampen coherence effects across the array; and the creation of quasiparticles, which have been unexpectedly detected at millikelvin temperatures, which exacerbate dissipation effects (van der Zant et al. 1996). As such, quantifying by way of direct conventional experimentation the nature of the superconductor–insulator phase transition outside of the parameter regime where these effects are negligible (P_0) is very difficult and, at certain fine grains, essentially impossible. But we might still have an expectation that the system can be characterized by H_{BH} outside of this constrained parameter regime P_0. In particular, we might expect that JJAs, as granular superconductors, will admit magnetic vortex states that display quantum dynamics.

This expectation is captured by condition (R). Upon establishing that an analogue of a zero-temperature quantum phase transition is occurring in the JJA system characterized by H_{BH}, our knowledge of general Bose–Hubbard systems then implies that such behavior will be exhibited in a broader regime, one that is inaccessible to probing by conventional experiment due to the complexity or intractability of the system in that regime.

Condition (Q) is established independently of the analogue simulation and is the key to the external validity of analogue *quantum* experiments. There is a multitude of independent lines of evidence that superconductors, and so JJAs, are well described by quantum theory, and perhaps even likely in a parameter regime much broader than *P*, but certainly within the parameter regime set by the limits of what can be probed by the cold-atom optical lattice source system. The interplay between theory and experiment that has allowed us to be relatively confident that Cooper pairs, and their behavior as bosons in superconductor–insulator phase transitions, can be described in the modeling framework of quantum theory reduces the inferential burden on external validation in analogue quantum simulations. Without this independent evidence of the quantum behavior of superconductors, we could not assume (Q) and so would again be in a relatively impoverished situation with regard to inductively supporting (C), as per claim 2.

This example demonstrates that the inferential structure of analogue quantum simulation relies on a kind of consilience between (Q), (H), and (R): (Q) sets the general empirical foundation on which we can use (R) to obtain a specific theoretical basis to support conclusions regarding the detailed physics of *T*, whereas (H) provides a more narrow experimental basis to support claims regarding the dynamical behavior of *T*. Although there may be phenomena in *T* that we are unable to probe or manipulate experimentally—such as quantum vortex states—we take superconductors to be well-understood systems within a prespecified range of scales established by a lengthy tradition of interplay between superconductor theory and experiment. Moreover, there is a sense in which the appropriate parameter regime in system *T* is being set by our knowledge and experience probing relevant phenomena in the source system *S*. We observe some phenomenon in *S*, like a quantized magnetic vortex state, only under a particular set of conditions, and we expect there to be an analogue set of conditions in *T*. This expectation is underpinned by the consilience between (Q), (H), and (R).

The combination of (Q) + (H) + (R) then allows us to argue inductively that the target system T_{JJA} is described by H_{BH} in a parameter regime *P* that is broader than the regime in which we have direct conventional empirical evidence (P_0). More specifically, it is the combination of these three conditions that provides the relevant inductive base for inferences about properties of inaccessible concrete phenomena in JJAs, such as quantum vortex states, based on the observation of such phenomena in the cold-atom system. Because we can validate the applicability of Bose–Hubbard dynamics in the target systems in some tractable regime, we are justified in making inferences about the Bose–Hubbard behavior of those systems in intractable regimes based on the behavior observed in the analogue simulation experiments. This inference underpins the claims typical of analogue quantum simulations that probing the accessible behavior of the source system can be taken to probe the inaccessible behavior of the target system.

Such arguments in effect justify simultaneous deidealization of a single abstract quantum model to source and target system within a designated range of applicability and, in so doing, provide justification for reasoning based on regularity within empirical types. As we noted earlier, the deidealization of the simulation model to the respective system models, which in practice establishes the operational prescriptions that underpin regularity across empirical types, is not particularly clear-cut. In short, our practical understanding of the physical systems, the nature of our models, the physical principles that underlie those models, how these constrain the parameter

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regimes within which they are applicable, and the contingencies of our empirical access to the respective systems in the laboratory all play key roles in deidealization. We intend much of the above discussion of such practical understandings and contingencies to provide a guide to the architecture of this process.

There are some caveats here, of course. It is important to note that there are limitations on the applicability of (Q); that is, there are limitations on the regime in which the JJA will be accurately described by quantum theory. At a certain level of coarse-grained abstraction, the JJA will behave classically. We do not expect the analogue quantum simulation to provide evidence for behavior in this extended parameter regime. But at the appropriate fine-grained description—at which one can generate confidence in the quantumness condition (Q)—we can then infer the relevant Bose–Hubbard model to be a suitable description.

We thus reach the remarkable conclusion that although the JJA and the cold-atom optical lattice are instances of wholly different material constitutions, we expect them to obey structurally similar phase space and critical point dynamics on account of the strength of the analogue simulation: an intertype uniformity principle becomes an intra-empirical-quantum-type uniformity principle, which then underpins the external validity of the analogue experiments.

5. Conclusion

This article has provided the first philosophical investigation of the epistemology of the novel experimental scientific practice of analogue quantum simulation. This practice can be understood as "wavefunction engineering" because it relies on both systems exemplifying the same empirical quantum type: despite having a different material constitution, the same quantum wavefunction can be used to accurately represent both systems in a relevant regime. We have argued that, in such contexts, limited empirical access to both source and target systems can be leveraged to external validation of analogue simulations by the independently and empirically established confidence in the validity of quantum theory in both systems. Crucial here is appeal to a quantum uniformity principle that can be understood as a meta-principle for modeling practice. As the practice of wavefunction engineering and analogue experimentation continues to thrive, the form and strength of such patterns of inference will become increasingly relevant to scientific practice and thus, we trust, to the philosophy of the scientific method.

One might wonder where the bulk of the work is done in this argument: on one hand is the underlying, but broad, uniformity principle, and on the other hand is the specific, but narrow, empirical evidence for the validity of the model due to direct observation. Specifically, one might ask whether the uniformity principle is adding a quantitative or a qualitative difference to the argument. After all, it is standard practice to confirm models by performing experiments in restricted parameter regimes. We argue that the difference is qualitative: we would not be able to conclude the broad validity of the simulation model in *both* source and target systems across the entire parameter regime of interest if we were not very confident in the validity of the modeling framework.

And indeed, there are prominent examples of analogue experimentation in which we do not have empirical evidence that the modeling framework is adequate for the

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target system. In particular, this is the case in the context of remote or entirely inaccessible phenomena like analogue gravity (Dardashti, Thébault, and Winsberg 2017), for which justificatory arguments are framed in terms of the universality of phenomena across different material types. It remains to be seen, however, how strongly the distinction between such cases and cases like those we have considered should be taken. On one hand, as Winsberg (2010) argued in the context of experimentation and classical computer simulation, if we want to characterize the difference between two methods, we should not focus on what objective relationship actually exists between the object of an investigation and its target. Rather, what distinguishes different methods is the character of the argument given for the legitimacy of the inference from object to target and the character of the background knowledge that grounds that argument. On such a view, the distinction between wavefunction engineering and analogue experimentation based on universality would be a robust one, as the type of argument to support the inference is distinct. However, on the other hand, at an ontological level, the distinction between *intra*-empirical-type uniformity and inter-material-type uniformity is not grounded in a clean or straightforward distinction.

Similarly, we can compare analogue simulation to both standard experimentation and simulation. Taking again Winsberg's view as the basis, the distinction between simulation and experimentation is grounded in what kind of evidence we refer to when justifying inferences. One could consider a speculative thesis, worthy of future consideration, along these lines as follows. First, one might think that arguments for the validity of a computer simulation are *model based*, whereas arguments for the validity of an experiment are *nomology based*. Then, second, given our empirical-quantum-type argument, analogue quantum simulation could be taken to be a practice that is genuinely intermediate between simulation and experimentation. Its justification is grounded simultaneously in both a model-based simulationist and a nomology-based experimentalist reasoning. Third and finally, we would then have that a model-based epistemology and nomology-based ontology of simulation and experimentation cannot be separated. Rather, because it is the mode of deidealization that is different in the two cases, we should not be trying to differentiate between what there is and what we know. As the practice of wavefunction engineering and analogue experimentation continues to thrive, such issues will become of increasing importance and thus warrant further investigation.

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