BOOK REVIEWS

ROCKAFELLAR, R. TYRELL, *Convex Analysis* (Princeton University Press, 1970), xviii+451 pp.

This book presents a branch of nonlinear several-real-variable analysis of growing importance in the study of optimisation problems in many areas of applied mathematics, in which differentiability assumptions are systematically replaced by convexity assumptions. It develops the theory of convex sets and functions from scratch with the sights aimed at convex programs and the relation between them and their duals. There are eight main sections: Basic Concepts; Topological Properties; Duality correspondences; Representations and Inequalities; Differential Theory; Constrained Extremum Problems; Saddle Functions and Minimax Theory; Convex Algebra.

The book is written at graduate level, but to quote the Preface " In view of the fact that economists, engineers and others besides pure mathematicians have become interested in convex analysis, an attempt has been made to keep the exposition on a relatively elementary technical level ", and in fact the only prerequisites are a sound knowledge of elementary real analysis and linear algebra, plus preferably some notion of the applications for which convex analysis is designed.

A central tool in the development is Fenchel's conjugacy operation for convex functions, while the later theory is unified by the neat and remarkably versatile notion of a (convex) *bifunction*, which is simply a convex function F on the product of two linear spaces, regarded as associating to each u in one space a convex function Fu on the other; heuristically Fu is the perturbation of an " original " objective function FO caused by changes in a set of initially zero parameters $u = (u_1, ..., u_m)$. Throughout, the author shows how hypotheses of theorems can be weakened in the presence of extra conditions, such as polyhedrality of a convex set or function. To help one through the resulting proliferation of detail there is an introductory section guiding one to the most significant results. (I still tended to get bogged down; a better page-layout and the judicious use of bold type would have helped.) The style is easily read and there are plenty of worked examples. There is an adequate index of some 500 entries, and a section on history and references, with a large bibliography.

The plethora of detail reflects the fact that this is a young subject, and that the book is a reference text oriented towards those who need the techniques of convex analysis in practical problems. The continual honing-down process of mathematics may furnish future introductions to the subject with a more streamlined approach, but this book should remain for some years the standard reference for anyone interested in convex analysis. I recommend it strongly.

J. D. PRYCE

ADAMS, J. F., *Algebraic Topology—A Student's Guide* (London Mathematical Society Lecture Note Series 4, Cambridge University Press, 1972), vi+300 pp.

This book is aimed specifically at research students in algebraic topology. The first part of the book, consisting of 31 pages, could be described as a statement of what every professional algebraic topologist should know together with the sources from which the material can be studied. Students should find the advice given in this section very useful, particularly the recommendations of which papers to read (and occasionally which papers not to read) for each particular topic.

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The remainder of the book consists of 24 articles illustrating the topics referred to in the first part. Two of these articles are summaries by the author on " Generalized homology and cohomology theories " and on " Complex cobordism " written specially for this book. Six others are taken from unpublished lecture notes, etc., which students may have difficulty in obtaining. These include the paper " On the construction FK " by J. W. Milnor and the notes of lectures by A. Dold and E. Dyer given at the Colloquium on Algebraic Topology, Aarhus, 1962.

I am less certain of the value of reprinting the remaining 16 articles all of which appeared in major journals and should be readily available to anyone with access to a University Library.

M. J. TOMKINSON

HINDLEY, J. R., LERCHER, B., and SELDIN, J. P., *Introduction to Combinatory Logic* (London Mathematical Society Lecture Note Series 7, Cambridge University Press, 1972), 170 pp., £2.

The authors devote an introductory chapter to lambda-conversion, presenting a modified version of Church's treatment. They then introduce the notion of a combinator and establish the basic structure used in the rest of the book. The connection between lambda-conversion and combinators is clearly indicated.

The natural numbers are then presented as sequences of combinators and Kleene's results showing the relation between combinatory and partially recursive functions are described with minor modifications. The authors then show how an analogue of Church's undecidability result can be constructed in combinatory logic.

The notion of extensional equality for combinators is then introduced and used to show the exact equivalence of lambda-conversion and the theory of combinators. Strong reduction for combinators is then defined and the Church-Rosser theorem is proved for this relation.

The next stage in the development of the subject is to show how combinators can be interpreted as set-theoretical functions. This requires the introduction of a theory of types and two ways in which this can be done are described. It is then shown how a formal logic based on combinators can be developed. The book concludes by showing how Gödel functions of finite type can be treated combinatorially.

This is a well-written text giving an up-to-date account of the present state of the art in Combinatory Logic. The only aspect of the subject which the authors have not covered is the application to programming languages but references to recent papers are given for any reader who wishes to explore the topic further.

M. T. PARTIS

KLINE, MORRIS, *Mathematical Thought from Ancient to Modern Times* (Oxford University Press, 1973), xvii + 1238 pp., £12.

In this imposing volume Professor Kline, well known for several books aimed at the general mathematical reader, ranges with easy mastery over an immense field, presenting a panoramic view of the evolution of mathematics from Babylonian, Egyptian and Greek times up to the first decades of the present century. As might be expected from the title, the emphasis is more on exposition than in many conventional histories of the subject. The arrangement is roughly chronological; in general the mathematical themes selected for treatment are not traced from their origins, but are taken at the stage or stages when they have attained sufficient maturity to influence the main stream of development; thus several topics recur at different periods. The