The standard of translation throughout is excellent. A second volume is promised, which will include the relation of graphs to surfaces, matrices and probability. It is eagerly awaited.

IAN ANDERSON

JOHNSTONE, P. T., *Topos Theory* (L.M.S. Monograph No. 10, Academic Press, London, 1977), xxiii+367 pp., £17·50.

The subject has evolved rapidly since the work of Lawvere and Tierney in 1969–70, many of its ideas being common knowledge to specialists but not easily available to others. Peter Johnstone's book is the first comprehensive publication of both the elements of the theory and their development and application; it is intended as an "introduction . . . for the research student with some experience in category theory" and as "a comprehensive reference work for the specialist". It succeeds admirably in these intentions, and with its wit and style should also interest many other mathematicians: from logic, topology, algebraic geometry, universal algebra, set theory. . . .

A topos is a cartesian closed category with finite limits and a "subobject classifier", which allows the construction of "classifying maps" of subobjects just as the two-element set allows that of characteristic functions of subsets in the topos $\mathcal{S}$ of sets. For instance, the category of set-valued functors on a small category, the category of presheaves or of sheaves over a space, the category of finite sets, or of sets acted on by a group—all are toposes; and, more significantly, each of these examples, and many others, can be imitated "internally" inside (almost) any topos, to create a new topos. From this elementary standpoint one may conveniently study the Grothendieck toposes of modern algebraic geometry, or universal algebra, which may be done in any topos with a natural number object $\mathbb{N}$ (for the construction of free algebras). Johnstone, however, does not regard topos theory as a machine for solving "major outstanding problems of mathematics", but rather as an idea which will "inevitably lead to the deeper understanding of the real features of a problem which is an essential prelude to its correct solution". And the essence of this idea is the "rejection of the idea that there is a fixed universe of 'constant' sets within which mathematics can and should be developed, and the recognition that the notion of 'variable structure' may be more conveniently handled within a universe of *continuously variable* sets than by the method, traditional since the rise of abstract set theory, of considering separately a domain of variation (i.e. a topological space) and a succession of constant structures attached to the points of this domain".

The book begins with a brief sketch of the categorical background, including Giraud's theorem characterising Grothendieck toposes as categories satisfying certain exactness and size conditions. The basic elements of the theory then follow: axioms, representability of partial maps, Pare's theorem that the opposite category of a topos E is equivalent to the category of algebras for the power-object monad on E, pull-back functors, image factorisations, internal categories and limits, commutation of finite limits with filtered colimits, topologies in a topos, sheaves and sheafification, geometric morphisms (between toposes) and their factorisation, Wraith's glueing construction, finite lax colimits in the 2-category of toposes, Diaconescu's theorem characterising geometric morphisms from a fixed topos to the topos of internal functors on a category inside another topos, the Mitchell-Diaconescu generalisation of Giraud's theorem, Boolean toposes and the double-negation topology, the axiom of choice, toposes generated by the subobjects of 1, and, perhaps most interesting for the general reader, the Mitchell-Benabou 'internal language of a topos', carrying further the idea that a topos should be considered as a universe of discourse rather than just yet another sort of category.

Chapters 6–9 independently cover the aspects of the subject most relevant to applications: first, in a topos with a natural number object, we can talk about finite cardinals, algebraic and geometric theories, and real numbers; second, we have Deligne's theorem (every "coherent" topos has enough points), one version of which is the completeness theorem for finitary geometric theories, and Barr's theorem that every Grothendieck topos is the image under "surjective" geometric morphism of a topos in which the axiom of choice is valid (example: the classical relationship between the toposes of sheaves over a topological space and over the same set discretely topologised). Third, cohomology with Abelian coefficients is studied in a Grothendieck topos, with the beginnings of Duskin's non-abelian version; Chapter nine discusses the various notions of finiteness in a topos, the equivalence of various extensions of topos theory to Zermelo-Fraenkel set theory, and Tierney's independence proof for the continuum hypothesis, which shows clearly that toposes are usefully considered to be "Heyting-valued models" of set theory; and finally, there is an appendix on Penon's locally internal categories.

The book is throughout clear and precise, with an interesting historical introduction and plenty of helpful remarks and illustrations. Further developments are indicated in over a hundred exercises, and there are excellent bibliographies (usefully including references to Mathematical Reviews) and indices of definitions, notations and names. And just as the significance of the subject is as an attitude rather than as a technique, so the importance of the book is not as an assembly of original or unpublished ideas in topos theory but as a coherent account, essential reading for any topos theorist wishing to understand and master his subject, and an excellent introduction for everyone else; it is not likely to be superseded by a better work for some years. I regret the shortage, occasionally, of examples to bring the theory down to earth—for example, of object classifiers, or of coherent toposes; and of applications of results such as Barr's theorem; the inexperienced reader will probably find the exercises rather hard. But on balance I am very happy with the level of treatment: it would be hard, in a more leisurely introduction, to give such a good idea of the scope, depth, and interest of this increasingly important subject.

R. DYCKHOFF

JORDAN, D. W. and SMITH, P., *Nonlinear Ordinary Differential Equations* (Oxford Applied Mathematics and Computing Series), 360 pp, £12·00 (hard cover).

The aim of this book is to provide an undergraduate text dealing with the techniques used to obtain exact or approximate solutions of ordinary differential equations. This topic is excellently motivated, wherever possible, by elementary dynamical situations giving a physical background to the mathematical theory. The reader is then left with a clear intuitive picture of what would otherwise be a purely abstract concept.

The book starts with the conventional phase plane analysis and then spends several chapters on perturbation methods. This extensive study covers the various techniques of singular perturbation theory, averaging, forced oscillations, harmonic and subharmonic response and differential equations with periodic coefficients. The book also covers Liapunov stability and has a section on existence of periodic solutions. A large number of these topics would not be out of place in a postgraduate course. However the authors have skilfully managed to introduce everything at an elementary level so that no final year undergraduate student should feel that the underlying principles are beyond him. This does not imply that the authors deal lightly with such topics—on the contrary, one is led through quite complicated mathematical detail with expert care. The text is also backed up by very good figures and many illustrative and instructive examples, some worked and some left as exercises. These have clearly been culled from many years of searching for new questions to complement the course of lectures from which the book has been developed.

The book is mainly methods oriented, the aim being to explain the mathematics behind the various techniques involved. This it does very well. It also covers some theorems regarding existence and uniqueness which is in any case rather limited in this sphere of mathematics. It does not however deal with some of the more difficult aspects—for example, error estimation for the various approximations. Some of these aspects have been covered in great detail by other authors and are probably best left out of a book of this nature.

J. G. BYATT-SMITH

PETRICH, MARIO, *Lectures in Semigroups* (Wiley, 1977), 164 pp.

If this text were handed to me without the author's name I should have had no difficulty in guessing correctly who had written it. The style is unmistakeable. To coin a new adjective, it is truly "petrich": minimally encyclopaedic (which, alas, naturally implies the adjective "dreich"). It is most certainly not an elementary text and reading it is not an easy task, the wealth of material that it contains rendering it difficult to digest quickly. Nevertheless, it will be enjoyed by a happy, albeit small, group of readers; for a considerable amount of material of Eastern European and Russian origin is presented here in English for the first time. The first chapter is introductory and the author begins Chapter II with the notion of a band (a semigroup in which $x^2 = x$ for all x).

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