

THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of *The Mathematical Gazette*, *Mathematics in School*, *Primary Mathematics* (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is **Charnwood Building, Holywell Park, Loughborough University Science and Enterprise Park, Leicestershire LE11 3AQ, UK (telephone 0116 221 0013)**. The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to **Teaching Committee** should be addressed to TC Chair. The **Association's Library** is housed in the University Library, Leicester.

Views expressed in the *Mathematical Gazette* by authors or advertisers are not necessarily those of the Association.

THE MATHEMATICAL GAZETTE

Editor:

Dr Gerry Leversha,
15 Maunders Road,
Hanwell,
London W7 3PN
g.leversha@btinternet.com

Problem Corner:

Mr Nick Lord,
Tonbridge School,
Tonbridge,
Kent TN9 1JP
njl@tonbridge-school.org

Production Editor:

Mr Bill Richardson,
Kintail,
Longmorn,
Elgin IV30 8RJ
wpr3145@gmail.com

Reviews Editor:

Mr Owen Toller,
4 Caldwell House,
48 Trinity Church Road,
London SW13 8EJ
owen.toller@btinternet.com

Potential advertisers e-mail Charlotte Dyason at: charlotted@media-shed.co.uk

Material for publication should be sent to the Editor.

Books for review should be sent to the Reviews Editor.

Advice to authors of notes and articles.

Study the format of articles in the *Gazette*. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the *Gazette* was produced on an Acorn machine using TechWriter and Draw.

10.1017/mag.2024.55

CONTENTS (continued)

Notes 108.01 to 108.18 (continued)

Remarks on perfect powers	<i>H. A. Shahali</i>	122
Digital root analysis of Smith numbers	<i>Shyam Sunder Gupta</i>	124
Ramanujan's proof of Bertrand's postulate	<i>Allan J. Silberger</i>	130
Simple bounds on a sum pertinent to primes	<i>Hazar Aydin</i>	135
De Moivre's theorem via difference equations	<i>T. N. Lucas</i>	136
Cone and Integral	<i>Angad Singh</i>	140
A visual proof that $b^e < e^b$ when $b > e$	<i>Bikash Chakraborty</i>	143
Proof without words: $\tan \frac{\pi}{12} = 2 - \sqrt{3}$, $\tan \frac{5\pi}{12} = 2 + \sqrt{3}$	<i>Martin Lukarevski</i>	143
Euler's limit—revisited	<i>Bikash Chakraborty, Sagar Chakraborty</i>	144
Proof without words: a lower bound for $n!$	<i>Mehdi Hassani</i>	146
Indeterminate exponentials without tears	<i>Reza Farhadian, Vadim Ponomarenko</i>	146
A triangle number identity	<i>Paul Stephenson</i>	148
The Madhava-Leibniz theorem	<i>Amrit Awasthi</i>	149
Golden triangles founded on Kepler's triangle	<i>Aldo Scimone</i>	152
On a generalisation of the Lemoine axis	<i>Hans Humenberger, Franz Embacher</i>	154
A one-line proof of the Finsler-Hadwiger inequality	<i>Nick Lord</i>	158
Teaching Note		
The Devil's Advocate and the binomial expansion	<i>Nick Lord</i>	161
A cautionary tale about the pole of polar coordinates	<i>Nick Lord</i>	162
Another appearance of the golden ratio	<i>Nick Lord</i>	163
Correspondence		
A biography of Frank Budden	<i>Chris Sangwin</i>	166
Sir Michael Atiyah	<i>Bill Richardson</i>	166
Problem Corner	<i>Nick Lord</i>	167
Student Problems	<i>Tuya Sa</i>	172
Reviews		175

CONTENTS

Articles

$xy = \cos(x + y)$ and other implicit equations that are surprisingly easy to plot	<i>Michael Jewess</i>	1
Singular matrices and pairwise-tangent circles	<i>A. F. Beardon</i>	12
Infinitely many composites	<i>Nick Lord, Des MacHale</i>	20
Relating constructions and properties through duality	<i>David L. Farnsworth, Steven J. Kilner</i>	27
Some generalisations and extensions of a remarkable geometry puzzle	<i>Quang Hung Tran</i>	36
A characterisation of regular n -gons via (in)commensurability	<i>Silvano Rossetto, Giovanni Vincenzi</i>	43
Extensions of Vittas' Theorem	<i>Nikolaos Dergiades, Quang Hung Tran</i>	53
The role of convexity in defining regular polyhedra	<i>Chris Ottewill</i>	61
Euler's prime-producing polynomial revisited	<i>Robert Heffernan, Nick Lord, Des MacHale</i>	69
ABC-triangles	<i>Jonny Griffiths</i>	78
Patterns among square roots of the 2×2 identity matrix	<i>Howard Sporn</i>	84
The Eureka theorem of Gauss	<i>Stan Dolan</i>	94
A slowly evolving conical pendulum	<i>Subhranil De</i>	105
Walk on a grid	<i>Manija Shahali, H. A. Shahali</i>	111

Notes 108.01 to 108.18

A use of Pythagorean triples in a problem in elementary geometry	<i>Alexander Kronberg</i>	118
Fermat-like equations for fractional parts	<i>Nguyen Xuan Tho, Nguyen Quynh Tram</i>	120

© The Mathematical Association 2024

Typeset by Bill Richardson

CAMBRIDGE
UNIVERSITY PRESS



Printed in Great Britain by Henry Ling Limited,
The Dorset Press, Dorchester, DT1 1HD
ISSN 0025-5572