THE k-EXTREMALLY DISCONNECTED SPACES AS PROJECTIVES

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1. Introduction. The letter k denotes an infinite cardinal. A *space* is a compact Hausdorff space unless otherwise indicated. A space is called *extremally disconnected* (*k-extremally disconnected*) if it is the Stone space for a complete (k-complete) Boolean algebra. A *map* is a continuous function from one space into another. A map $f:X \rightarrow Y$ is called *minimal* if f is onto, but $f(M)$ is properly contained in Y for each closed proper subset M of X. A space F is called *free* if F has a dense subset X such that every space-valued function on X extends to a map on all of F or, equivalently, if F is the Stone-Čech compactification of some discrete topological space X. If $\mathfrak{C}$ denotes a category of spaces and maps and X is a space in $\mathfrak{C}$, we say X is *projective in* $\mathfrak{C}$ if given spaces and maps $f:A \rightarrow B$, $g:X \rightarrow B$ in $\mathfrak{C}$ with f onto, there is a map $h:X \rightarrow A$ in $\mathfrak{C}$ such that $fh = g$.

Gleason **(2)** proved that every space X is the image under a minimal map of a unique extremally disconnected space $m(X)$. An immediate consequence is that the projectives and extremally disconnected spaces coincide in the category of spaces and minimal maps (Gleason proves this fact in the category of spaces and all maps). For each space X, $m(X)$ is called the *minimal projective extension* of X. In this paper, we define the category $\mathfrak{C}(k)$ of k-spaces and k-maps, and we show that every k-space is the image under a minimal k-map of a unique k-extremally disconnected space. It then follows that in the category $\mathfrak{M}(k)$ of k-spaces and minimal k-maps, the projectives and k-extremally disconnected spaces coincide (we have not been able to prove this in $\mathfrak{C}(k)$).

2. k-open sets. Let R denote the real numbers and $C(X)$ the set of all R-valued continuous functions on a space X. A subset N of X is called a *zero set of* X if $N = f^{-1}(0)$ for some f in $C(X)$; this is equivalent to $N = \{x:f(x) \geqslant r\}$ for some f in $C(X)$ and r in R, or even $N = f^{-1}(D)$ for some f in $C(X)$ and closed subset D of R. The complement of a zero set is called a *cozero set of* X. A k-set (*k-family*) is a set (family) whose cardinal does not exceed k. A *k-open* subset of X is the union of a k-family of cozero sets of X; a *k-closed* subset of X is the intersection of a k-family of zero sets of X. The complement of a k-open (k-closed) subset of X is k-closed (k-open). The cozero sets of X form a base for the open sets. A simultaneously open and closed set will be called cl-*open*;

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a cl-open subset of X is easily seen to be a cozero set. We note, finally, that a cozero set is k-open.

THEOREM 1. (**5**, Theorem 11). *If X is a space, and if $C(X)$ is a k-complete lattice in its natural ordering, then every cozero set of X is the union of a countable family of* cl-*open subsets of X.*

THEOREM 2 (Stone). *The following statements are equivalent for a space X:*

(a) *The closure of every k-open subset of X is open.*

(b) *$C(X)$ is a k-complete lattice in its natural ordering.*

(c) *The* cl-*open subsets of X form a base for the open sets, and the closure of the union of a k-family of* cl-*open sets is open.*

(d) *X is k-extremally disconnected, i.e., the Stone space of a k-complete Boolean algebra.*

Proof. The equivalence of (b), (c), and (d) is proved in (**5**, Theorems 17 and 18). Assuming (a), the first part of (c) follows from the regularity of X and the fact that the k-open subsets form a base; the fact that a cl-open set is k-open gives the second part of (c). Conversely, assume (b), (c), and (d). By (d) and Theorem 1, a k-open set is expressible as the union of a k-family of cl-open sets; and such a union, by (c), has an open closure.

Remark. Let k_0 denote the first infinite cardinal. The k_0-extremally disconnected spaces are usually called basically disconnected. Of course, each k-extremally disconnected space is basically disconnected.

Remark. It is shown in (**1**, Section 1.14) that the zero sets and the k_0-closed sets coincide or, what is the same thing, that the cozero sets of a space and the k_0-open sets coincide.

LEMMA 1. *Let X be a space. Then:* (1) *The intersection of two k-open sets of X is k-open.* (2) *Let T be a closed subset of X. A subset of T is k-open in T if and only if it is of the form $V \cap T$ for some k-open subset V of X.* (3) *Let T be a* cl-*open subset of X. A subset S of T is k-open in T if and only if it is k-open in X.*

Proof. The validity of these three statements about k-open sets follows immediately from their validity for cozero sets. (1) The intersection of two cozero sets is a cozero set because the union of two zero sets is a zero set (**1**, Section 1.10). (2) The intersection of a zero set of X with T is obviously a zero set of T; and since each R-valued continuous function on T extends continuously to all of X, each zero set of T is the intersection with T of a zero set of X. The statement for cozero sets follows by taking complements. (3) It suffices to prove that a non-null zero set of T, $Z \equiv \{t \in T: f(t) = 0\}$, is a zero subset of X. Let r be a real number different from each value of $f \in C(T)$. Define $F(x) = f(x)$ if x is in T and $F(x) = r$ if x is not in T. Then F is in $C(X)$ and $Z = \{x \in X: F(x) = 0\}$, a zero set of X.

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3. Minimal maps. Professor M. Henriksen kindly brought the following lemma to my attention.

LEMMA 2 (E. Weinberg). *A map $f:X \to Y$ is minimal if and only if every non-empty open subset of X contains the pre-image of some non-empty open subset of Y.*

Proof. Immediate.

THEOREM 3. *The minimal projective extension of a space X is free if and only if X has a dense discrete subset* (a dense subset whose points are open in X).

Proof. Suppose the minimal projective extension of X is free. Then there is a minimal map $f:\beta D \to X$, where βD is the Stone-Čech compactification of a discrete space D. For each d in D, $\{d\}$ is open in βD, and the only subset of X whose pre-image could be $\{d\}$ is $\{f(d)\}$; hence $\{f(d)\}$ is open in X by Lemma 2. Thus, $f(D)$ is a dense discrete subset of X.

Conversely, suppose C is a dense discrete subset of X. Let j be a function carrying a set D 1-1 onto C. Then j extends to a map $f:\beta D \to X$. Given d in D, $\{j(d)\}$ is open; hence, $V \equiv f^{-1}(j(d)) \cap (\beta D \setminus \{d\})$ is open in βD. But, being contained in the nowhere dense subset $\beta D \setminus D$, V is void. Consequently, $f^{-1}(j(d)) = \{d\}$ and $\{d\}$ is the pre-image of an open set. Since every open subset of βD meets D, Lemma 2 implies f is minimal.

LEMMA 3 (2). *If $f:X \to Y$ is a minimal map and U is open in X, then $f(U) \subset \text{cl}(Y \setminus f(X \setminus U))$.*

COROLLARY 1. *If $f:X \to Y$ is minimal and U is open in X, then*

$$\text{cl}(f(U)) = f(\text{cl}(U)) = \text{cl}(Y \setminus f(X \setminus U)).$$

Proof. The first equality is true for any map; the second equality follows from Lemma 3 and the inclusion $Y \setminus f[X \setminus U] \subset f[U]$.

4. k-Spaces. We omit the straightforward proof of the following lemma.

LEMMA 4. *Let V and W be open subsets of a space X. The following statements are equivalent*:

(*a*) *$V \cap W$ is void and $V \cup W$ is dense in X.*

(*b*) $\text{cl}(X \setminus \text{cl}(V)) = \text{cl}(W)$.

(*c*) $\text{cl}(X \setminus \text{cl}(W)) = \text{cl}(V)$.

Definition. Two open subsets V and W of a space X which satisfy (a), (b), and (c) will be called *complementary open subsets* of X, and one is called an *open complement* of the other; e.g., for each open subset V of X, $X/\text{cl}(V)$ is an open complement of V. A space in which every k-open subset has a k-open complement will be called a k-space.

Example A. Every k-extremally disconnected space X is a k-space; for if V is k-open in X, $X \setminus \text{cl}(V)$ is cl-open and therefore a cozero set.

Example B. Any space in which every open set is a cozero set (for instance, any compact metric space) is a k-space.

LEMMA 5. *The disjoint union of two k-spaces is a k-space; a subspace M of a k-space is a k-space provided* $\mathrm{cl}(\mathrm{int}\ M) = M$.

Proof. The following easily proved fact will be useful, both in this and later proofs:

(1) *If V, E, and F are subsets of a space X with V open and if* $\mathrm{cl}(E) = \mathrm{cl}(F)$, *then* $\mathrm{cl}(V \cap E) = \mathrm{cl}(V \cap F)$.

Let M be a subspace of the k-space X such that $\mathrm{cl}(\mathrm{int}\ M) = M$. Let V be k-open in M. By Lemma 1, there is a k-open subset V' of X such that $V' \cap M = V$. Let W' be a k-open complement in X of V' so that $W \equiv W' \cap M$ is k-open in M. Clearly, $V \cap W$ is void. Furthermore, using (1),

$$\begin{aligned}\mathrm{cl}(V \cup W) &= \mathrm{cl}(M \cap (V' \cup W')) = \mathrm{cl}(\mathrm{cl}(\mathrm{int}\ M) \cap (V' \cup W'))\\ &= \mathrm{cl}(\mathrm{int}\ M \cap (V' \cup W')) = \mathrm{cl}(\mathrm{int}\ M \cap \mathrm{cl}(V' \cup W')) = \mathrm{cl}(\mathrm{int}\ M \cap X)\\ &= \mathrm{cl}(\mathrm{int}\ M) = M.\end{aligned}$$

Thus W is a k-open complement in M of V.

Now suppose X is the disjoint union of the k-spaces X_i $(i = 1, 2)$, and for any subset N of X, set $N_i = N \cap X_i$ so that N is open in X if and only if N_i is open in X_i for each i. Let V be a k-open subset of X. By Lemma 1, V_i is k-open in X_i for each i. Let W_i be a k-open complement in X_i of V_i; by Lemma 1, W_i is k-open in X for each i. Therefore, $W \equiv W_1 \cup W_2$ is k-open in X. And

$$W \cap V = (W_1 \cup W_2) \cap (V_1 \cup V_2) = \cup_{i,j}(W_i \cap V_j) = \emptyset.$$

Every non-void open subset U of X must meet X_i for some i; hence, $W_i \cup V_i$ for some i. This implies that $U \cap (W \cup V)$ is not empty. Therefore, $W \cup V$ is dense in X, and W is a k-open complement in X of V.

Remark. Lemma 5 shows how new k-spaces can be constructed from those of the preceding examples. Thus, there are plenty of k-spaces.

5. The main results.

Definition. A map $f : X \to Y$ will be called a k-map if given U k-open in X, there is a subset V of $f(X)$ k-open in $f(X)$ such that $f(\mathrm{cl}(U)) = \mathrm{cl}(V)$.

THEOREM 4. *Every k-space X is the image under a minimal k-map f of a k-extremally disconnected space M; if M' is another k-extremally disconnected space and $f' : M' \to X$ is a minimal k-map, there is a homeomorphism $h: M'$ such that $f'h = f$.*

Our proof, following Gleason's construction of minimal projective extensions, depends on the Stone representation theory for Boolean algebras. Let $\mathfrak{A}$ denote

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a Boolean algebra. The set $S(\mathfrak{A})$ of all maximal proper filters $\mathfrak{x}$ of $\mathfrak{A}$ (i.e., subsets $\mathfrak{x}$ not containing 0 and maximal with respect to the property that A, B in $\mathfrak{x}$ implies $A \wedge B \in \mathfrak{x}$) is made into a space by taking as a base for the open sets those subsets of $S(\mathfrak{A})$ of the form $\rho(A) \equiv \{\mathfrak{x} \in S(\mathfrak{A}): A \in \mathfrak{x}\}$ where A is in $\mathfrak{A}$. Furthermore, $A \rightarrow \rho(A)$ is an isomorphism of $\mathfrak{A}$ onto the Boolean algebra of all cl-open subsets of $S(\mathfrak{A})$. We shall say that a space is *totally disconnected* whenever it has a base of cl-open sets. The category of totally disconnected spaces and their maps is dual to the category of Boolean algebras and homomorphisms (**4**, Section 11). If X and Y are totally disconnected spaces and μ is a homomorphism from the Boolean algebra $\mathfrak{A}(X)$ of cl-open subsets of X into the Boolean algebra $\mathfrak{A}(Y)$ of cl-open subsets of Y, the canonical map $\rho(\mu): Y \rightarrow X$ is determined as follows: given y in Y, the set $\mathfrak{x}$ of all cl-open subsets N of X such that $\mu(N)$ contains y is a maximal filter of $\mathfrak{A}(X)$; this implies that $\bigcap \mathfrak{x}$ is a singleton. $\rho(\mu)(y)$ is this single element of $\bigcap \mathfrak{x}$. However, the only maximal filter of $\mathfrak{A}(X)$ whose intersection is a given point x of X is the set of all cl-open subsets of X containing x. Therefore,

(2) $\quad$ if $\rho(\mu)(y) \in N$, N cl-open, then $y \in \mu(N)$.

A closed subset M of a space X is called *regular* if $M = \text{cl}(\text{int } M)$ or, equivalently, if $M = \text{cl}(V)$ for some open set V. If $M = \text{cl}(V)$ for some k-open subset V of X, then M is called k-regular. Whenever a regular (k-regular) closed set $\text{cl}(V)$ is given, it will be understood that V is open (k-open). Let $\text{Reg}(X)$ ($\text{Reg}(k, X)$) denote the set of all regular (k-regular) closed subsets of X. It is well known that $\text{Reg}(X)$, partially ordered by inclusion, is a complete Boolean algebra under the following definitions of join, meet, and complement. Let $\{D_i\} = \{\text{cl}(V_i)\}$ be any subset of $\text{Reg}(X)$. Set

(3) $\bigvee_i D_i = \text{cl}(\bigcup_i \text{int } D_i) = \text{cl}(\bigcup_i V_i)$,

(4) $\bigwedge_i D_i = \text{cl}(\text{int} \bigcap D_i)$,

(5) $-D = \text{cl}(X \setminus D) = \text{cl}(X \setminus \text{cl}(V))$ for each $D = \text{cl}(V) \in \text{Reg}(X)$.

For two regular closed sets:

(6) $D_1 \vee D_2 = \text{cl}(V_1 \cup V_2) = D_1 \cup D_2$, and

(7) $D_1 \wedge D_2 = \text{cl}(\text{int}(\text{cl } V_1 \cap \text{cl } V_2)) = \text{cl}((\text{int cl } V_1) \cap (\text{int cl } V_2))$
$= \text{cl}(V_1 \cap V_2)$ using (1).

Since the k-open subsets of X are closed under the formation of finite intersections and finite unions, (6) and (7) imply that $\text{Reg}(k, X)$ is a sublattice of $\text{Reg}(X)$. The union of a k-family of k-open sets is k-open, so (3) implies that $\text{Reg}(k, X)$ is closed under the formation of joins of k-subsets. If X is a k-space, then (5) implies that $\text{Reg}(k, X)$ contains the Boolean complement of each of its elements. Therefore, $\text{Reg}(k, X)$ is a k-complete Boolean algebra for each k-space X.

Proceeding to the proof of Theorem 4, the Stone space $S \equiv S(\text{Reg}(k, X))$, where X is the given k-space, is k-extremally disconnected by definition. Observe:

(8) *If* $\mathfrak{x} \in S$, $x \in \bigcap \mathfrak{x}$, *and* V *a k-open neighbourhood of* x, *then* $\text{cl}(V)$ *is in* $\mathfrak{x}$.

For if W is a k-open complement of V, either $\text{cl}(V)$ or $\text{cl}(W)$ is in $\mathfrak{x}$ (since $\mathfrak{x}$ is a maximal filter) and if $\text{cl}(W) \in \mathfrak{x}$, then x is in $V \cap \text{cl}(W)$, a contradiction. Now each $\mathfrak{x}$ is a family of closed subsets of the space X with the finite intersection property; so $\bigcap \mathfrak{x}$ has a member, say x. If y is an element of X distinct from x, there is a k-open neighbourhood V of x such that $y \notin \text{cl}(V)$. By (8), $\text{cl}(V)$ is in $\mathfrak{x}$; hence, y is not in $\bigcap \mathfrak{x}$. Define $f(\mathfrak{x})$, for each $\mathfrak{x}$ in S, to be the single element of $\bigcap \mathfrak{x}$ and note that (8) now reads: if $f(\mathfrak{x})$ belongs to the k-open set V, then $\text{cl}(V)$ is in $\mathfrak{x}$ and therefore $\mathfrak{x}$ is in $\rho(\text{cl}(V))$. In other words:

(9) *If* V *is k-open in* X, $f^{-1}(V) \subset \rho(\text{cl}(V))$ and $V \subset f(\rho(\text{cl}(V)))$. But $\mathfrak{x} \in \rho(\text{cl}(V))$ implies $\text{cl}(V) \in \mathfrak{x}$, which implies $f(\mathfrak{x}) \in \text{cl}(V)$; i.e.:

(10) *If* V *is k-open in* X, *then* $f(\rho(\text{cl}(V))) \subset \text{cl}(V)$.

To prove that f is continuous at $\mathfrak{x}$, let U be a neighbourhood of $f(\mathfrak{x})$ and V a k-open neighbourhood of $f(\mathfrak{x})$ such that $\text{cl}(V) \subset U$. Then by (9) and (10), $\rho(\text{cl}(V))$ is an open neighbourhood of $\mathfrak{x}$ whose image under f is contained in $\text{cl}(V)$. To prove f is onto, let x be an element of X. The set of all $\text{cl}(V)$ such that V is a k-open neighbourhood of x is contained in a maximal proper filter $\mathfrak{x} \in S$, and

$$f(\mathfrak{x}) \in \bigcap \mathfrak{x} \subset \bigcap \{\text{cl}(V) \colon V \text{ is a } k\text{-open neighbourhood of } x\}$$
$$\subset \{V \colon V \text{ is a } k\text{-open neighbourhood of } x\} = \{x\}.$$

Each non-void open subset of S contains a set $\rho(\text{cl}(V))$, V non-void, which in turn, by (9), contains $f^{-1}(V)$; so by Lemma 2, f is minimal. And f is a k-map, since if N is a k-regular closed subset of S, then N is of the form $\rho(\text{cl}(V))$ for some k-open subset V of X; hence, $V \subset f(\rho(\text{cl}(V))) \subset \text{cl}(V)$. This implies that $f(N) = \text{cl}(V)$, as desired. This proves the existence part of Theorem 4; the following lemma facilitates the uniqueness proof.

LEMMA 6. *Let* $f\colon X \to Y$ *be a minimal map. Then* $\mu(A) \equiv f(A)$ *is a Boolean algebra isomorphism of* $\text{Reg}(X)$ *onto* $\text{Reg}(Y)$ *whose inverse is given by*

$$\mu^{-1}(\text{cl}(V)) = \text{cl}(f^{-1}(V)) = \text{cl}(\text{int}\, f^{-1}(\text{cl}(V))).$$

If X *and* Y *are k-spaces and* f *is a minimal k-map, then* μ *carries* $\text{Reg}(k, X)$ *onto* $\text{Reg}(k, Y)$.

Proof. Using Corollary 1, for each A in $\text{Reg}(X)$ we have $f(A) \in \text{Reg}(Y)$ and

$$\mu(-A) = f(\text{cl}(X \setminus A)) = \text{cl}(Y \setminus f[X \setminus (X \setminus A)])$$
$$= \text{cl}(Y \setminus f[A]) = -f(A) = -\mu(A).$$

Also, $\mu(A \vee B) = \mu(A) \vee \mu(B)$ so that μ is a homomorphism. If $\mu(\text{cl}(U))$ is void, then so is $f(\text{cl}(U)) = \text{cl}(Y \setminus f(X \setminus U))$. Thus $Y \setminus f(X \setminus U)$ is void and $f(X \setminus U) = Y$. Since f is minimal, $X \setminus U$ must be all of X: hence, U is void. Therefore, $\text{cl}(U)$ is void. So f is 1–1. Given V open in Y, one easily checks that

$$f(\text{cl}(f^{-1}(V))) = f(\text{cl}(\text{int} f^{-1}(\text{cl}(V)))) = \text{cl}(V)$$

and this proves the first statement of the lemma. The definition of a k-map implies that μ carries $\text{Reg}(k, X)$ into $\text{Reg}(k, Y)$. Since, in general, the inverse image under a map of a k-open set is k-open, μ^{-1} carries $\text{Reg}(k, Y)$ into $\text{Reg}(k, X)$.

Turning to the uniqueness assertion of Theorem 4, suppose $f_i: M_i \to X$ is a minimal k-map carrying the k-extremally disconnected space M_i onto the k-space X $(i = 1, 2)$. $\text{Reg}(k, M_i)$ consists of the cl-open subsets of M_i and by Lemma 6 there is a natural isomorphism μ_i from this Boolean algebra onto $\text{Reg}(k, X)$ $(i = 1, 2)$. By the previously mentioned duality, the isomorphism $\psi \equiv \mu_2^{-1}\mu_1$ of $\text{Reg}(k, M_1)$ onto $\text{Reg}(k, M_2)$ induces a homeomorphism $h: M_2 \to M_1$.

If $f_1 h \neq f_2$, then there is an element p of M_2 such that $f_1 h(p) \neq f_2(p)$. To reach a contradiction, choose V a k-open neighbourhood of $f_1 h(p)$ such that

$$(*)\ f_2(p) \notin \text{cl}(V) \equiv A.$$

Now $f_1 h(p) \in V$ implies that $h(p) \in \text{cl}(f_1^{-1}(V)) = \mu_1^{-1}(A)$. According to (2), p must belong to

$$\psi\mu_1^{-1}(A) = \mu_2^{-1}(A) = \text{cl}(\text{int} f_2^{-1}(A)) \subset f_2^{-1}(A).$$

Therefore, $f_2(p)$ is in A, contradicting (*). Consequently, $f_1 h = f_2$, and this concludes the proof.

COROLLARY 2. *In the category $\mathfrak{M}(k)$ of k-spaces and minimal k-maps, a space is projective if and only if it is k-extremally disconnected.*

Proof. The composition of k-maps is a k-map, so $\mathfrak{M}(k)$ is, indeed, a category. Now suppose X is projective in $\mathfrak{M}(k)$. Let M be k-extremally disconnected and $f: M \to X$ a minimal k-map. Because X is projective, there is a k-map $h: X \to M$ such that $fh = \text{id}_X$. Consequently, h is 1-1. If $h(X)$ is properly contained in M, then $X = fh(X)$ is a proper subset of X by the minimality of f, a contradiction. Therefore, h carries X homeomorphically onto M and X is k-extremally disconnected.

Conversely, suppose X is k-extremally disconnected. Let $f: A \to B$ and $g: X \to B$ be minimal k-maps. Let $e: M \to A$ be a minimal k-map with M k-extremally disconnected. Then $fe: M \to B$ and $g: X \to B$ are two minimal k-maps with M and X k-extremally disconnected, so there is a homeomorphism $h: X \to M$ such that $feh = g$. Thus eh is the required map in $\mathfrak{M}(k)$.

Remarks. Consider the category $\mathfrak{C}(k)$ of k-spaces and all k-maps. As in the first part of the proof of Corollary 2, every projective in $\mathfrak{C}(k)$ is easily seen to be k-extremally disconnected. The converse would follow if we knew that each k-map from a k-extremally disconnected space X onto a k-space B could be restricted to a subspace M of X such that $M \to B$ were a minimal k-map and M were a k-space. We have not been able to prove this. To carry out a programme in $\mathfrak{C}(k)$ similar to that of Rainwater **(3)** in the category $\mathfrak{C}$ of all spaces and maps, it would be necessary to prove the following: given X k-extremally disconnected, there is a map $f:P \to X$ in $\mathfrak{C}(k)$ such that P is projective in $\mathfrak{C}(k)$, f is onto, and f can be restricted to a minimal k-map onto X. In the category $\mathfrak{C}$, every extremally disconnected space is easily seen to be the image of a free space. By analogue, we define the k-free space kD, D a set, to be the Stone space of the Boolean algebra of subsets A of D such that either A or $D \setminus A$ is a k-set. However, we have not been able to prove that the k-free spaces have properties in $\mathfrak{C}(k)$ analogous to the properties possessed by the free spaces in $\mathfrak{C}$.

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