A Giant Step: from Milli- to Micro-arcsecond Astrometry
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Preliminary result of the Earth's free oscillations by Galerkin method

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Abstract. We use a Galerkin method to compute the eigenfunctions and eigenperiods of some of the Earths spheroidal and toroidal modes. The boundary conditions are treated using a Tau method. We show that for a realistic Earth model the difference between the computed and observed periods is less than 1.4%. We conclude that a Galerkin method may be an effective tool for the studies of the Earth's normal modes.

Keywords. methods: analytical, Earth

1. Method

Galerkin method is an efficient method to convert an operator problem to a discrete problem (Li 2006; Seyed-Mahmoud 1994). Consider

$$L[\chi(x)] + \phi(x) = 0 \text{ over the interval } a \leqslant x \leqslant b \, . \tag{1.1}$$

where L is a linear differential operator, and χ and ϕ are linear functions. Let $S = \{y_i(x)\}_{i=1}^{\infty}$ define the set of all linear independent functions. Any function $\chi(x)$ can then be written uniquely as a linear combination of $\chi(x) = \sum_{i=1}^{N} a_i y_i(x)$. Application of a Galerkin then makes the RHS of equation (1.1) as null as possible by requiring that

$$\int_a^b y_j(x) \left\{ L\left[\sum_{i=1}^{N} a_i y_i(x)\right] + \phi(x) \right\} dx = 0 \text{ for j=1,...,N} \tag{1.2}$$

This leads to a system of N equations in N unknowns (a_i) which we can solve uniquely.

As a test in this work, we consider a spherical non-rotating elastic isotropic (SNREI) earth model in hydrostatic equilibrium. In solid layers, linear controlling equations are:

$$\rho_0 \omega^2 \overrightarrow{u} + \rho_0 \nabla V_1 + \rho_0 \nabla(\overrightarrow{u} \cdot \overrightarrow{g_0}) - \rho_0 \overrightarrow{g_0}(\nabla \cdot \overrightarrow{u}) + \nabla \cdot \tilde{\Gamma} = 0 \tag{1.3}$$

and for the isotropic small oscillations of an inviscous liquid core:

$$\rho_0 w^2 \vec{u} - \nabla p_1 + \rho_0 \nabla V_1 + \rho_1 \vec{g_0} = 0 \tag{1.4}$$

$$\nabla^2 V_1 + 4\pi G \rho_1 = 0 \tag{1.5}$$

$$\rho_1 = -\nabla \cdot (\rho_0 \vec{u}) \tag{1.6}$$

$$p_1 = -\vec{u} \cdot \nabla P_0 + \alpha^2 \rho_1 + \alpha^2 \vec{u} \cdot \nabla \rho_0 \tag{1.7}$$

where, the displacement vector field $\overrightarrow{u}$ is expanded to spheroidal and toroidal fields.

$$\overrightarrow{u}(r,\theta,\phi) = \sum_{n,m} \{u_n^m(r) Y_n^m(\theta,\phi)\hat{r} + v_n^m(r)\nabla_1 Y_n^m(\theta,\phi) - w_n^m(r)\hat{r} \times \nabla_1 Y_n^m(\theta,\phi)\} \tag{1.8}$$

And the boundary conditions for $\overrightarrow{u}$, the stress field $\tilde{\Gamma}$, and the incremental potential V_1 and its gradiant are: $\hat{n} \cdot \tilde{\Gamma}$, $\hat{n} \cdot (\nabla V_1 - 4\pi G \rho_0 \overrightarrow{u})$, V_1 be continuous on all boundaries; $\overrightarrow{u}$ be continuous cross welded boundaries, and $\hat{n} \cdot \overrightarrow{u}$ be continuous across solid-fluid boundary. We use a Tau method to solve boundary conditions.

2. Results & conclusion

Figure 1 are some results of the eigenfunctions (displacements) of the simple Earth as a solid sphere:

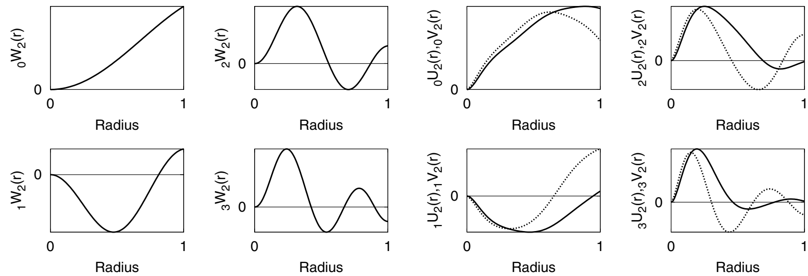

Figure 1. Eigenfunctions (u,v,w) of fundamental (n=0) and three overtones (n=1,2,3) of a solid sphere. The radius are normalized to be 1 at surface. In the right half 4 subplots, bold and dotted lines are for u(r) and v(r), respectively.

The eigen-periods are calculated for two earth models and listed in Table 1. The preliminary reference Earth model(PREM) is used in which the Earth is treated as inner solid core, fluid outer core and solid mantle. There is a maximum 4.1% or 1.4% difference if we treat the mantle(+crust) as 1 or 10 layers, respectively.

Table 1. Periods of toroidal and spheroidal modes in minutes, the observed values are from Lapwood & Usami (1981).

Modes		Toroidal			Spheroidal		
Legendre degree	Overtone	Observed	Calculated		Observed	Calculated	
			1-layer	10-layer		1-layer	10-Layer
2	0	44.01	45.53	43.63	53.89	51.84	53.43
	1	12.61	13.06	12.53	24.51	25.35	24.18
3	0	28.43	29.58	28.15	35.56	36.36	35.26
	1	11.59	11.90	11.48	17.68	18.06	17.54

Conclusion: As more realistic Earth model is considered, the numerical results for the periods of the Earth's acoustic modes converge to those of the observed ones. The remained 1.4% discrepancies between the numerical and observed periods may be reduced quickly by a more realistic Earth model (such as rotation or/and ellipticity). We conclude that the Galerkin method is a powerful tool for the studies of the Earth's normal modes.

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