ELECTROMAGNETIC DETECTORS OF GRAVITATIONAL WAVES*

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Our group is investigating highfrequency gravitational waves (GW). The most promising approach to detection and laboratory generation of such GW seems to be through the transformation of GW into electromagnetic waves (EMW), and the reverse process: EMW→GW. The effects are small of course.

The generation, EMW→GW, depends on the gravitational effect of the density of electromagnetic energy, which is equal to $(E^2+B^2)/8\pi c^2$ and is of order 10^{-12} g cm^{-3} for $B=10^5$ G. The detection depends on h – the GW perturbations of the metric. To obtain $h=1$ one needs an energy flux $W=c^5/G=10^{59}$ erg s^{-1}.

On the other hand, there are factors which multiply the effect and inspire some hope. They are the resonance and coherence of waves. Although we give no final answer, the situation (ignoring technical difficulties) seems better than it did some years ago. Gertsenstein (*JETP*, 1961) made an important contribution to the theory of generation process. He considered an EMW propagating through a constant magnetic field B_0, so that the magnetic field of the wave B is parallel to B_0. A rigourous treatment was given by Boccaletti, de Sabbata, Fortini, Gualdi in *Nuovo Cimento.* In what follows we don't write tensor indices (see this paper for such details).

Due to the equality of the propagation velocities of GW and EMW, an EMW generates a GW with the same wave vector $\mathbf{K}$ and frequency ω. This is called coherence. The amplitude of the GW is proportional to the interaction length l; the coefficient q of energy transformaty on is proportional to l^2:

$$q=\frac{W(h)}{W(B_0)}=\frac{GB_0^2l^2}{c^4}.$$

The equations for reciprocal transformations in a constant magnetic field have a very similar appearance. We use quantities h' and B' to describe GW and EMW, normalised in order to obtain equal coefficients in the energy flux:

$$W_{(\mathrm{GW})}\ \mathrm{erg\ cm^{-2}\ s^{-1}}=(h')^2$$
$$W_{(\mathrm{EMW})}\ \mathrm{erg\ cm^{-2}\ s^{-1}}=(B')^2$$

The equations are similar:

$$\Box h'=qB', \qquad \Box B'=qh'$$

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where $\Box$ is the d'Alambert operator. One can introduce 'normal waves' f and g which are uncoupled:

$$h' + B' = f, \quad h' - B' = g.$$

Then

$$\Box f = qf, \quad \omega^2 = c^2k^2 - q$$
$$\Box g = qg, \quad \omega^2 = c^2k^2 + q.$$

Oscillations of the form 100% GW→100% EMW are predicted. But the domain of one complete oscillation is enourmous: it is the gravitational radius corresponding to the mass density of the constant magnetic field. The transformation coefficient q is proportional to the wave vector $|\mathbf{k}|$ therefore no superlight velocity occurs in the dispersion equation:

$$\omega = ck \pm \frac{q}{2ck}, \quad \frac{\partial\omega}{\partial k} = c = \text{const}.$$

The numerical calculations are shown for laboratory conditions, for pulsars and so on:

TABLE I

	B_0	l	q
Laboratoty	10^5	10^3	10^{-33}
Pulsar	10^{13}	10^6	10^{-11}
Cosmology $z=0$	$\leqslant 10^{-7}$	10^{28}	10^{-7}
Cosmology $z=10^3$	10^{-1}	10^{22}	10^{-4}

The effects are meagre. The last entries are a guess for a 'magnetic Universe': the greatest imaginable homogeneous field is used, whose energy density is equal to that of 2.7 K blackbody radiation and changes according to the same law: $\varepsilon \sim B^2 \sim (1+z)^4$ with z – the cosmological redshift.

The last entry is promising, but the heretofore neglected interaction of EMW with electrons and atoms destroys the coherence: using the amended equation $\Box B' = qh' + rB'$ we see that the aformentioned effect fails to occur.

Now we consider the case of a closed resonator for EMW. A resonator may be used as a source as well as a detector of GW. The possible types of EMW are classified as a set of eigen-solutions with definite frequencies. An EMW in a resonator with frequency ω produces a GW with frequency 2ω. The state and phase of EMW oscillations in a system of resonators may be ajusted in such a way that the whole system is working coherently.

To describe a resonator as a detector consider a change of the EM field in the resonator due to a GW with a frequency ω_g. At first, we neglect damping. EM field equations in a resonator without GW are:

$$B = a_n(t)\, f_n(x)\,; \qquad \frac{d^2 a_n}{dt^2} = -\omega_n^2 a_n\,; \qquad a_n = a_{n0} e^{-i\omega_n t}.$$

Due to a GW they become

$$\frac{d^2 a_n}{dt^2} = -\omega_n^2 a_n + \omega_n^2 b_n e^{-i\omega_n t}, \qquad b_n \sim a_n h, \qquad \omega_n = \omega_m \pm \Omega.$$

Notice that the equations are written for time dependent amplitudes. We will not dwell here on the underlying Maxwell equations in a space curved by the gravitational wave. Thus these considerations give results which must be multiplied by numerical coefficients of order unity. Sometimes these coefficients are zero – but these exceptional cases (and the corresponding forms and positions of the resonators) should be avoided.

The GW introduces a mixture of different modes of oscillations. Resonance occurs if the GW frequency is equal to the difference of two EMW frequences.

Two particular cases should be mentioned: (1) a static initial field, $\omega_m = 0$, $a_n = a_{n0}$; and (2) parametric resonance with one type of oscillation, where $\Omega = 2\omega_n$. For the first case, with zero initial wave amplitude a_n, we introduce the energy transformation coefficient q as the ratio of EMW energy gain to the GW energy from through the resonator. In this case we have:

$$a_n \sim t,\ E_n \sim t^2, \qquad q = \frac{E_n(t)}{I_g S t} = \frac{G B_0^2 t}{c^2 \omega_n}.$$

Comparing with the open case, we see that the gain due to the resonator is equal to ct/l (it is a pity that no reflection occurs and that no resonator can be used for GW – otherwise, our problems would be solved!) Here t is the duration of GW action. By cooling the resonator to a low temperature, one can avoid the spontaneous birth of EMW photons (resonator excitation). Still, the time t in the formulae for q is bound due to losses in the resonator.

In principle the energy gain is augmented if the initial amplitude of the two EMW are nonzero. In this case ΔE of one of them is proportional to the first power of the small quantity h the amplitude of the GW. But now the energy gain ΔE must be measured with respect to the background of already excited oscillations. No net gain is achieved.

Particularly in the case of parametric resonance, the effect is proportional to h. We have

$$\omega_m = \tfrac{1}{2}\Omega, \qquad a_n \sim t, \qquad \Delta E = E t \sqrt{\frac{16\pi G I_g}{c^3}}.$$

One can exploit the amplitude change

$$\frac{da_n}{dt} = \dot{h}_{2\omega} a_n \sin\theta$$

or phase change

$$\frac{d\varphi_n}{dt} = \dot{h}_{2\omega} \cos\theta$$

by the choice of phase difference θ between GW and EMW.

Here also the problem occurs.

The best parameters feasible give again of the order of 10^{-5} photons during a 1000 s cycle. When the generator and detector are separated one meter and $a_n = 0$ at $t = t_0$. Thus without some supplementary idea the detection scheme does not work! And nobody knows if the new idea will employ an EMW resonator.

Interesting in principle, although not the best for energy gain, is the situation when the resonator is in the form of a unidimensional waveguide. In this case, neglecting dispersion, one can consider a wave packet with definite front and rear ends – 1 and 2. Geometrical optics can be used; one knows that the mode number in the region between 1 and 2 is constant; therefore, if a systematic change of the length $l_2 - l_1$ occurs, the frequency is shifted according to

$$\Delta\omega/\omega = \Delta l_{21}/l_{21} .$$

The case of a annular waveguide is typical. The propagation is along the φ coordinate; $r = \text{const}$. The part of the metric with dt and $d\varphi$ (but $dr = 0$, $dz = 0$) is

$$ds^2 = c^2\, dt^2 - r^2(1 - h_{22})\, d\varphi^2 .$$

The metric perturbation due to a circularly polarized GW is included. The resonance case occurs if motion of packet is always in phase with the metric distortion. In this case the packet length or frequency depends on time linearly.

Another treatment of the problem could be given by decomposing a wave of finite length a superposition of elementary eigenoscillations, with

$$\omega_{n+1} = \omega_n + \frac{c}{r} .$$

The frequency shift due to GW is smaller than the frequency difference of two adjacent eigenoscillations. Therefore the action of GW could be described as the transfer of energy from one mode to another.

The two treatments are equivalent. What is worth mentioning in the geometrical optics approach is the selection rules. A straight waveguide with mirrors on the ends gives no systematical effects if it is orientes along the propagation direction of the GW. This is due to the transverse character of GW. But the straight waveguide also does not work in the perpendicular plane and this is a non trivial selection rule. One must go to an annular waveguide or incline the straight waveguide. But here we are going into details important for obtaining the best gain from resonators. This has meaning only in the case when order of magnitude estimates suggest that on experiment is possible, which is unfortunately not yet the case.

So it is appropriate to end the discussion with the slogan 'New ideas are badly needed'.