the complexity of a learner (in terms of *Turing reducibility*) affects the number of mind changes for learning a given family.

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NICHOLAS PISCHKE. ***Proof-Theoretical Aspects of Nonlinear and Set-Valued Analysis***. Technische Universität Darmstadt, Darmstadt, Germany. 2024. Supervised by Prof. Dr. Ulrich Kohlenbach. MSC: 03F10, 47H04, 47H20, 47J25. Keywords: proof mining, set-valued operators, nonlinear semigroups, duality in Banach spaces.

Abstract

This thesis is concerned with extending the underlying logical approach as well as the breadth of applications of the proof mining program to various (mostly previously untreated) areas of nonlinear analysis and optimization, with a particular focus being placed on topics which involve set-valued operators.

For this, we extend the current logical methodology of proof mining by new systems and corresponding so-called logical metatheorems that cover these more involved areas of nonlinear analysis. Most of these systems crucially rely on the use of intensional methods, treating sets with potentially high quantifier complexity in the defining matrix via characteristic functions and axioms that describe only their properties and do not completely characterize the elements of the sets.

The applicability of all of these metatheorems is then substantiated by a range of case studies for the respective areas which in particular also highlight the naturalness of the use of intensional methods in the design of the corresponding systems.

The first new area covered thereby is the theory of nonlinear semigroups induced by corresponding evolution equations for accretive operators. In that context, we present (besides an initial foray into the area from 2015) essentially the first applications of proof mining to the theory of partial differential equations. Concretely, we provide quantitative versions of four central results on the asymptotic behavior of solutions to such equations.

The second new area unlocked in this thesis is that of the continuous dual of a Banach space and its norm (which are also approached via intensional methods). This in particular relies on a proof-theoretically tame treatment of suprema over (certain) bounded sets in this intensional context which is further exploited later on. These systems, which give access to this until now untreated fundamental notion from functional analysis, are then used to provide further substantial extensions to treat various notions from convex analysis like the Fréchet derivative of a convex function, Fenchel conjugates, Bregman distances, and monotone operators on Banach spaces in the sense of Browder.

These systems are then utilized to provide applications in the context of Picard- and Halpern-style iterations of so-called Bregman strongly nonexpansive mappings where we provide both new quantitative and qualitative results.

Lastly, we discuss the key notion of extensionality of a set-valued operator and its relation to set-theoretic maximality principles in more depth (which was already singled out—to some degree—in previous work). We thereby exhibit an issue arising with treating full extensionality in the context of these intensional approaches to set-valued operators and present useful fragments of the full extensionality statement where these issues are avoided.

Corresponding to these fragments, we discuss a range of uniform continuity statements for set-valued operators beyond the usual notion involving the Hausdorff-metric. In particular, in that context, we utilize the previous tame treatment of suprema over bounded sets to also

provide the first proof-theoretic treatment of that Hausdorff-metric in the context of systems for proof mining.

The applicability of this treatment of the Hausdorff-metric is then in particular substantiated by a last case study where we provide quantitative information for a Mann-type iteration of set-valued mappings which are nonexpansive w.r.t. the Hausdorff-metric.

The abstract was taken directly from the thesis.

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