ANALYSIS OF LLR DATA BY THE PROGRAM SYSTEM ERA

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Abstract. Some results of analysis of lunar ranging data of 1969–1995 are presented. A relevant dynamic theory has been constructed by numerical integration of the equations of motion of the major planets and the Moon and of lunar libration equations. To make the dynamical theory completely self-consistent an optional approach is tested in which the equations of Earth rotation are integrated simultaneously. Partial derivatives with respect to estimated parameters are also obtained by numerical integrations of variational equations. Preliminary results of evaluation of a large set of parameters involved in the lunar ranging data are presented.

1. Introduction

Lunar laser ranging observations (LLR) take a special place among other observational techniques of high precision as they provide valuable data for multi-disciplinary investigations. Firstly, LLR contributes to geodynamics making it possible to monitor Earth rotation, especially on the long-term time scale. Secondly, LLR-derived selenodynamical results make it possible to simplify reduction of astrometric observations of the Moon.

While modelling LLR observations the most serious difficulties arise due to complicated features of the lunar dynamics. In the lunar and planetary DE/LE ephemerides the problem has been overcome by a straightforward and efficient approach based on simultaneous numerical integration of the equations of lunar orbital and rotational motion. The main ideas of the approach are documented in the paper (Newhall *et al.*, 1983) which becomes a millstone for further investigations.

The aim of this work is to develop a similar approach in the frame of the applied program package ERA (Krasinsky *et al.*, 1989; Krasinsky and

I. M. Wytrzyszczak, J. H. Lieske and R. A. Feldman (eds.),
Dynamics and Astrometry of Natural and Artificial Celestial Bodies, 227, 1997.

Vasilyev, 1996) designed for multi-disciplinary applications of astronomical observations of high precision, and then to process available LLR data as a first step to combined treating lunar and planetary observations.

2. Mathematical Model and Observations Used

There are 4 active reflectors on different places on the surface of the Moon. The data used in this analysis are presented in Table 2. The 5^{th} column of the table gives the mean apriori errors of the observations of each group. These values were calculated according to an algorithm proposed by Newhall (1995) from apriori errors as they are given by observers.

The dynamical model has been constructed by a simultaneous numerical integration of the orbital motion of the Moon and planets and the equations of the rotation of the Moon. In comparing with the method used in DE102/DE200 ephemerides as described in Newhall (1983) the following amendments were made:

1. The influence of the Earth's gravitational potential on the lunar orbit is computed taking into account all harmonics of the potential up to 4^{th} order (both zonal and tesseral). Analysis shows that tesseral harmonics generate short-periodic perturbations of the elements as well as an additional secular trend in the lunar mean longitude of the order $0.8''$/century. The amplitude of the semi-diurnal perturbations is of about 0.03 mas that corresponds to 5 cm in the coordinates.

2. Perturbations of lunar librations from the nonsphericity of the lunar gravitational potential may be computed by taking into account spherical harmonics of any order. In fact, the integration was carried out with harmonics up to 4^{th} order, but from analysis of observations, Stokes' coefficients for harmonics of 5^{th} order were also estimated.

3. The equations of motion of the planets, the Moon and the lunar rotation were integrated simultaneously with the Poisson equations of motion of the Earth's rotational kinetic moment.

Differential equations for lunar physical librations depend on the principal moments of inertia of the Moon A, B, C and on the Stokes coefficients of the gravitational field of the Moon C_{kj} and S_{kj}. Parameters of lunar physical libration β, γ can be written in terms of A, B, C as:

$$\begin{aligned} \beta &= (C-A)/B \\ \gamma &= (B-A)/C. \end{aligned} \tag{1}$$

The constants β, γ are related with C_{20}, C_{22} in the following way:

$$\begin{aligned} \beta &= \frac{(4C_{22}-2C_{20})/g}{(4C_{22}+2C_{20})/g+2} \\ \gamma &= 4C_{22}/g, \end{aligned} \tag{2}$$

where $g = C/mR_M^2$, and where m and R_M represent the mass and mean radius of the Moon. The coefficients C_{kj} and S_{kj} appear in the equations of motion in the form of C_{kj}/g and S_{kj}/g only; just these combinations can be evaluated from LLR observations. So, we have fixed the value of g and adjusted the parameters C_{kj}, S_{kj}. The adopted value is $g = 0.390689526131941$.

Note, that the values of β, γ and C_{20}, C_{22} recommended by the IERS Standards (McCarthy, 1992) disagree since they don't satisfy the relations (2). For integration, the values of β, γ were taken from the IERS Standards but the corresponding values of C_{20}, C_{22} were computed according to (2).

The integration was carried out by Everhart's method (which is built-in to the ERA package) with the automatically operated choice of the step of integration. For computing partial derivatives with respect to estimated parameters the variational equations were simultaneously integrated. The set of parameters include the lunar initial coordinates and velocities, libration angles and their velocities, Stokes coefficients of the selenopotential, the angle of the tide delay, and the elements of the Earth's orbit.

3. Some Results from LLR-Analysis

The main goal of the current stage of the investigation is to test the mathematical model used in ERA for the analysis of LLR data. The initial values of the equations of motion of the Moon and major planets were taken from DE200-ephemeris. Initial values for equations of rotation of the Moon are known to a rather high degree of uncertainty. Probably that is the reason why initial residuals were significant and sometimes exceeded 200 ns. Thus, as the first step a reference orbit was constructed which fits all the observations with a minimum number of the estimated parameters. While constructing this reference ephemeris the following parameters were estimated: (1) coordinates and velocities of the Moon for the initial Julian date 2460000.5; (2) Euler angles of the selenocentric coordinate system (and corresponding time derivatives) for 2460000.5; (3) coordinates of LLR-stations and reflectors; (4) delay angle δ of the tidal bulge and the Stokes coefficients of the selenopotential C_{20} and C_{22}; (5) time delay biases for all the stations excluding MLRS2.

Because LLR observations are not sensitive to rotations of the coordinate frame, the parameters cannot be evaluated all together. So, the selenocentric longitude and latitude of Apollo-15 were not fitted. An unexpected outcome of the research is a statistically significant estimate of the Stokes coefficients C_{21} and S_{22} which due to this reason also have been included to the set of unknowns. Four iterations were carried out to reach coincidence between pre- and post-fit residuals for the reference orbit. Table 1 gives the starting value of the tide-delay angle δ and the estimated value for the reference orbit obtained after the iterations. The table presents corresponding value for the tidal lunar deceleration $\dot{n}_M$ calculated in accordance with the well-known relation:

$$\dot{n}_M = -4.5\,(m/(m+M))\,(R_E/r_M)^5\, n_M^2\, k_2 \sin(2\delta) \qquad (3)$$

where $g = C/mR_M^2$, and where m and R_M represent the mass and the mean radius of the Moon, respectively. R_E is the Earth's radius, r_M is the lunar distance, n_M is the lunar mean motion and k_2 is the Love number for the Earth. Table 1 gives also the estimates obtained for the Stokes coefficients of the selenopotential.

TABLE 1. Tidal parameters and coefficients of selenopotential.

	Adopted value	Estimated	value
δ (deg)	2.332	2.525	±0.062
$\dot{n}_M$ (″/cy²)	−24.41	−26.43	±0.64
C_{20}	−0.0002021505	−0.0002021203	±0.0000000013
C_{22}	0.0000222697	0.0000222883	±0.0000000006
C_{21}	0.0	0.00000000758	±0.00000000007
S_{22}	0.0	0.000000000158	±0.000000000010

Random square errors of residuals are presented in Table 2 for each group. Column 6 corresponds to the reference orbit, column 7 to the post-fit residuals of a more complete analysis in which the following additional parameters were estimated: (1) the coefficients of the lunar gravitational potential up to degree 5; (2) corrections to the precession and nutation angles, in-phase and out-of-phase coefficients for 18.6-year, 9.3-year, annual, semi-annual, monthly and fortnight terms in the nutation; (3) linear trend in sidereal time.

Comparison of the residuals for the reference ephemeris with the post-fit residuals shows a reduction of the rms by a factor of 2. As an illustration in Figure 1 there are plots of the residuals for CERGA (all observations of the reflector Apollo-15).

Comparing the post-fit rms in Table 2 with apriori estimated accuracy one can see that the former are 4-5 times larger. Probably that is due to some unmodelled effects of the dynamical theory. More detailed analysis shows that the signatures in rms are short-periodic on the level about 0.5 ns (see Figure 2, where the pattern of residuals for a 48-hour time interval are given for the MLRS2 station). The short-periodic effects are considerably less in the post-fit residuals but nevertheless they are clearly seen. Since

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TABLE 2. Observational data and residuals.

Station	Ref.	Time-span	Num.of obs.	σ_0 (ns)	RMS(ns) pre-fit	RMS(ns) post-fit
McDonald	1	1970–1982	468	1.038	5.443	6.432
	2	1971–1982	495	1.114	5.248	6.505
	3	1971–1985	2356	0.974	10.396	4.271
	4	1973–1981	132	1.105	10.502	4.745
MLRS	1	1986–1988	10	0.105	2.350	2.195
	2	1986–1987	26	0.121	3.970	1.659
	3	1985–1988	236	0.180	1.450	0.849
	4	1986–1987	3	0.134	2.940	1.300
CERGA	1	1984–1995	419	0.235	3.087	1.373
	2	1984–1995	414	0.248	2.745	1.868
	3	1984–1995	3577	0.231	1.732	0.677
	4	1984–1995	221	0.341	2.487	1.436
Haleakala	1	1985–1989	20	0.248	2.122	1.254
	2	1985–1988	23	0.202	2.782	1.028
	3	1984–1990	633	0.174	1.413	0.947
	4	1985–1988	18	0.319	1.578	0.748
MLRS2	1	1988–1995	66	0.163	3.678	1.505
	2	1988–1995	82	0.173	3.682	1.401
	3	1988–1995	790	0.181	1.848	0.745
	4	1988–1995	11	0.158	2.678	0.876

1 – Apollo-11, 2 – Apollo-14, 3 – Apollo-15, 4 – Lunochod-2

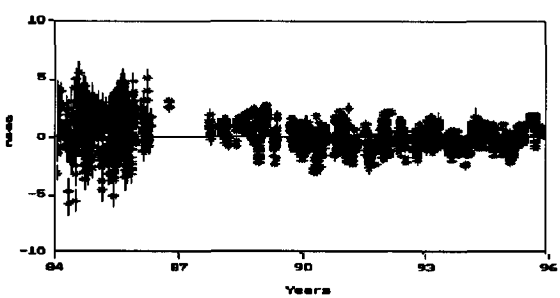

Figure 1. Global solution. Residuals for Apollo-15, CERGA.

geodynamical parameters generate signatures of the same type, the obtained estimates of these parameters may be corrupted by systematic errors and cannot be considered as reliable.

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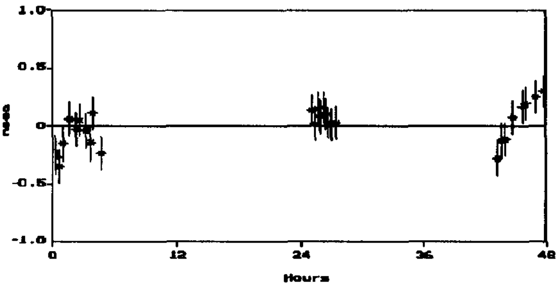

Figure 2. Global solution. Short arc residual pattern.

4. Conclusion

Here, we present just the initial results of the analysis of LLR data obtained with the universal applied-program package ERA. The investigation has proved a necessity of some refinement of the lunar dynamical model. In particular, it is necessary to take into account effects of energy dissipation of lunar rotation and find out a source of the marked short-periodic signatures. That is why at this stage we do not present results for geo- and selenodynamics.

Acknowledgements. We thank Drs. P. Shelus and J. Ries from MLRS who supplied us with LLR data, and Dr. X X Newhall from JPL for valuable help and advice.

References

Krasinsky, G.A., Novikov, F.A., and Scripnichenko, V.I.: 1989, "Problem oriented language for ephemeris astronomy and its realization in the system ERA", *Celest. Mech.* **45**, 219–229.
Krasinsky, G.A. and Vasilyev, M.V.: 1996, "Universal programming system ERA for high precision applications of dynamical and ephemeris astronomy", this volume.
McCarthy, D. (ed.): 1992, "IERS Standards", *IERS Tech. Note 13*, Obs. de Paris.
Newhall, X X, Standish, E.M., and Williams, J.G.: 1983, "DE102: A numerical integrated ephemeris of the Moon and planets spanning for forty-four centuries", *Astron. Astrophys.* **125**, 150–167.
Newhall, X X: 1995, private communication.