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CORRECTIVE TERMS IN THE GLACIOLOGICAL BALANCE

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ABSTRACT. Two corrective terms in the mass balance between two cross-sections are computed. The first results from the fact that balances are measured at moving stakes. The second correction arises since the discharges are calculated through summer cross-sections which are smaller than their mean values during the whole year.

RÉSUMÉ. *Termes correctifs dans le bilan glaciologique*. On calcule deux termes correctifs dans le bilan de masse entre les deux sections transversales. Le premier provient du fait que les bilans sont mesurés à des balises mouvantes, le deuxième de ce qu'on calcule les débits à travers des sections transversales estivales, moindres que les sections moyennes pour l'année entière.

ZUSAMMENFASSUNG. *Korrektionsglieder bei der glaziologischen Massenbilanz*. Zwei Korrektionsglieder in der Massenbilanz zwischen zwei Querschnitten werden berechnet. Die erste Korrektion berücksichtigt die Tatsache, dass die Bilanzen an bewegten Pegeln gemessen werden. Die zweite Korrektion kommt daher, dass der Durchfluss in Sommer-Querschnitten berechnet wird, die kleiner sind als die ganzjährigen Mittelwerte.

EITHER because the balances have not been measured over a whole glacier, or in order to check them, or in order to estimate the area of cross-sections, the glaciological balance is very often calculated for part of a valley glacier.

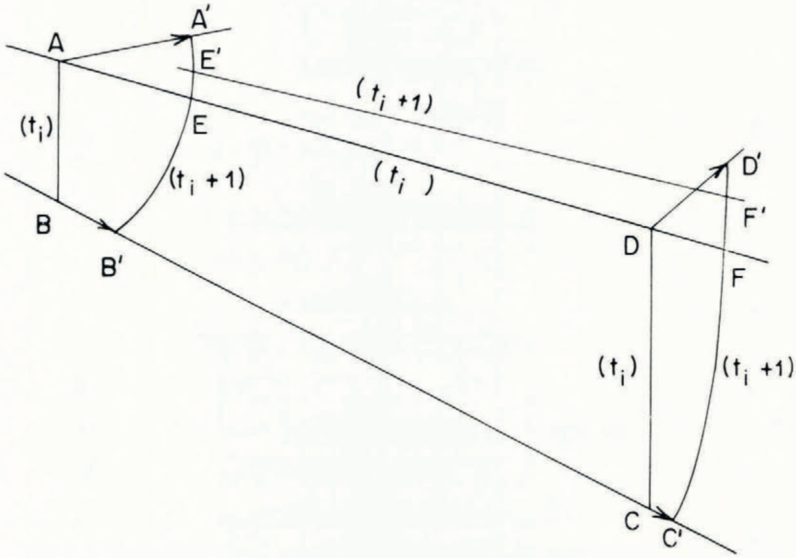

Fig. 1.

Let X, Y be fixed geographical coordinates, $Z(t)$ the altitude of the surface of the glacier at this point (assumed to be ice, not firn) and ΔZ its variation during the year (t_1, t_1+1). The (net) balance during the same year, in equivalent height of ice will be denoted b, and the annual discharge at two cross-sections AB and CD (Fig. 1), q_{AB} and q_{CD}. The mass balance between both cross-sections may be written (in volume of ice):

$$q_{AB}-q_{CD} = \iint_{A}^{D} (\Delta Z - b)\, dX dY. \tag{1}$$

In this relation b is defined at a point of given coordinates (X, Y). Actually b is normally measured at stakes moving with the ice. The slightly different value so measured will be denoted b'. In order to introduce the measured values b' into Equation (1), a correction must be made.

A longitudinal section of the glacier is sketched in Fig. 1. After one year the vertical planes AB and CD have become the curved surfaces A′B′ and C′D′, and the new surface is E′F′. We must then compare the volume ABCD, equal to the volume A′B′C′D′, and the volume E′B′C′F′:

$$\mathrm{ABB'A'} - \mathrm{AEA'} + \mathrm{EB'CD} = \mathrm{EB'CD} + \mathrm{DCC'D'} - \mathrm{DFD'} + \mathrm{A'EFD'}. \tag{2}$$

ABB′A′ and DCC′D′ are the discharges q_{AB} and q_{CD}. The distance between A′D′ and E′F′ is the measured balance b' (negative in the case of Fig. 1). The distance between E′F′ and EF is the variation of altitude ΔZ (positive in the case of Fig. 1). Therefore:

$$q_{\mathrm{AB}} - q_{\mathrm{CD}} = \int_r^l \tfrac{1}{2}\mathrm{AA'}(\Delta Z - b')\,\mathrm{d}s + \int_{\mathrm{E'}}^{\mathrm{F'}} (\Delta Z - b')\,\mathrm{d}X\mathrm{d}Y - \int_r^l \tfrac{1}{2}\mathrm{DD'}(\Delta Z - b')\,\mathrm{d}s, \tag{3}$$

$\int_r^l$ denoting a sum along a cross-section (not the same in each case) and s the distance in this transversal direction. So it appears that *in Equation (1), b' may be used instead of b, if the discharges are measured at two cross-sections* A *and* D, *and if the sum of* $(\Delta Z - b')$ *is extended between lines half-way between* A *and* D *and their position one year after*. If for instance A and D are 1 km apart and the mean velocity along a cross-section is 100 m/a at A, 40 m/a at B, the surface over which the sum must be done is diminished by 3%.

Whichever may be the definition adopted for the balance (at a fixed point or at a stake), a second correction arises from the fact that the area of the cross-section fluctuates within the year. In general the topographic work is done at instants t_i which are near the end of the ablation season, when the area of a cross-section is near a minimum. Actually, during the remainder of the year the area is bigger, the surface being raised (in equivalent height of ice) by $b(X, Y, t)$, the transient balance. The true annual discharge is then:

$$q' = \int_r H_i(\overline{\overline{U}}\,\mathrm{d}X + \overline{\overline{V}}\,\mathrm{d}Y) + \int_{t_i}\int_r^l b(X, Y, t)(U_S\,\mathrm{d}X + V_S\,\mathrm{d}Y)\,\mathrm{d}t, \tag{4}$$

where $\overline{\overline{U}}$ and $\overline{\overline{V}}$ denote average values through a vertical of the velocities in the X and Y direction, U_S and V_S the surface velocities and H_i the ice thickness at the point X, Y.

Assuming a linear increase with time of the transient balance $b(X, Y, t)$ during the accumulation season $(t_i, t_i + t_1)$, and next a linear decrease with time during the ablation season $(t_i + t_1,\ t_i + 1)$; b_m denoting the maximum transient balance, at instant $t_i + t_1$, and $t_2 = 1 - t_1$ the length of the ablation season (in years):

$$\int_{t_i}^{t_i+1} b\,\mathrm{d}t = \frac{b_m t_1}{2} + \frac{(b_m + b)t_2}{2} = \frac{b_m}{2} + \frac{bt_2}{2} \tag{5}$$

b' and b'_m, measured at moving stakes, may be used instead of b and b_m, since this approximation introduces a second-order correction only. Therefore:

$$q' = \int_r^l H_i(\overline{\overline{U}}\,\mathrm{d}X + \overline{\overline{V}}\,\mathrm{d}Y) + \int_r^l \frac{b'_m + b't_2}{2}(U_S\,\mathrm{d}X + V_S\,\mathrm{d}Y) \tag{6}$$

In order to estimate the importance of this correction, a result found by Nye (1965) will be used. Assuming either no sliding on the rock-bed, or Weertman's law of friction, this author has shown that the discharge in a parabolic channel is, within a few per cent, equal to

$$\Delta q = \int_r^l H_i(U_S\,\mathrm{d}X + V_S\,\mathrm{d}Y). \tag{7}$$

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Thus due to the second correction H_i should be changed to

$$H_i+\tfrac{1}{2}(b'_m+b't_2).$$

In the Alps, b'_m is of the order of 2 m of ice. Near the snout b' can reach −6 m or more and $t_2 \approx \frac{1}{3}$. When dealing with small glaciers, the mean thickness of which is 30 to 100 m, these corrections are not negligible.

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REFERENCE

Nye, J. F. 1965. The flow of a glacier in a channel of rectangular, elliptic or parabolic cross-section. *Journal of Glaciology*, Vol. 5, No. 41, p. 661–90.