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INFINITE SYSTEMS OF DIFFERENTIAL EQUATIONS II

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1. Introduction. This paper is a continuation of earlier work [**6**], in which we studied the existence and the stability of solutions to the infinite system of nonlinear differential equations

$$(1.1)\quad \begin{cases} x_i'(t) = \sum_{j=1}^{\infty} a_{ij}(t)x_j(t) + f_i(t, \underset{\sim}{x}(t)), & t \in R_s \\ x_i(s) = c_i \end{cases}$$

$i = 1, 2, \ldots$. Here s is a nonnegative real number, $R_s = \{t \in R: t \geqq s\}$, and $\underset{\sim}{x}(t) = \{x_1(t), x_2(t), \ldots\}$ denotes a sequence-valued function. Conditions on the coefficient matrix $A(t) = [a_{ij}(t)]$ and the nonlinear perturbation $\underset{\sim}{f}(t, \underset{\sim}{x}) = \{f_i(t, \underset{\sim}{x})\}$ were established which guarantee that for each initial value $\underset{\sim}{c} = \{c_i\} \in l^1$, the system (1.1) has a strongly continuous l^1-valued solution $\underset{\sim}{x}(t)$ (i.e., each $x_i(t)$ is continuous and $\|\underset{\sim}{x}(t)\| = \sum_{i=1}^{\infty}|x_i(t)|$ converges uniformly on compact subsets of R_s). A theorem was also given which yields the exponential stability for the nonlinear system (1.1). For more information we refer the reader to [**6**].

Our purpose here is to give conditions which ensure the existence of a strongly continuous solution $\underset{\sim}{x}(t)$ to (1.1) which also converges to a limit as $t \to \infty$. It is easy to see that the continuous functions on R_s which have this property are precisely the restrictions of the continuous functions on the one point compactification $[s, \infty]$ of R_s. We say such functions are *convergent at* ∞, and call a solution to (1.1) with this property a *convergent solution*.

Our interest in this problem was motivated by a theorem for an abstract linear nonhomogeneous equation given in [**5**, p. 153]. However, we found that the hypotheses of the abstract result were difficult to formulate in terms of the matrix entries. Moreover, examination of similar results for finite systems (see [**1**] and [**3**]) suggested that some of the hypotheses required for the abstract equation might be unnecessary in the case of the infinite system. For example, our diagonal dominance conditions (3.2) and (3.3) imply that $A(t)$ has a bounded inverse for all t, but do not imply that $A(t)A^{-1}(s)$ is even bounded when $t \neq s$ (cf. assumptions (B_3) on p. 108 and (i) on p. 153 of [**5**]).

We note that with some modifications, the l^1 results in [**6**] can be extended to the cases of l^p $(1 \leqq p < \infty)$ and c_0. All results in the present paper are established for these sequence spaces. For other l^p results on infinite systems of linear differential equations, see [**2**] and [**7**].

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2. Assumptions. Throughout the paper, all entries in the matrix $A(t) = [a_{ij}(t)]$ are assumed to be continuous on R_0 and convergent at ∞. Put

(2.1) $\alpha(t) = \sup \{ \sum_{i\neq j}|a_{ij}(t)|: j = 1, 2, \ldots\}$

(2.2) $\beta(t) = \sup \{ \sum_{j\neq i}|a_{ij}(t)|: i = 1, 2, \ldots\}.$

We assume that $A(t)$ satisfies the following conditions.

(A_1) The sum in (2.1) is uniformly convergent on R_0 for each j, and the function $\alpha(t)$ is bounded on R_0.

(A_2) The sum in (2.2) is uniformly convergent on R_0 for each i, and the function $\beta(t)$ is bounded on R_0.

Let $D(t) = \text{diag}\,[a_{ii}(t)]$ and $B(t) = A(t) - D(t)$ so that $A(t)$ is decomposed into its diagonal and off-diagonal parts. It can be shown that under the above conditions, $B(t)$ is a bounded linear operator on l^p $(1 \leqq p < \infty)$ and c_0 for each $t \in R_0$, and that the mapping $t \mapsto B(t)$ defines a strongly continuous operator-valued function which is also convergent at ∞.

On the perturbation $\underset{\sim}{f}(t, \underset{\sim}{x}) = \{f_i(t, \underset{\sim}{x})\}$, we impose the following conditions, depending on the sequence space in which we intend to work. For the space l^p $(1 \leqq p < \infty)$, we require

$(F_{1,p})$ For each i, f_i: $R_0 \times l^p \rightarrow C$ is continuous, and $f_i(t, \underset{\sim}{x})$ converges in C as $t \rightarrow \infty$, uniformly for $\underset{\sim}{x}$ in bounded subsets of l^p.

$(F_{2,p})$ If B is a bounded subset of l^p, then $\sum_i|f_i(t, \underset{\sim}{x})|^p$ converges uniformly on $R_0 \times B$.

$(F_{3,p})$ There is a continuous function g: $R_0 \rightarrow R_0$ such that $\|\underset{\sim}{f}(t, \underset{\sim}{x})\|_p \leqq g(t)\|\underset{\sim}{x}\|_p$ for all t and all $\underset{\sim}{x}$.

For the existence of convergent c_0-solutions to (1.1) we require instead of $(F_{1,p})$ and $(F_{3,p})$, the conditions $(F_{1,\infty})$ and $(F_{3,\infty})$ obtained by replacing l^p by c_0 in $(F_{1,p})$ and p by ∞ in $(F_{3,p})$, and instead of $(F_{2,p})$, the condition

$(F_{2,\infty})$ If B is a bounded subset of c_0, then $\lim_{i\rightarrow\infty}|f_i(t, \underset{\sim}{x})| = 0$ uniformly on $R_0 \times B$.

Condition $(F_{2,p})$ implies that $\underset{\sim}{f}(R_0 \times B)$ is totally bounded in l^p whenever B is bounded in l^p. Furthermore, $(F_{1,p})$ and $(F_{2,p})$ together imply that $\underset{\sim}{f}$ is (the restriction of) a strongly continuous l^p-valued function on $[0, \infty] \times l^p$. This has a consequence which will be useful later. Similar remarks apply to the case of c_0.

It should be pointed out that for the extensions to l^p of the l^1 results in [**6**], weaker forms of the above assumptions are sufficient. We impose these stronger conditions in order to obtain convergent solutions. The precise formulation of the weaker assumptions is left to the reader (compare those in [**6**] and in the present paper).

3. Stability theorem. We first consider the system of linear equations

$$(3.1) \quad \begin{cases} x_i'(t) = \sum_{j=1}^{\infty} a_{ij}(t) x_j(t) & t \in R_s \\ x_i(s) = c_i, \end{cases}$$

$i = 1, 2, \ldots$. Our next result is concerned with the exponential stability of the system (3.1).

An infinite matrix $A(t) = [a_{ij}(t)]$ is said to be *column diagonally dominant* if there is a positive number δ_1 such that

$$(3.2) \quad -\operatorname{Re} a_{jj}(t) \geqq \sum_{i \neq j} |a_{ij}(t)| + \delta_1$$

for $j = 1, 2, \ldots$ and $t \in R_0$. It is said to be *row diagonally dominant* if there is a positive number δ_2 such that

$$(3.3) \quad -\operatorname{Re} a_{ii}(t) \geqq \sum_{j \neq i} |a_{ij}(t)| + \delta_2$$

for $i = 1, 2, \ldots$ and $t \in R_0$.

The following result is needed in Section 4.

THEOREM 1. *Assume that conditions* (A_1) *and* (A_2) *hold.* (*i*) *If* $A(t)$ *is column diagonally dominant then, for each* $\underset{\sim}{c} \in l^1$, *the solution* $\underset{\sim}{x}(t)$ *of* (3.1) *satisfies*

$$(3.4) \quad \|\underset{\sim}{x}(t)\|_1 \leqq \|\underset{\sim}{c}\|_1 e^{-\delta_1(t-s)} \quad t \in R_s,$$

where δ_1 *is as in* (3.2). (*ii*) *If* $A(t)$ *is row diagonally dominant then, for each* $\underset{\sim}{c} \in c_0$. *the solution of* (3.1) *satisfies*

$$(3.5) \quad \|\underset{\sim}{x}(t)\|_\infty \leqq \|\underset{\sim}{c}\|_\infty e^{-\delta_2(t-s)} \quad t \in R_s,$$

where δ_2 *is as in* (3.3). (*iii*) *If* $A(t)$ *is both row and column diagonally dominant then, for each* $\underset{\sim}{c} \in l^p$, *the solution of* (3.1) *satisfies*

$$(3.6) \quad \|\underset{\sim}{x}(t)\|_p \leqq \|\underset{\sim}{c}\|_p e^{-\delta(t-s)} \quad t \in R_s,$$

where $\delta = \min(\delta_1, \delta_2)$ *and* $1 < p < \infty$.

To prove the above assertions, we need some preliminaries. For any finite matrix A the limit

$$(3.7) \quad \mu_p(A) = \lim_{h \to 0^+} \frac{\|I + hA\|_p - 1}{h} \quad (1 \leqq p \leqq \infty)$$

exists. Also it is known [**4**] that

$$(3.8) \quad \mu_1(A) = \sup_j \left(\operatorname{Re} a_{jj} + \sum_{i \neq j} |a_{ij}|\right)$$

$$(3.9) \quad \mu_\infty(A) = \sup_i \left(\operatorname{Re} a_{ii} + \sum_{j \neq i} |a_{ij}|\right).$$

We shall prove

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LEMMA 1. *For* $1 < p < \infty$

$$\text{(3.10)} \quad \mu_p(A) \leqq \frac{1}{p}\mu_1(A) + \frac{1}{q}\mu_\infty(A),$$

where q is the conjugate exponent of p.

Proof. Let δ_{ij} be the Kronecker delta. It can be shown that

$$\begin{aligned}\|I + hA\|_p &\leqq \sup_j [(\textstyle\sum_i|\delta_{ij} + ha_{ij}|)]^{1/p} [\sup_i (\textstyle\sum_j|\delta_{ij} + ha_{ij}|)]^{1/q} \\ &= [\|I + hA\|_1]^{1/p} [\|I + hA\|_\infty]^{1/q}.\end{aligned}$$

Thus, for $h > 0$,

$$\frac{\|I + hA\|_p - 1}{h} \leqq \frac{[\|I + hA\|_1]^{1/p}[\|I + hA\|_\infty]^{1/q} - 1}{h}.$$

Letting $h \rightarrow 0^+$ and using the definitions of μ_p, μ_1, and μ_∞, now produces the required inequality.

Proof of Theorem 1. Assertion (*i*) is proved in [**6**]. To prove assertions (*ii*) and (*iii*), we let $\underset{\sim}{x}^{(n)}(t)$ denote the solution of the finite system

$$\begin{cases} x_i'(t) = \sum_{j=1}^n a_{ij}(t)x_j(t) & (t \in R_s) \\ & i = 1, \ldots n, \\ x_i(s) = c_i \end{cases}$$

obtained from (3.1) by truncation. We shall express $x^{(n)}(t)$ as an infinite sequence with all but the first n entries being zero. By Theorem 3, p. 58 in [**4**], we have

$$\|\underset{\sim}{x}^{(n)}(t)\|_p \leqq \|\underset{\sim}{c}_n\|_p \exp\left(\int_s^t \mu_p[A_n(\tau)]d\tau\right) \quad t \in R_s,$$

where $\underset{\sim}{c}_n$ and $A_n(t)$ are, respectively, the truncation of $\underset{\sim}{c}$ after n terms and $A(t)$ after n rows and columns. For $p = \infty$ we get from (3.9) and the row diagonal dominance (3.3)

$$\|\underset{\sim}{x}^{(n)}(t)\|_\infty \leqq \|\underset{\sim}{c}_n\|_\infty e^{-\delta_2(t-s)} \quad t \in R_s.$$

For $1 < p < \infty$ Lemma 1 gives

$$\|\underset{\sim}{x}^{(n)}(t)\|_p \leqq \|\underset{\sim}{c}_n\|_p e^{-\delta(t-s)} \quad t \in R_s.$$

Letting $n \rightarrow \infty$ in the last two estimates and using an extension of Theorem 3.1 in [**6**], we obtain (3.5) and (3.6). This completes the proof of Theorem 1.

4. Convergent solutions. For the following discussion, we shall drop the indices p, 0 and ∞, and write l to denote any one of the sequence spaces l^p $(1 \leqq p < \infty)$ or c_0, and (F_1), (F_2), (F_3) for the corresponding conditions $(F_{1,p})$, $(F_{2,p})$ and $(F_{3,p})$. In this section we shall prove the following major result.

THEOREM 2. *Assume that conditions* (A_1) *and* (A_2) *hold, and that the coefficient matrix satisfies the diagonal dominance conditions* (3.2) *and* (3.3). *Assume also that the nonlinear perturbation* $\underset{\sim}{f}(t, \underset{\sim}{x})$ *satisfies* (F_1), (F_2), *and* (F_3). *Then for any* $\underset{\sim}{c} \in l$, *the system* (1.1) *has a strongly continuous l-valued solution which is convergent at* ∞.

In order to make our argument concise, we shall introduce some notation. If K is a compact Hausdorff space and X is a Banach space, we write $C(K, X)$ for the Banach space of all (strongly) continuous X-valued functions on K. If $f: K \times X \rightarrow X$, and if x is an X-valued function on K, then we write Fx for the X-valued function $t \rightarrow f(t, x(t))$ on K. Then we have the following lemma.

LEMMA 2. *If* $f: K \times X \rightarrow X$ *is continuous, then* $x \mapsto Fx$ *defines a continuous map on* $C(K, X)$ *into itself.*

Proof. It is immediate that Fx belongs to $C(K, X)$ whenever x does. Hence we need only prove the continuity of the mapping F.

Fix $x \in C(K, X)$ and $\epsilon > 0$. For each $t \in K$ and each $\delta \geqq 0$, there is a neighborhood $U(t, \delta)$ of t in K such that

(4.1) $\quad s \in U(t, \delta)$ implies $\|x(s) - x(t)\| < \delta$.

Since f is continuous, for each $t \in K$, there is a neighborhood $W(t, \epsilon)$ of $(t, x(t))$ in $K \times X$ such that

(4.2) $\quad (s, y) \in W(t, \epsilon)$ implies $\|f(s, y) - f(t, x(t))\| < \epsilon/2$.

Without loss of generality, we can suppose that $W(t, \epsilon)$ is of the form

$$W(t, \epsilon) = V(t, \epsilon) \times \{y \in X: \|y - x(t)\| < \delta(t, \epsilon)\},$$

where $V(t, \epsilon)$ is a neighborhood of t in K, $\delta(t, \epsilon) > 0$ and

(4.3) $\quad V(t, \epsilon) \subseteq U(t, \delta(t, \epsilon)/2)$.

Choose finitely many points $t_1, \ldots, t_n$ in K such that the neighborhoods $V_i = V(t_i, \epsilon)$ cover K, and put

(4.4) $\quad \delta = \min \{\delta(t_i, \epsilon)/2: i = 1, \ldots, n\}$.

Let $y \in C(K, X)$ with

$$\|y - x\| = \sup_{t \in K} \|y(t) - x(t)\| < \delta.$$

Any $t \in K$ belongs to some V_i, and we have

$$\|x(t) - x(t_i)\| < \delta(t_i, \epsilon)/2,$$

by (4.3) and (4.1). So,

$$\begin{aligned} \|Fy(t) - Fx(t)\| &= \|f(t, y(t)) - f(t, x(t))\| \\ &\leqq \|f(t, y(t)) - f(t_i, x(t_i))\| \\ &+ \|f(t_i, x(t_i)) - f(t, x(t))\| \\ &< \epsilon, \end{aligned}$$

by (4.2). That proves the lemma.

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The consequence of this which is of interest to us is that conditions (F_1) and (F_2) imply that $\underset{\sim}{x} \to F\underset{\sim}{x}$ (where, of course, $\underset{\sim}{F}\underset{\sim}{x}(t) = \underset{\sim}{f}(t, \underset{\sim}{x}(t))$) is a continuous mapping on $C([s, \infty], l)$ into itself, for any $s \geqq 0$.

Let Δ be the triangle $\{(t, s): 0 \leqq s \leqq t\}$. We say that a function $\underset{\sim}{x}(t, s)$ defined on Δ has *property* (C) if $\lim_{t\to\infty} \underset{\sim}{x}(t, t - \tau)$ exists for each $\tau \geqq 0$, and if also the convergence is uniform for τ in bounded subsets of R_0.

For $j = 1, 2, \ldots$, let $\underset{\sim}{e}_j = \{\delta_{ij}: i = 1, 2, \ldots\} \in l$, and let $\underset{\sim}{u}_j(t, s) = \{u_{ij}(t, s): i = 1, 2, \ldots\}$ be the unique strongly continuous solutions of (3.1) with the initial value $\underset{\sim}{c} = \underset{\sim}{e}_j$. As in [**6**], we define the *fundamental matrix* for the system (3.1) to be $U(t, s) = [u_{ij}(t, s)]$. Clearly $U(s, s) = I$, $s \in R_0$, where $I = [\delta_{ij}]$ is the infinite identity matrix. Furthermore, it can be shown that for each $\underset{\sim}{c} \in l$, the function $t \mapsto U(t, s)\underset{\sim}{c}$ on R_s is the unique strongly continuous solution of (3.1) with values in l.

LEMMA 3. *Suppose that conditions (A_1) and (A_2) hold, and that $A(t)$ is row and column diagonally dominant. Then, for each $\underset{\sim}{c} \in l$, the function $U(t, s)\underset{\sim}{c}$ has property (C).*

Proof. Put

$$(4.5) \qquad K(t, s) = \operatorname{diag}\left[\exp\left(\int_s^t a_{ii}(\tau)d\tau\right)\right]$$

for s and t in R_0, and define the integral operator I by

$$I\underset{\sim}{x}(t, s) = K(t, s)\underset{\sim}{c} + \int_s^t K(t, \tau)B(\tau)\underset{\sim}{x}(\tau, s)d\tau$$

where $B(t)$ is the off-diagonal part of the matrix $A(t)$. It can be shown, as in [**6**, § 2], that the iterates $I^n[K(t, s)\underset{\sim}{c}]$ converge as $n \to \infty$ to the solution $U(t, s)\underset{\sim}{c}$ of (3.1) uniformly in the strip $S_T = \{(t, s): 0 \leqq s \leqq t \text{ and } t - s \leqq T\}$, for each $T \geqq 0$. We shall complete the proof of the lemma by showing:

(*i*) $K(t, s)\underset{\sim}{c}$ has property (C),

(*ii*) if $\underset{\sim}{x}(t, s)$ is continuous on Δ and has property (C), then $I\underset{\sim}{x}(t, s)$ has property (C).

Since each $a_{ii}(t)$ is convergent at ∞, it is easy to see that each of the scalar-valued functions $\exp\left(\int_s^t a_{ii}(\tau)d\tau\right)$ has property (C), and (*i*) follows from this and (4.5).

Now we prove (*ii*). In view of (*i*), we need only show that the function $(t, s) \mapsto \int_s^t K(t, \tau)\, B(\tau)\underset{\sim}{x}(\tau, s)d\tau$ has property (C). Since $B(t)$ converges strongly as $t \to \infty$, and since $\{\underset{\sim}{x}(t, t - \tau): t \geqq \tau, 0 \leqq \tau \leqq T\}$ has compact closure in l for each $T \geqq 0$, it is straightforward to show that $(t, s) \to B(t)\underset{\sim}{x}(t, s)$ has property (C). Then, using (*i*), one can show that $K(t, t - \sigma)\, B(t - \sigma)$ $\underset{\sim}{x}(t - \sigma, t - \tau)$ converges as $t \to \infty$, uniformly with respect to τ and σ

satisfying $0 \leqq \sigma \leqq \tau \leqq T$. From this, it follows that

$$\int_0^T K(t, t-\sigma)B(t-\sigma)\underset{\sim}{x}(t-\sigma, t-\tau)d\sigma$$

converges as $t \to \infty$, uniformly with respect to τ in $[0, T]$. That proves (*ii*), and the lemma.

Proof of Theorem 2. As in [**6**, Thm. 4.1], a strongly continuous solution of (1.1) is just a fixed point of the integral operator

$$T\underset{\sim}{x}(t) = U(t, s)\underset{\sim}{c} + \int_s^t U(t, \tau)\underset{\sim}{f}(\tau, \underset{\sim}{x}(\tau))d\tau,$$

and we must show that, under our hypotheses, T has a fixed point in $C([s, \infty], l)$. We shall show first that T defines a continuous mapping of $C([s, \infty], l)$ into itself.

Now, $T = S \circ \underset{\sim}{F}$, where $\underset{\sim}{F}$ is the map associated with $\underset{\sim}{f}$ as in Lemma 2, and S is the integral operator defined by

$$S\underset{\sim}{y}(t) = U(t, s)\underset{\sim}{c} + \int_s^t U(t, \tau)\underset{\sim}{y}(\tau)d\tau.$$

Since $\underset{\sim}{F}$ is already continuous on $C([s, \infty],l)$ into itself by Lemma 2, we just have to show the same thing for S. Since $U(t, s)\underset{\sim}{c}$ is continuous for $t \geqq s$ and convergent as $t \to \infty$ by Theorem 1, we need only consider the operator R defined by

$$R\underset{\sim}{y}(t) = \int_s^t U(t, \tau)\underset{\sim}{y}(\tau)d\tau.$$

It is easy to see that $R\underset{\sim}{y}(t)$ is continuous on $[s, \infty)$, so it suffices to show that $R\underset{\sim}{y}(t)$ is convergent at ∞. From Theorem 1, we have that $\|U(t, \tau)\| \leqq e^{-\delta(t-\tau)}$. Also, since $\underset{\sim}{y}(t) \in C([s, \infty], l)$, we have $\|\underset{\sim}{y}\|_\infty = \sup_{t \geqq s}\|\underset{\sim}{y}(t)\| < \infty$. Now, for any $t_1 \geqq s$ and $t_2 \geqq s$, and any $\rho \geqq 0$ such that both $t_1 - \rho \geqq s$ and $t_2 - \rho \geqq s$,

$$\begin{aligned} \|R\underset{\sim}{y}(t_1) - R\underset{\sim}{y}(t_2)\| \leqq & \left\| \int_s^{t_1-\rho} U(t_1, \tau)\underset{\sim}{y}(\tau)d\tau \right\| \\ & + \left\| \int_s^{t_2-\rho} U(t_2, \tau)\underset{\sim}{y}(\tau)d\tau \right\| \\ & + \left\| \int_{t_1-\rho}^{t_1} U(t_1, \tau)\underset{\sim}{y}(\tau)d\tau - \int_{t_2-\rho}^{t_2} U(t_2, \tau)\underset{\sim}{y}(\tau)d\tau \right\| . \end{aligned}$$

A simple estimate shows that the first and the second terms on the right are both dominated by $e^{-\delta\rho}\|\underset{\sim}{y}\|_\infty/\delta$. Given $\epsilon > 0$, we can pick and fix ρ such that $2e^{-\delta\rho}\|\underset{\sim}{y}\|_\infty/\delta < \epsilon/2$. The third term on the right is equal to

$$\left\| \int_0^\rho [U(t_1, t_1-\tau)\underset{\sim}{y}(t_1-\tau) - U(t_2, t_2-\tau)\underset{\sim}{y}(t_2-\tau)]d\tau \right\| .$$

From Lemma 3 and the fact that $\underset{\sim}{y}(t) \in C([s, \infty], l)$, it follows easily that $U(t, \tau)\underset{\sim}{y}(\tau)$ has property (C). Therefore, for all sufficiently large t_1 and t_2, the inequality

$$\|U(t_1, t_1 - \tau)\underset{\sim}{y}(t_1 - \tau) - U(t_2, t_2 - \tau)\underset{\sim}{y}(t_2 - \tau)\| < \epsilon/2\rho$$

holds for all $\tau \in [0, \rho]$. A combination of these estimates yields

$$\|R\underset{\sim}{y}(t_1) - R\underset{\sim}{y}(t_2)\| < \epsilon.$$

for all sufficiently large t_1 and t_2. Thus $R\underset{\sim}{y}(t) \in C([s, \infty], l)$, as required. It is easy to see that the operator R is linear and bounded on $C([s, \infty], l)$, with $\|R\| \leqq \delta^{-1}$. This completes the proof of the fact that T is a continuous mapping on $C([s, \infty], l)$ into itself.

The proof of the theorem is now completed by an application of Tychonoff's fixed point theorem, as in [**6**] (see Theorem 4.2). Only slight modifications are required due to the fact that we are now working with the Banach spaces $C([s, \infty], l)$.

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