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THE ESSCHER PREMIUM PRINCIPLE: A CRITICISM*

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1. INTRODUCTION AND SUMMARY

The Esscher premium principle has recently had some exposure, namely, with the works of BÜHLMANN (1980) and GERBER (1980).

BÜHLMANN (1980) devised the principle and coined the name for it within the framework of utility theory and risk exchange. GERBER (1980), on the other hand, gives further insight into the principle by studying it within the realm of forecasting in much the same spirit as credibility theorists forecast premiums. However, there is an important distinction: the choice of loss function.

The present note sets out to criticize this relatively embryonic principle using decision theoretic arguments and indicates that the Esscher premium is essentially a small perturbation of the well established linearized credibility premium BÜHLMANN (1970).

2. ESSCHER PRINCIPLE AND CRITICISM

Let H denote the Esscher premium principle with loading $h > 0$. That is, if X is an observable random variable and Y is a parameter (a risk or a random quantity) to be forecasted then the Esscher premium is given by

$$H(Y \mid X) = \frac{E(Ye^{hY} \mid X)}{E(e^{hY} \mid X)}$$

That is, $H(Y \mid X)$ is the Bayes decision rule for estimating Y given the data X relative to the loss function

$$L_2(Y,a) = (Y-a)^2\, e^{hY}$$

where a is the estimate of Y, and of course the loading h is greater than zero.

Now, for the clincher. This loss function is nonsensical from the point of view of estimation. It indicates a loss (or error) to the forecaster that is essentially the antithesis of relative loss.

We illustrate: a forecaster commits a larger error if he estimates Y by 110 when its true value is 100 than if he extimates Y by 11 when the true value is 1.

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Gerber (1980) demonstrates, inter alia that if $X_1, X_2, \ldots, X_n$ are observable random variables such that conditional on θ (a risk parameter) fixed, $X_1, \ldots, X_{n+1}$ are i.i.d. and X_{n+1} is to be predicted then

$$H(X_{n+1} \mid X_1 \ldots X_n) = \text{Bayes rule relative to loss function } L_1 \times \{1 + o(h)\} \text{ as } h \downarrow 0,$$

where L_1 is squared error loss given by

$$L_1(X_{n+1}, a) = (X_{n+1} - a)^2.$$

The above demonstration is carried out under certain regularity conditions but nevertheless it means that the

$$\text{Esscher premium} = \text{linear credibility premium} \times \{1 + o(h)\} \text{ as } h \downarrow 0$$

if the sampling distribution is a one parameter exponential family and the prior distribution for θ is the conjugate prior. See Jewell (1974) for exact credibility.

Finally, since in practice h would be chosen to be close to 0, the Esscher premium is a small perturbation of the linearized credibility premium.

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