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COMMUTANT LIFTING, TENSOR ALGEBRAS, AND FUNCTIONAL CALCULUS

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Abstract A non-commutative multivariable analogue of Parrott's generalization of the Sz.-Nagy–Foiaş commutant lifting theorem is obtained. This yields Tomita-type commutant results and interpolation theorems (e.g. Sarason, Nevanlinna–Pick, Carathéodory) for $F_n^\infty \bar{\otimes} \mathcal{M}$, the weakly-closed algebra generated by the spatial tensor product of the non-commutative analytic Toeplitz algebra F_n^∞ and an arbitrary von Neumann algebra $\mathcal{M}$. In particular, we obtain interpolation theorems for bounded analytic functions from the open unit ball of $\mathbb{C}^n$ into a von Neumann algebra.

A variant of the non-commutative Poisson transform is used to extend the von Neumann inequality to tensor algebras, and to provide a generalization of the functional calculus for contractive sequences of operators on Hilbert spaces. Commutative versions of these results are also considered.

Keywords: commutant lifting; tensor algebras; functional calculus; interpolation

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1. Introduction

Let $F^2(H_n) = \mathbb{C}1 \oplus \oplus_{m\geqslant 1} H_n^{\otimes m}$ be the full Fock space on n generators, where H_n is an n-dimensional complex Hilbert space with orthonormal basis $\{e_1, e_2, \dots, e_n\}$ if n is finite, and $\{e_1, e_2, \dots\}$ if $n = \infty$. For each $i = 1, 2, \dots,$ define the left creation operator by $S_i\xi := e_i \otimes \xi$, $\xi \in F^2(H_n)$. We shall denote by $\mathcal{P}$ the set of all $p \in F^2(H_n)$ which are finite sums of tensor monomials. Define F_n^∞ as the set of all $g \in F^2(H_n)$ such that

$$\|g\|_\infty := \sup\{\|g \otimes p\|_{F^2(H_n)} : p \in \mathcal{P},\ \|p\|_{F^2(H_n)} \leqslant 1\} < \infty.$$

We denote by $\mathcal{A}_n$ the closure of $\mathcal{P}$ in $(F_n^\infty, \|\cdot\|_\infty)$. The Banach algebra F_n^∞ (respectively, $\mathcal{A}_n$) can be viewed as a non-commutative analogue of the Hardy space $H^\infty(\mathbb{D})$ (respectively, disc algebra $A(\mathbb{D})$); when $n = 1$ they coincide.

In [**20**, Theorem 3.1] we proved that $\mathcal{A}_n$ is completely isometrically isomorphic to the norm-closed algebra generated by any sequence $V_1, \dots, V_n$ of isometries with $V_1V_1^* + \dots + V_nV_n^* \leqslant I$, and the identity. It follows from [**18**, Theorem 4.3] that the non-commutative analytic Toeplitz algebra F_n^∞ can be identified with the weakly-closed (WOT-closed)

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algebra generated by the left creation operators $S_1, \dots, S_n$, and the identity. The algebras F_n^∞ and $\mathcal{A}_n$ were introduced by the author in [**16**] in connection with a non-commutative von Neumann inequality, and have been studied in several papers [**2**, **15**, **18**–**20**, **22**], and recently in [**3**, **6**–**8**, **21**].

We established a strong connection between the algebra F_n^∞ and the function theory on the open unit ball $\mathbb{B}_n$ of $\mathbb{C}^n$ through the non-commutative von Neumann inequality [**16**] (see also [**18**, **20**, **22**]). In particular, we proved that there is a completely contractive homomorphism

$$\Phi : F_n^\infty \to H^\infty(\mathbb{B}_n), \quad f(S_1, \dots, S_n) \mapsto f(\lambda_1, \dots, \lambda_n),$$

where $(\lambda_1, \dots, \lambda_n) \in \mathbb{B}_n$. A characterization of the analytic functions in the range of the map Φ was obtained in [**3**, **7**]. Arveson proved that Φ is not surjective [**4**] and the functions in its range are the multipliers of a certain function Hilbert space. In [**3**, **7**], it was proved that $F_n^\infty / \ker \Phi$ is an operator algebra which can be identified with $\mathcal{W}_n^\infty := P_{F_s^2} F_n^\infty|_{F_s^2}$, the compression to the symmetric Fock space $F_s^2 \subseteq F^2(H_n)$.

In [**1**, **3**, **4**, **7**, **22**], a good case is made that the appropriate commutative multivariable analogue of $H^\infty(\mathbb{D})$ is the algebra $\mathcal{W}_n^\infty$, which is the WOT-closed algebra generated by $B_i := P_{F_s^2} S_i|_{F_s^2}$, $i = 1, \dots, n$, and the identity. In this paper, we provide further evidence that F_n^∞ (respectively, $\mathcal{W}_n^\infty$) is a non-commutative (respectively, commutative) multivariate analogue of $H^\infty(\mathbb{D})$.

The main result of § 2 (see Theorem 2.1) is a non-commutative multivariable analogue of Parrott's generalization [**12**] of the Sz.-Nagy–Foiaş commutant lifting theorem [**24**] (see also [**9**]). We also identify (see Theorem 2.3) the commutant of sets of the form $F_n^\infty \otimes \mathcal{S}$, where $\mathcal{S}$ is a subset of $B(\mathcal{H})$, the algebra of bounded linear operators on a Hilbert space $\mathcal{H}$, and contains the identity operator. These results are used to extend Sarason's interpolation result [**23**] to $F_n^\infty \otimes \mathcal{M}$, where $\mathcal{M} \subseteq B(\mathcal{H})$ is a self-adjoint set containing the identity (see Theorem 2.4). This will lead to Tomita-type commutant results. In particular, Corollary 2.6 shows that if J is a WOT-closed two-sided ideal in F_n^∞, then

$$[(F_n^\infty/J) \otimes \mathcal{M}]' = (F_n^\infty/J)' \,\bar{\otimes}\, \mathcal{M}'$$

where the prime denotes the commutant and $\mathcal{A} \,\bar{\otimes}\, \mathcal{B}$ denotes the WOT-closed algebra generated by the spatial tensor product of the two algebras. Another consequence of Theorem 2.4 is a Nevanlinna–Pick type interpolation theorem for analytic functions from the unit ball of $\mathbb{C}^n$ into a von Neumann algebra $\mathcal{M}$, which extends results from [**3**, **7**, **11**, **13**, **21**, **23**]. On the other hand, the non-commutative Carathéodory interpolation problem [**19**, Corollary 4.4] is extended to $F_n^\infty \,\bar{\otimes}\, \mathcal{M}$.

In § 3, we consider a variant of the non-commutative Poisson transform introduced in [**22**] and provide extensions of the von Neumann type inequalities from [**4**, **12**, **16**, **20**, **22**, **26**]. This will lead to a generalization of the F_n^∞-functional calculus for contractive sequences of class C_0 [**18**], which also extends the Sz.-Nagy–Foiaş H^∞-functional calculus for $C_{\cdot 0}$-contractions [**25**].

More precisely, let $\mathbb{F}_n^+$ be the unital free semigroup on n generators $s_1, \dots, s_n$, and let e be its neutral element. For any $\sigma := s_{i_1} \cdots s_{i_k} \in \mathbb{F}_n^+$ we define its length $|\sigma| := k$, and

$|e| = 0$. On the other hand, if $T_i \in B(\mathcal{H})$, $i = 1, \dots, n$, we denote $T_\sigma := T_{i_1} \cdots T_{i_k}$ and $T_e := I_{\mathcal{H}}$. We show that if $\mathcal{T} := [T_1, \dots, T_n]$ is C_0-row contraction (see § 3 for terminology) and $\Delta := I_{\mathcal{H}} - \sum_{i=1}^n T_i T_i^*$, then every $f \in F_n^\infty \bar{\otimes} \{T_1, \dots, T_n, \Delta\}'$ has a unique Fourier expansion $f \sim \sum_{\alpha \in \mathbb{F}_n^+} S_\alpha \otimes W_{(\alpha)}$ with $W_{(\alpha)} \in \{T_1, \dots, T_n, \Delta\}'$, and the map

$$f \mapsto f(T_1, \dots, T_n) := \text{SOT-}\lim_{r \to 1} \sum_{\alpha \in \mathbb{F}_n^+} r^{|\alpha|} W_{(\alpha)} T_\alpha$$

is a WOT-continuous, completely contractive homomorphism from the algebra $F_n^\infty \bar{\otimes} \{T_1, \dots, T_n, \Delta\}'$ to $B(\mathcal{H})$. A commutative version of this result is also obtained. We mention that all the results of this paper are true if $n = \infty$, in a slightly adapted version.

After this paper was submitted for publication, Muhly and Solel published a paper [**10**] which contains a multivariable commutant lifting result (see their Theorem 4.4). Their result is close to our Theorem 2.1, but seems to be different. In any case, our proof is different, and is based on the geometric structure of the non-commutative minimal isometric dilation [**14**] and Parrott's Lemma [**12**].

2. Commutant lifting, tensor algebras, and interpolation

Let us recall from [**14, 15, 17**] a few results concerning the non-commutative dilation theory for n-tuples of operators. A sequence of operators $\mathcal{T} := [T_1, \dots, T_n]$, $T_i \in B(\mathcal{H})$, $i = 1, \dots, n$, is called contractive (or row contraction) if $T_1 T_1^* + \cdots + T_n T_n^* \leqslant I_{\mathcal{H}}$. We say that a sequence of isometries $\mathcal{V} := [V_1, \dots, V_n]$ on a Hilbert space $\mathcal{K} \supseteq \mathcal{H}$ is a minimal isometric dilation of $\mathcal{T}$ if the following properties are satisfied:

(i) $V_1 V_1^* + \cdots + V_n V_n^* \leqslant I_{\mathcal{K}}$;

(ii) $V_i^*|_{\mathcal{H}} = T_i^*$, $i = 1, \dots, n$;

(iii) $\mathcal{K} = \bigvee_{\alpha \in \mathbb{F}_n^+} V_\alpha \mathcal{H}$.

The minimal isometric dilation of $\mathcal{T}$ is uniquely determined up to an isomorphism. Let us consider a realization of it on Fock spaces. As in [**14**], let us define $D_{\mathcal{T}} : \oplus_{j=1}^n \mathcal{H} \to \oplus_{j=1}^n \mathcal{H}$ by $D_{\mathcal{T}} := (I_{\oplus_{j=1}^n \mathcal{H}} - \mathcal{T}^* \mathcal{T})^{1/2}$, and set $\mathcal{D} := \overline{D_{\mathcal{T}}(\oplus_{j=1}^n \mathcal{H})}$. Let $D_i : \mathcal{H} \to F^2(H_n) \otimes \mathcal{D}$ be defined by

$$D_i h := 1 \otimes D_{\mathcal{T}}(\underbrace{0, \dots, 0}_{i-1 \text{ times}}, h, 0, \dots, 0) \oplus 0 \oplus 0 \cdots .$$

Consider the Hilbert space $\mathcal{K} := \mathcal{H} \oplus (F^2(H_n) \otimes \mathcal{D})$ and define $V_i : \mathcal{K} \to \mathcal{K}$ by

$$V_i(h \oplus (\xi \otimes d)) := T_i h \oplus (D_i h + (S_i \otimes I_{\mathcal{D}})(\xi \otimes d))$$

for any $h \in \mathcal{H}$, $\xi \in F^2(H_n)$, $d \in \mathcal{D}$. Notice that

$$V_i = \begin{bmatrix} T_i & 0 \\ D_i & S_i \otimes I_{\mathcal{D}} \end{bmatrix}$$

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with respect to the decomposition $\mathcal{K} = \mathcal{H} \oplus (F^2(H_n) \otimes \mathcal{D})$. It was proved in [**14**] that the sequence $\mathcal{V} := [V_1, \dots, V_n]$ is the minimal isometric dilation of $\mathcal{T}$. Let $\mathcal{H}_0 := \mathcal{H}$ and

$$\mathcal{H}_k := \mathcal{H}_{k-1} \bigvee \biggl(\bigvee_{|\alpha|=1} V_\alpha \mathcal{H}_{k-1} \biggr) \quad \text{if } k \geqslant 2. \tag{2.1}$$

Notice that $\mathcal{K} = \bigvee_{k=0}^{\infty} \mathcal{H}_k$, $\mathcal{H}_k \subset \mathcal{H}_{k+1}$, and all subspaces $\mathcal{H}_k$ are invariant to each V_i^*, $i = 1, \dots, n$. On the other hand, we have $\mathcal{H}_1 = \mathcal{H} \oplus \mathcal{D}$ and

$$\mathcal{H}_k = \mathcal{H} \oplus \bigoplus_{|\alpha| \leqslant k-1} e_\alpha \otimes \mathcal{D} \quad \text{if } k \geqslant 2, \tag{2.2}$$

where $\{e_\alpha\}_{\alpha \in \mathbb{F}_n^+}$ is the canonical basis of $F^2(H_n)$ generated by $e_1, \dots, e_n$, i.e. $e_\alpha := e_{i_1} \otimes \cdots \otimes e_{i_k}$ if $\alpha := s_{i_1} \cdots s_{i_k} \in \mathbb{F}_n^+$, and $e_\alpha = 1$ if $\alpha = e$. Denote $\mathcal{V}_0 := \mathcal{T}$ and $\mathcal{V}_k := [V_{1,k}, \dots, V_{n,k}]$ if $k \geqslant 1$, where $V_{i,k} := P_{\mathcal{H}_k} V_i|_{\mathcal{H}_k}$, $i = 1, \dots, n$, and $P_{\mathcal{H}_k}$ is the orthogonal projection from $\mathcal{K}$ onto $\mathcal{H}_k$. Notice that $V_{i,k}$, $i = 1, \dots, n$, are partial isometries with orthogonal final spaces and initial space $\mathcal{H}_{k-1}$. It is easy to see that $\mathcal{V}$ is also the minimal isometric dilation of $\mathcal{V}_k$, $V_i^*|_{\mathcal{H}_k} = V_{i,k}^*$, and $V_i^* = \text{SOT-lim}_{k\to\infty} V_{i,k}^* P_{\mathcal{H}_k}$.

On the other hand, let us mention that $\mathcal{V}_{k+1}$ is the one-step dilation of $\mathcal{V}_k$, i.e. $\mathcal{H}_{k+1} = \mathcal{H}_k \oplus D_{\mathcal{V}_k}(\oplus_{j=1}^n \mathcal{H}_k)$, and, for each $i = 1, \dots, n$,

$$V_{i,k+1}(x \oplus y) = V_{i,k}x \oplus D_{\mathcal{V}_k}(\underbrace{0, \dots, 0}_{i-1 \text{ times}}, x, 0, \dots, 0)$$

for any $x \in \mathcal{H}_k$ and $y \in D_{\mathcal{V}_k}(\oplus_{j=1}^n \mathcal{H}_k)$. Given $A \in B(\mathcal{H})$ with $A \in C^*(T_1, \dots, T_n)'$, the commutant of the C^*-algebra generated by $T_1, \dots, T_n$, there exists a unique extension $\tilde{A}$ of A to the Hilbert space $\mathcal{K} = \mathcal{H} \oplus (F^2(H_n) \otimes \mathcal{D})$ such that

$$\tilde{A} V_i = V_i \tilde{A}, \quad i = 1, \dots, n. \tag{2.3}$$

Indeed, since $A \in C^*(T_1, \dots, T_n)'$, it is easy to see that

$$(\oplus_{j=1}^n A) D_{\mathcal{T}}(\underbrace{0, \dots, 0}_{i-1 \text{ times}}, h, 0, \dots, 0) = D_{\mathcal{T}}(\underbrace{0, \dots, 0}_{i-1 \text{ times}}, Ah, 0, \dots, 0)$$

for any $h \in \mathcal{H}$ and $i = 1, \dots, n$. Since $\mathcal{K} = \bigvee_{\alpha \in \mathbb{F}_n^+} V_\alpha \mathcal{H}$, one can see that $\tilde{A}$ is uniquely determined by condition (2.3), and, moreover,

$$\tilde{A} \biggl(\sum_{\alpha \in \mathbb{F}_n^+} e_\alpha \otimes d_{(\alpha)} \biggr) = \sum_{\alpha \in \mathbb{F}_n^+} e_\alpha \otimes (\oplus_{j=1}^n A) d_{(\alpha)}$$

for any

$$\sum_{\alpha \in \mathbb{F}_n^+} e_\alpha \otimes d_{(\alpha)} \in F^2(H_n) \otimes \mathcal{D}.$$

It is easy to see that the mapping $A \mapsto \tilde{A}$ from $C^*(T_1, \dots, T_n)'$ to $C^*(V_1, \dots, V_n)'$ is a $*$-monomorphism of C^*-algebras. Notice also that if we set $A_k := \tilde{A}|_{\mathcal{H}_k}$, $k = 1, 2, \dots$, then A_k is the unique extension of A_{k-1} to $\mathcal{H}_k$ such that $A_k \in C^*(V_{1,k}, \dots, V_{n,k})'$.

Let $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{K}_1$, $\mathcal{K}_2$ be Hilbert spaces and denote $\mathcal{H} := \mathcal{H}_1 \oplus \mathcal{H}_2$ and $\mathcal{K} =: \mathcal{K}_1 \oplus \mathcal{K}_2$. Any operator $T \in B(\mathcal{H}, \mathcal{K})$ can be written as an operator matrix

$$T = \begin{bmatrix} A & B \\ C & X \end{bmatrix}$$

with respect to the corresponding decompositions of $\mathcal{H}$ and $\mathcal{K}$. It was proved in [**12**] that

$$\inf_{X \in B(\mathcal{H}_2, \mathcal{K}_2)} \left\| \begin{bmatrix} A & B \\ C & X \end{bmatrix} \right\| = \max \left\{ \left\| \begin{bmatrix} A \\ C \end{bmatrix} \right\|, \left\| \begin{bmatrix} A & B \end{bmatrix} \right\| \right\}. \tag{2.4}$$

Moreover, there are sequences of real numbers $\{c_k\}$, $\{d_k\}$ such that

$$Y := \text{WOT-}\lim_{k \to \infty} -c_k C(I - d_k A^* A)^{-1} A^* B$$

exists and the above-mentioned infimum is equal to

$$\left\| \begin{bmatrix} A & B \\ C & Y \end{bmatrix} \right\|.$$

In what follows, we find a non-commutative multivariable analogue of Parrott's generalization [**12**] of the Sz.-Nagy–Foiaş commutant lifting theorem [**24**].

Theorem 2.1. *Let $\mathcal{T} := [T_1, \dots, T_n]$ be a contractive sequence of operators on a Hilbert space $\mathcal{H}$ and let $\mathcal{V} := [V_1, \dots, V_n]$ be its minimal isometric dilation on the Hilbert space $\mathcal{K} \supseteq \mathcal{H}$. If $X \in B(\mathcal{H})$ and $XT_i = T_i X$ for any $i = 1, \dots, n$, then there exists $X_\infty \in B(\mathcal{K})$ satisfying the following properties:*

(i) $X_\infty V_i = V_i X_\infty$, *for any* $i = 1, \dots, n$;

(ii) $X_\infty \tilde{A} = \tilde{A} X_\infty$ *for any* $A \in C^*(T_1, \dots T_n, X)'$;

(iii) $X_\infty^*|_{\mathcal{H}} = X^*$ *and* $P_{\mathcal{H}} \tilde{A} V_\alpha X_\infty^{*m} V_\beta^*|_{\mathcal{H}} = A T_\alpha X^{*m} T_\beta^*$, *for any* $\alpha, \beta \in \mathbb{F}_n^+$, $A \in C^*(T_1, \dots T_n, X)'$, *and* $m \in \mathbb{N}$;

(iv) $\|X_\infty\| = \|X\|$.

Proof. We use the notation and preliminaries preceding the theorem. Let us construct a sequence of operators $X_k \in B(\mathcal{H}_k)$, $k = 1, 2, \dots$, with the following properties:

(a) $X_k V_{i,k} = V_{i,k} X_k$, for any $i = 1, \dots, n$;

(b) $X_k A_k = A_k X_k$ for any $A \in C^*(T_1, \dots T_n, X)'$;

(c) $X_k^*|_{\mathcal{H}_{k-1}} = X_{k-1}^*$ and $P_{\mathcal{H}_{k-1}} A_k V_{\alpha,k} X_k^{*m} V_{\beta,k}^*|_{\mathcal{H}_{k-1}} = A_{k-1} V_{\alpha,k-1} X_{k-1}^{*m} V_{\beta,k-1}^*$, for any $A \in C^*(T_1, \dots T_n, X)'$ and $\alpha, \beta \in \mathbb{F}_n^+$, $m \in \mathbb{N}$;

(d) $\|X_k\| = \|X_{k-1}\| = \|X\|$.

Once this is established, it will be a routine exercise to show that the limit $X_\infty := \text{SOT-lim}_{k\to\infty} X_k P_{\mathcal{H}_k}$ exists and X_∞ has the stated properties in the theorem.

Our first step is to show that X_1 exists with the above-mentioned properties, when $k = 1$. For each $i = 1, 2, \dots, n$, let $V_{i,1} : \mathcal{H}_1 := \mathcal{H} \oplus \mathcal{D} \to \mathcal{H}_1$ be defined by

$$V_{i,1}(h \oplus d) = T_i h \oplus D(\underbrace{0, \dots, 0}_{i-1 \text{ times}}, h, 0, \dots, 0).$$

Notice that $V_{i,1}$, $i = 1, 2, \dots, n$, are partial isometries with orthogonal ranges and initial space $\mathcal{H} \oplus \{0\}$. Define $Z \in B(\mathcal{H}_1)$ by $Z(h \oplus d) := Xh \oplus 0$. Let

$$\mathcal{N}_1 := \mathcal{H} \oplus \{0\}, \qquad \mathcal{N}_2 := \mathcal{H}_1 \ominus \mathcal{N}_1 = \{0\} \oplus \mathcal{D}, \qquad \mathcal{M}_1 := V_{1,1}\mathcal{N}_1 + \dots + V_{n,1}\mathcal{N}_1,$$

and $\mathcal{M}_2 := \mathcal{H}_1 \ominus \mathcal{M}_1$.

Now we shall prove that the set of all operators $X_1 \in B(\mathcal{H}_1)$ satisfying the relations $P_{\mathcal{H}} X_1|_{\mathcal{H}} = X$ and $X_1 V_{i,1} = V_{i,1} X_1$, for any $i = 1, \dots, n$, is non-empty and is exactly $L_1 + B(\mathcal{M}_2, \mathcal{N}_2)$ where $B(\mathcal{M}_2, \mathcal{N}_2)$ is embedded in $B(\mathcal{H}_1)$, and $L_1 \in B(\mathcal{H}_1)$ is defined by

$$L_1 := \sum_{i=1}^n V_{i,1} X V_{i,1}^* + Z P_{\mathcal{M}_2}. \tag{2.5}$$

Since $\sum_{i=1}^n V_{i,1} V_{i,1}^* = P_{\mathcal{M}_1}$, $V_{i,1}^* V_{i,1} = P_{\mathcal{N}_1}$ for any $i = 1, 2, \dots, n$, and $V_{i,1}^* V_{j,1} = 0$ if $i \neq j$, we infer that

$$L_1 P_{\mathcal{M}_1} = \left(\sum_{i=1}^n V_{i,1} X V_{i,1}^*\right) P_{\mathcal{M}_1} = \left(\sum_{i=1}^n V_{i,1} X V_{i,1}^*\right)\left(\sum_{j=1}^n V_{j,1} V_{j,1}^*\right) = \sum_{i=1}^n V_{i,1} X V_{i,1}^*.$$

Therefore,

$$L_1 P_{\mathcal{M}_1} = \sum_{i=1}^n V_{i,1} X V_{i,1}^*. \tag{2.6}$$

On the other hand, we have

$$\begin{aligned} P_{\mathcal{N}_1} L_1 &= P_{\mathcal{N}_1} L_1 P_{\mathcal{M}_1} + P_{\mathcal{N}_1} L_1 P_{\mathcal{M}_2} \\ &= P_{\mathcal{N}_1}\left(\sum_{i=1}^n V_{i,1} X V_{i,1}^*\right) + P_{\mathcal{N}_1}\left(\sum_{i=1}^n V_{i,1} X V_{i,1}^* + Z P_{\mathcal{M}_2}\right) P_{\mathcal{M}_2} \\ &= \sum_{i=1}^n T_i X V_{i,1}^* + P_{\mathcal{N}_1} Z P_{\mathcal{M}_2} = \sum_{i=1}^n X T_i V_{i,1}^* + P_{\mathcal{N}_1} Z P_{\mathcal{M}_2} \\ &= X P_{\mathcal{N}_1} P_{\mathcal{M}_1} + P_{\mathcal{N}_1} Z P_{\mathcal{M}_2} = P_{\mathcal{N}_1} Z (P_{\mathcal{M}_1} + P_{\mathcal{M}_2}) = P_{\mathcal{N}_1} Z = Z. \end{aligned}$$

Therefore,

$$P_{\mathcal{N}_1} L_1 = Z, \tag{2.7}$$

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which implies that $P_{\mathcal{N}_1}L_1|_{\mathcal{N}_1} = X$. Using the above-mentioned relations and $V_iP_{\mathcal{N}_1} = V_i$, $i = 1, \ldots, n$, we deduce

$$\begin{aligned} L_1V_{i,1} &= L_1P_{\mathcal{M}_1}V_{i,1} = \left(\sum_{j=1}^n V_{j,1}XV_{j,1}^*\right)V_{i,1} \\ &= V_{i,1}XV_{i,1}^*V_{i,1} = V_{i,1}XP_{\mathcal{N}_1} = V_{i,1}Z \\ &= V_{i,1}P_{\mathcal{N}_1}L_1. \end{aligned}$$

Therefore,

$$L_1V_{i,1} = V_{i,1}L_1, \quad \text{for any } i = 1, \ldots, n.$$

Now, suppose that $X_1 \in B(\mathcal{H}_1)$ satisfies $P_{\mathcal{H}}X_1|_{\mathcal{H}} = X$ and $X_1V_{i,1} = V_{i,1}X_1$ for any $i = 1, \ldots, n$. Since $X_1 - L_1$ commutes with each $V_{i,1}$, $i = 1, \ldots, n$, and $\mathcal{N}_2 = \ker V_{i,1}$, we infer that $\mathcal{N}_2$ is invariant for $X_1 - L_1$. Taking into account that $P_{\mathcal{N}_1}(X_1 - L_1)|_{\mathcal{N}_1} = 0$, we deduce that the range of $X_1 - L_1$ is in $\mathcal{N}_2$.

On the other hand, using $V_{i,1}P_{\mathcal{N}_1} = V_{i,1}$, we obtain

$$\begin{aligned} (X_1 - L_1)P_{\mathcal{M}_1} &= (X_1 - L_1)\sum_{i=1}^n V_{i,1}V_{i,1}^* = \sum_{i=1}^n V_{i,1}(X_1 - L_1)V_{i,1}^* \\ &= \sum_{i=1}^n V_{i,1}P_{\mathcal{N}_1}(X_1 - L_1)|_{\mathcal{N}_1}V_{i,1}^* = 0. \end{aligned}$$

This shows that $X_1 - L_1 \in B(\mathcal{M}_2, \mathcal{H}_1)$. Summing up, we deduce that $X_1 - L_1 \in B(\mathcal{M}_2, \mathcal{N}_2)$, which shows that $X_1 = L_1 + Y$ for some $Y \in B(\mathcal{M}_2, \mathcal{N}_2)$.

Conversely, let $Y \in B(\mathcal{M}_2, \mathcal{N}_2)$. It is clear that $P_{\mathcal{N}_1}(L_1 + Y)|_{\mathcal{N}_1} = X$, and, since $V_{i,1}Y = YV_{i,1} = 0$, $i = 1, \ldots, n$, we infer that $L_1 + Y$ commutes with each $V_{i,1}$, $i = 1, \ldots, n$. Therefore, we proved that the set

$$\{X_1 \in B(\mathcal{H}_1) : P_{\mathcal{H}}X_1|_{\mathcal{H}} = X \text{ and } V_{i,1}X_1 = X_1V_{i,1}\}$$

is non-empty and equal to $L_1 + B(\mathcal{M}_2, \mathcal{N}_2)$, where L_1 is given by (2.5). According to (2.4), there exists X_1 with

$$\|X_1\| = \max\{\|L_1P_{\mathcal{M}_1}\|, \|P_{\mathcal{N}_1}L_1\|\}.$$

Taking into account (2.6), (2.7), we obtain $\|X_1\| = \|X\|$. Moreover, if we set

$$\begin{aligned} \Lambda &:= P_{\mathcal{N}_1}L_1P_{\mathcal{M}_1} = ZP_{\mathcal{M}_1}, \\ \Delta &:= P_{\mathcal{N}_2}L_1P_{\mathcal{M}_1} = P_{\mathcal{N}_2}\left(\sum_{i=1}^n V_{i,1}ZV_{i,1}^*\right), \\ \Gamma &:= P_{\mathcal{N}_1}L_1P_{\mathcal{M}_2} = ZP_{\mathcal{M}_2}, \end{aligned}$$

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then we may choose X_1 to be of the form

$$X_1 = \Lambda + \Delta + \Gamma + \text{WOT-}\lim_{k\to\infty} -c_k\Delta(I - d_k\Lambda\Lambda^*)^{-1}\Lambda^*\Gamma$$

for some sequences of real numbers $\{c_k\}$, $\{d_k\}$.

Now, let $A \in C^*(T_1,\dots,T_n,X)'$ and A_1 be its canonical extension to $\mathcal{H}_1$. Since $A_1 \in C^*(V_{1,1},\dots,V_{n,1})'$ and $P_{\mathcal{M}_1} = \sum_{i=1}^n V_{i,1}V_{i,1}^*$, it is clear that $\mathcal{M}_1$ and $\mathcal{M}_2$ are invariant under A_1. It follows from the definition of A_1 that $\mathcal{N}_1$ and $\mathcal{N}_2$ are also invariant to A_1. Since $A_1 \in C^*(V_{1,1},\dots,V_{n,1},Z)'$, it follows that A_1 commutes with Λ, Δ, Γ, and, hence, with X_1. Let us show that

$$P_{\mathcal{H}}A_1V_{\alpha,1}X_1^{*m}V_{\beta,1}^*|_{\mathcal{H}} = AT_\alpha X^{*m}T_\beta^*, \tag{2.8}$$

for any $A \in C^*(T_1,\dots T_n,X)'$ and $\alpha,\beta \in \mathbb{F}_n^+$, $m \in \mathbb{N}$. Since $\mathcal{N}_2$ is invariant for X_1 and $P_{\mathcal{H}}X_1 = Z$, we have

$$X_1 = \begin{bmatrix} X & 0 \\ * & * \end{bmatrix}$$

with respect to the orthogonal decomposition $\mathcal{H}_1 = \mathcal{H} \oplus \mathcal{H}^\perp$, where $\mathcal{H}^\perp := \mathcal{H}_1 \ominus \mathcal{H}$. Notice that $X_1^*|_{\mathcal{H}} = X^*$. On the other hand, the matrices of $V_{i,1}$ and A_1 are of the form

$$V_{i,1} = \begin{bmatrix} T_i & 0 \\ * & 0 \end{bmatrix} \quad \text{and} \quad A_1 = \begin{bmatrix} A & 0 \\ 0 & * \end{bmatrix}.$$

Hence,

$$A_1V_{\alpha,1}X_1^mV_{\beta,1}^*|_{\mathcal{H}} = \begin{bmatrix} AT_\alpha X^mT_\beta^* & 0 \\ * & 0 \end{bmatrix},$$

which proves (2.8).

Now, since X_1 always exists, one can replace $T_1,\dots,T_n$ and X by $V_{1,1},\dots,V_{n,1}$ and X_1 and iterate the process, obtaining at the kth step an operator $X_k \in B(\mathcal{H}_k)$ satisfying properties (a)–(d). The proof is complete. □

As in the classical case, we expect Theorem 2.1 to yield interpolation theorems for analytic functions from the open unit ball of $\mathbb{C}^n$ into a von Neumann algebra. We need to consider some preliminary results. According to Theorem 1.2 from [**19**], the commutant of F_n^∞, which we denote by R_n^∞, is equal to $U^*F_n^\infty U$, where U is the unitary operator on $F^2(H_n)$ defined by $U(e_{i_1}\otimes e_{i_2}\otimes\cdots\otimes e_{i_k}) = e_{i_k}\otimes\cdots\otimes e_{i_2}\otimes e_{i_1}$. Moreover, the commutant of R_n^∞ is equal to F_n^∞.

For each $\mathcal{S} \subseteq B(\mathcal{H})$, we denote its commutant by $\mathcal{S}'$. Define the isometries $Q_\alpha : \mathcal{H} \to F^2(H_n)\otimes\mathcal{H}$, $\alpha \in \mathbb{F}_n^+$, by $Q_\alpha h := e_\alpha \otimes h$, $h \in \mathcal{H}$.

Lemma 2.2. *If $\mathcal{S} \subseteq B(\mathcal{H})$ and f is in the commutant of*

$$\{U^*S_iU \otimes I_{\mathcal{H}} : i = 1,\dots,n\} \cup \{I_{F^2(H_n)} \otimes Y : Y \in \mathcal{S}\},$$

then the operators $Q_\alpha^* f Q_e$, $\alpha \in \mathbb{F}_n^+$, are in $\mathcal{S}'$, and f has a formal Fourier expansion

$$f \sim \sum_{\alpha\in\mathbb{F}_n^+} S_\alpha \otimes Q_\alpha^* f Q_e.$$

Proof. Since f commutes with $U^* S_i U \otimes I_{\mathcal{H}}$, $i = 1, \dots, n$, we infer that it is uniquely determined by

$$f(1 \otimes h) = \sum_{\alpha\in\mathbb{F}_n^+} e_\alpha \otimes Q_\alpha^* f Q_e h, \quad h \in \mathcal{H}.$$

Indeed, notice that

$$\begin{aligned} f(e_\beta \otimes h) &= f(U^* S_\beta U \otimes I_{\mathcal{H}})(1 \otimes h) = (U^* S_\beta U \otimes I_{\mathcal{H}}) f(1 \otimes h) \\ &= \sum_{\alpha\in\mathbb{F}_n^+} e_{\alpha\beta} \otimes Q_\alpha^* f Q_e h. \end{aligned}$$

On the other hand, since f commutes with each $I_{F^2(H_n)} \otimes Y$, $Y \in \mathcal{S}$, we have

$$\begin{aligned} (I_{F^2(H_n)} \otimes Y) f(1 \otimes h) &= \sum_{\alpha\in\mathbb{F}_n^+} e_\alpha \otimes Y Q_\alpha^* f Q_e h \\ &= f(I_{F^2(H_n)} \otimes Y)(1 \otimes h) = \sum_{\alpha\in\mathbb{F}_n^+} e_\alpha \otimes Q_\alpha^* f Q_e Y h. \end{aligned}$$

Hence, $Q_\alpha^* f Q_e \in \mathcal{S}'$. □

We denote by $F_n^\infty \otimes \mathcal{S}$ (respectively, $R_n^\infty \otimes \mathcal{S}$) the spatial tensor product, and by $F_n^\infty \bar{\otimes} \mathcal{S}'$ (respectively, $R_n^\infty \bar{\otimes} \mathcal{S}'$) the WOT-closed algebra generated by the spatial tensor product of the two algebras. The following result is a Tomita-type theorem in a non-self-adjoint setting.

Theorem 2.3. *If $\mathcal{S} \subseteq B(\mathcal{H})$ and $I_{\mathcal{H}} \in \mathcal{S}$, then*

$$(R_n^\infty \otimes \mathcal{S})' = F_n^\infty \bar{\otimes} \mathcal{S}' \quad \text{and} \quad (F_n^\infty \otimes \mathcal{S})' = R_n^\infty \bar{\otimes} \mathcal{S}'.$$

Proof. Since $(R_n^\infty)' = F_n^\infty$, it is easy to see that $F_n^\infty \bar{\otimes} \mathcal{S}' \subseteq (R_n^\infty \otimes \mathcal{S})'$. Conversely, assume that f is in $(R_n^\infty \otimes \mathcal{S})'$. Then f belongs to the commutant of the set $\{U^* S_i U \otimes I_{\mathcal{H}} : i = 1, \dots, n\} \cup \{I \otimes Y : Y \in \mathcal{S}\}$. According to Lemma 2.2, the operators $W_{(\alpha)} := Q_\alpha^* f Q_e$, $\alpha \in \mathbb{F}_n^+$, are in $\mathcal{S}'$, and f has a formal Fourier expansion

$$f \sim \sum_{\alpha\in\mathbb{F}_n^+} S_\alpha \otimes W_{(\alpha)}.$$

For each $0 < r < 1$, define

$$f_r := \sum_{\alpha\in\mathbb{F}_n^+} r^{|\alpha|} S_\alpha \otimes W_{(\alpha)}.$$

Notice that the convergence of this series is in the uniform norm. Indeed, since

$$f(1 \otimes h) = \sum_{\alpha\in\mathbb{F}_n^+} e_\alpha \otimes W_{(\alpha)} h, \quad h \in \mathcal{H},$$

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we infer that

$$\sum_{\alpha\in\mathbb{F}_n^+} W_{(\alpha)}^* W_{(\alpha)} \leqslant \|f\| I_{\mathcal{H}}. \tag{2.9}$$

On the other hand, we have

$$\begin{aligned}\sum_{k=0}^{\infty} r^k \Big\| \sum_{|\alpha|=k} S_\alpha \otimes W_{(\alpha)} \Big\| &= \sum_{k=0}^{\infty} r^k \Big\| \sum_{|\alpha|=k} W_{(\alpha)}^* W_{(\alpha)} \Big\|^{1/2} \\ &\leqslant \Big(\sum_{k=0}^{\infty} r^k \Big) \Big\| \sum_{\alpha\in\mathbb{F}_n^+} W_{(\alpha)}^* W_{(\alpha)} \Big\|^{1/2}.\end{aligned}$$

Therefore, $f_r \in F_n^\infty \bar{\otimes} \mathcal{S}'$. Moreover, f_r is in the norm-closure of the algebra generated by the operators $S_i \otimes I$, $I \otimes Z$, where $Z \in \mathcal{S}'$, $i = 1, \dots, n$. According to the non-commutative von Neumann inequality, we have $\|f_r\| \leqslant \|f\|$ for any $0 < r < 1$. Now, let us prove that

$$\text{SOT-}\lim_{r\to 1} f_r = f. \tag{2.10}$$

For any $h \in \mathcal{H}$, $\beta \in \mathbb{F}_n^+$, we have

$$\begin{aligned}\|f_r(e_\beta \otimes h) - f(e_\beta \otimes h)\|^2 &= \Big\| \sum_{\alpha\in\mathbb{F}_n^+} (r^{|\alpha|} - 1) e_{\alpha\beta} \otimes W_{(\alpha)} h \Big\|^2 \\ &= \sum_{\alpha\in\mathbb{F}_n^+} (r^{|\alpha|} - 1)^2 \|W_{(\alpha)} h\|^2 \\ &= \sum_{k=1}^{\infty} (r^k - 1)^2 \sum_{|\alpha|=k} \|W_{(\alpha)} h\|^2.\end{aligned}$$

Using (2.9), we deduce that $\|f_r(e_\beta \otimes h) - f(e_\beta \otimes h)\| \to 0$, as $r \to 1$. Since $\|f_r\| \leqslant \|f\|$, a standard argument shows that (2.10) follows. Hence, $f \in F_n^\infty \bar{\otimes} \mathcal{S}'$ and the proof is complete. □

One can easily see from the proof of Theorem 2.3 that $F_n^\infty \bar{\otimes} \mathcal{S}'$ is the WOT-closed algebra generated by the operators $S_i \otimes I$, $I \otimes Z$, where $i = 1, \dots, n$, and $Z \in \mathcal{S}'$, and any $f \in F_n^\infty \bar{\otimes} \mathcal{S}'$ has a formal Fourier expansion. On the other hand, if $\mathcal{M}$ is a von Neumann algebra, then $(F_n^\infty \bar{\otimes} \mathcal{M})'' = F_n^\infty \bar{\otimes} \mathcal{M}$.

A complete description of the invariant subspace structure of F_n^∞ was obtained in [**15**, Theorem 2.2] (even in a more general setting). A subspace $\mathcal{N}$ of $F^2(H_n)$ is invariant under $S_1, \dots, S_n$ if and only if $\mathcal{N} = \bigoplus_{\lambda\in\Lambda} U^* \varphi_\lambda U[F^2(H_n)]$, for some family $\{\varphi_\lambda \in F_n^\infty : \lambda \in \Lambda\}$ of isometries with orthogonal ranges (see also [**8**, **19**]).

In what follows we use Theorems 2.1 and 2.3 in order to extend Sarason's interpolation result [**23**] to our setting.

Theorem 2.4. *Let $\mathcal{M} \subseteq B(\mathcal{K})$ be such that $I_{\mathcal{H}} \in \mathcal{M} = \mathcal{M}^*$ and let $\mathcal{N} \subseteq F^2(H_n)$ be an invariant subspace under $S_1^*, \dots, S_n^*$. If $X \in B(\mathcal{N} \otimes \mathcal{K})$ commutes with $P_{\mathcal{N}\otimes\mathcal{K}}(F_n^\infty \otimes$*

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$\mathcal{M})|_{\mathcal{N}\otimes\mathcal{K}}$, then there exists $\Phi \in R_n^\infty \bar{\otimes} \mathcal{M}'$ such that

$$P_{\mathcal{N}\otimes\mathcal{K}}\Phi|_{\mathcal{N}\otimes\mathcal{K}} = X, \quad \|\Phi\| = \|X\|,$$

where $P_{\mathcal{N}\otimes\mathcal{K}}$ is the orthogonal projection of $F^2(H_n)\otimes\mathcal{K}$ onto $\mathcal{N}\otimes\mathcal{K}$. If, in addition, $U\mathcal{N}$ is an invariant subspace under $S_1^*,\dots,S_n^*$, then

$$[P_{\mathcal{N}\otimes\mathcal{K}}(F_n^\infty \otimes \mathcal{M})|_{\mathcal{N}\otimes\mathcal{K}}]' = P_{\mathcal{N}\otimes\mathcal{K}}[(F_n^\infty)' \bar{\otimes} \mathcal{M}']|_{\mathcal{N}\otimes\mathcal{K}}.$$

Proof. Let $B \in \mathcal{M}$, and define $\tilde{B} \in B(F^2(H_n)\otimes\mathcal{K})$ by $\tilde{B} := I \otimes B$. Since X commutes with $P_{\mathcal{N}\otimes\mathcal{K}}(F_n^\infty \otimes \mathcal{M})|_{\mathcal{N}\otimes\mathcal{K}}$ and $I_{\mathcal{H}} \in \mathcal{M}$, we deduce that X commutes with $T_i := P_{\mathcal{N}}S_i|_{\mathcal{N}} \otimes I_{\mathcal{K}}$, $i = 1,\dots,n$, and also with $\tilde{B}|_{\mathcal{N}\otimes\mathcal{K}} = I_{\mathcal{N}} \otimes B$ for any $B \in \mathcal{M}$. Since $\mathcal{M} = \mathcal{M}^*$, we have $\tilde{B}|_{\mathcal{N}\otimes\mathcal{K}} \in C^*(T_1,\dots,T_n,X)'$. On the other hand, $\mathcal{N}$ is invariant under each $S_1^*,\dots,S_n^*$, and $[P_{\mathcal{N}}S_1|_{\mathcal{N}},\dots,P_{\mathcal{N}}S_n|_{\mathcal{N}}]$ is a C_0-row contraction. Using [**14**, Proposition 2.3] we infer that its minimal isometric dilation is $[S_1,\dots,S_n]$. Therefore, the minimal isometric dilation of $[T_1,\dots,T_n]$ is $[S_1\otimes I_{\mathcal{K}},\dots,S_n\otimes I_{\mathcal{K}}]$. On the other hand, $\tilde{B}$ commutes with $\{S_i\otimes I_{\mathcal{K}}, S_i^*\otimes I_{\mathcal{K}},\ i=1,\dots,n\}$. Notice that $\tilde{B}$ must be the canonical extension of $\tilde{B}|_{\mathcal{N}\otimes\mathcal{K}}$. According to Theorem 2.1, there exists $X_\infty \in B(F^2(H_n)\otimes\mathcal{K})$ such that

(i) $X_\infty(S_i\otimes I_{\mathcal{K}}) = (S_i\otimes I_{\mathcal{K}})X_\infty$, for any $i = 1,\dots,n$;

(ii) $X_\infty\tilde{B} = \tilde{B}X_\infty$ for any $B \in \mathcal{M}$;

(iii) $\|X_\infty\| = \|X\|$;

(iv) $X_\infty^*|_{\mathcal{N}\otimes\mathcal{K}} = X^*$.

Using Theorem 2.3, we find Φ in $R_n^\infty \bar{\otimes} \mathcal{M}'$ such that $X_\infty = \Phi$. Now, assume that $U\mathcal{N}$ is an invariant subspace under $S_1^*,\dots,S_n^*$, and let $X := P_{\mathcal{N}\otimes\mathcal{K}}\Psi|_{\mathcal{N}\otimes\mathcal{K}}$ with $\Psi \in R_n^\infty \bar{\otimes} \mathcal{M}'$. Notice that X commutes with $P_{\mathcal{N}\otimes\mathcal{K}}(F_n^\infty \otimes \mathcal{M})|_{\mathcal{N}\otimes\mathcal{K}}$. The proof is complete. □

Corollary 2.5. *Let $\mathcal{M}$ be a von Neumann algebra acting on a Hilbert space $\mathcal{K}$ and let $\mathcal{N} \subseteq F^2(H_n)$ be a subspace with the property that $\mathcal{N}$ and $U\mathcal{N}$ are invariant under $S_1^*,\dots,S_n^*$. If $\Psi \in R_n^\infty \bar{\otimes} \mathcal{M}$, then there is $\Phi \in R_n^\infty \bar{\otimes} \mathcal{M}$ such that $\|\Phi\| = \|P_{\mathcal{N}\otimes\mathcal{K}}\Psi|_{\mathcal{N}\otimes\mathcal{K}}\|$ and $P_{\mathcal{N}\otimes\mathcal{K}}\Phi|_{\mathcal{N}\otimes\mathcal{K}} = P_{\mathcal{N}\otimes\mathcal{K}}\Psi|_{\mathcal{N}\otimes\mathcal{K}}$.*

Proof. Since $X := P_{\mathcal{N}\otimes\mathcal{K}}\Psi|_{\mathcal{N}\otimes\mathcal{K}}$ commutes with $P_{\mathcal{N}\otimes\mathcal{K}}(F_n^\infty \bar{\otimes} \mathcal{M}')|_{\mathcal{N}\otimes\mathcal{K}}$, we can use Theorem 2.4 to find $\Phi \in R_n^\infty \bar{\otimes} (\mathcal{M}')'$ with the required properties. Since $(\mathcal{M}')' = \mathcal{M}$, according to the double commutant theorem, the result follows. □

Let J be a WOT-closed two-sided ideal of F_n^∞. Define $J(1) := \{\psi(1); \psi \in J\}$ and $\mathcal{N}_J := F^2(H_n) \ominus J(1)$. Let us remark that $\mathcal{N}_J$ and $U\mathcal{N}_J$ are invariant subspaces under S_i^*, $i = 1,\dots,n$. It was proved in [**3**, **7**] that the quotient algebra F_n^∞/J is completely isometricaly isomorphic to $P_{\mathcal{N}_J}F_n^\infty|_{\mathcal{N}_J}$. Using this result and Theorem 2.4, one can deduce the following Tomita-type commutant result.

Corollary 2.6. *If J is a WOT-closed two-sided ideal in F_n^∞ and $\mathcal{M} \subseteq B(\mathcal{K})$ such that $I_\mathcal{H} \in \mathcal{M} = \mathcal{M}^*$, then*

$$[(F_n^\infty/J) \otimes \mathcal{M}]' = (F_n^\infty/J)' \,\bar{\otimes}\, (\mathcal{M})'.$$

In the particular case when J is the WOT-closed two-sided ideal in F_n^∞ generated by $\{e_j \otimes e_i - e_i \otimes e_j : 1 \leqslant i < j \leqslant n\}$, we have $\mathcal{N}_J = F_s^2$, the symmetric Fock space, and F_n^∞/J can be identified with $\mathcal{W}_n^\infty := P_{F_s^2} F_n^\infty|_{F_s^2}$.

Corollary 2.7. *Let $\mathcal{M} \subseteq B(\mathcal{K})$ with $I_\mathcal{H} \in \mathcal{M} = \mathcal{M}^*$ and let $\mathcal{E} \subseteq F_s^2$ be an invariant subspace under each $B_i^* := S_i^*|_{F_s^2}$, $i = 1, \dots, n$. If $X \in B(\mathcal{E} \otimes \mathcal{K})$ commutes with $P_{\mathcal{E}\otimes\mathcal{K}}(\mathcal{W}_n^\infty \otimes \mathcal{M})|_{\mathcal{E}\otimes\mathcal{K}}$, then there exists $g \in \mathcal{W}_n^\infty \,\bar{\otimes}\, \mathcal{M}'$ such that*

$$P_{\mathcal{E}\otimes\mathcal{K}} g|_{\mathcal{E}\otimes\mathcal{K}} = X, \quad \|g\| = \|X\|.$$

Moreover, $[P_{\mathcal{E}\otimes\mathcal{K}}(\mathcal{W}_n^\infty \otimes \mathcal{M})|_{\mathcal{E}\otimes\mathcal{K}}]' = P_{\mathcal{E}\otimes\mathcal{K}}[\mathcal{W}_n^\infty \,\bar{\otimes}\, \mathcal{M}']|_{\mathcal{E}\otimes\mathcal{K}}$.

Proof. Since F_s^2 is invariant to S_i^*, $i = 1, \dots, n$, it is easy to see that $\mathcal{E}$ has the same property. Taking into account that $\mathcal{W}_n^\infty$ is the compression of F_n^∞ to the symmetric Fock space, we can see that X commutes with $P_{\mathcal{E}\otimes\mathcal{K}}(F_n^\infty \otimes \mathcal{M})|_{\mathcal{E}\otimes\mathcal{K}}$. Applying Theorem 2.4, we find $f \in F_n^\infty \,\bar{\otimes}\, \mathcal{M}'$ such that $P_{\mathcal{E}\otimes\mathcal{K}}(U^* \otimes I) f (U \otimes I)|_{\mathcal{E}\otimes\mathcal{K}} = X$ and $\|X\| = \|f\|$. Hence, $P_{\mathcal{E}\otimes\mathcal{K}} f|_{\mathcal{E}\otimes\mathcal{K}} = X$, and, if we set $g := P_{F_s^2\otimes\mathcal{K}} f|_{F_s^2\otimes\mathcal{K}}$, then $P_{\mathcal{E}\otimes\mathcal{K}} g|_{\mathcal{E}\otimes\mathcal{K}} = X$ and $\|X\| \leqslant \|g\| \leqslant \|f\| = \|X\|$. This shows that $\|X\| = \|g\|$, and the proof is complete. □

Let $\mathcal{M}$ be a von Neumann algebra acting on a Hilbert space $\mathcal{K}$. According to the non-commutative von Neumann inequality, if $\phi \in F_n^\infty \,\bar{\otimes}\, \mathcal{M}$, then the map $\lambda \mapsto \phi(\lambda)$ is in $H^\infty(\mathbb{B}_n) \,\bar{\otimes}\, \mathcal{M}$, where $\mathbb{B}_n := \{z \in \mathbb{C}^n : |z| < 1\}$. A consequence of Theorem 2.4 is the following Nevanlinna–Pick-type interpolation problem for the algebra $F_n^\infty \,\bar{\otimes}\, \mathcal{M}$.

Corollary 2.8. *Let $\lambda_1, \dots, \lambda_k$ be k distinct points in $\mathbb{B}_n$ and let $W_1, \dots, W_k$ be in the unit ball of a von Neumann algebra $\mathcal{M}$ acting on a Hilbert space $\mathcal{K}$. Then there exists $\Phi \in F_n^\infty \,\bar{\otimes}\, \mathcal{M}$, such that $\|\Phi\| \leqslant 1$ and $\Phi(\lambda_j) = W_j$, $j = 1, 2, \dots, k$, if and only if the operator matrix*

$$\left[\frac{I_\mathcal{K} - W_j W_i^*}{1 - \langle \lambda_j, \lambda_i \rangle}\right]_{i,j=1,2,\dots,k}$$

is positive definite.

Proof. Construct $\Psi \in R_n^\infty \,\bar{\otimes}\, \mathcal{M}$ such that $\Psi(\lambda_j) = W_j$, $j = 1, 2, \dots, k$. Define $X := P_{\mathcal{N}\otimes\mathcal{K}} \Psi|_{\mathcal{N}\otimes\mathcal{K}}$, with $\mathcal{N}$ as defined in Theorem 2.4 from [**3**], and apply Corollary 2.5. The rest of the proof is similar to [**3**, Theorem 2.4]. □

Let $\mathcal{P}_m$ be the set of all polynomials in $F^2(H_n)$ of degree less than or equal to m, and let $\mathcal{P}_m^\infty := \{p(S_1, \dots, S_n) : p \in \mathcal{P}_m\}$. Let $J_{>m}^\infty$ be the WOT-closed two-sided ideal of F_n^∞ generated by $\{S_\alpha : \alpha \in \mathbb{F}_n^+,\ |\alpha| = m + 1\}$. Another consequence of Theorem 2.4 (when $\mathcal{N} := \mathcal{P}_m$) is the following result that generalizes the non-commutative Carathéodory interpolation problem [**19**, Corollary 4.4] to our setting.

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Corollary 2.9. *Let $\mathcal{M}$ be a von Neumann algebra acting on a Hilbert space $\mathcal{K}$, and let $f \in \mathcal{P}_m^\infty \otimes \mathcal{M}$. Then*

$$\operatorname{dist}(f, J_{>m}^\infty \bar{\otimes} \mathcal{M}) = \|P_{\mathcal{P}_m \otimes \mathcal{K}} f|_{\mathcal{P}_m \otimes \mathcal{K}}\|.$$

3. Non-commutative Poisson transforms and functional calculus

A variant of the non-commutative Poisson transform (see [**22**, § 8]) is used to extend the von Neumann-type inequalities from [**4**, **12**, **16**, **20**, **22**, **26**]. This will lead to a generalization of the functional calculus for contractive sequences of class C_0 (see [**18**]), which also extends the Sz.-Nagy–Foiaş H^∞-functional calculus for $C_{\cdot 0}$ (see [**25**]).

A sequence $\mathcal{T} := [T_1, \dots, T_n]$, $T_i \in B(\mathcal{H})$, is called C_0-row contraction if $\mathcal{T}$ is a row contraction and

$$\text{SOT-}\lim_{k\to\infty} \sum_{\alpha \in \mathbb{F}_n^+,\ |\alpha| = k} T_\alpha T_\alpha^* = 0.$$

For example, if $\sum_{i=1}^n T_i T_i^* \leqslant a I_{\mathcal{H}}$ for some $a < 1$, then $\mathcal{T}$ is a C_0-row contraction. Suppose that $\mathcal{T}$ is a C_0-row contraction and let $\Delta := I_{\mathcal{H}} - \sum_{i=1}^n T_i T_i^*$. The Poisson kernel $K(\{T_i\})$ associated to $\mathcal{T}$ is the linear map

$$K(\{T_i\}) : \mathcal{H} \to F^2(H_n) \otimes \mathcal{H} \quad \text{defined by} \quad K(\{T_i\})h := \sum_{\alpha \in \mathbb{F}_n^+} e_\alpha \otimes \Delta^{1/2} T_\alpha^* h.$$

Since $\sum_{\alpha \in \mathbb{F}_n^+} T_\alpha \Delta T_\alpha^* = I_{\mathcal{H}}$, the Poisson kernel is an isometry. Notice that

$$(S_\alpha^* \otimes C^*) K(\{T_i\}) h = K(\{T_i\}) T_\alpha^* C^* h$$

for any $\alpha \in \mathbb{F}_n^+$, and $C \in \{T_1, \dots, T_n, \Delta\}'$ (the prime stands for the commutant). Hence,

$$K(\{T_i\})^* (S_\alpha S_\beta^* \otimes A C^*) K(\{T_i\}) = A T_\alpha T_\beta^* C^* \tag{3.1}$$

for any $\alpha, \beta \in \mathbb{F}_n^+$ and $A, C \in \{T_1, \dots, T_n, \Delta\}'$. We define the Poisson transform associated with $\mathcal{T}$ to be the map

$$\Psi_{\mathcal{T}} : B(F^2(H_n) \otimes \mathcal{H}) \to B(\mathcal{H}), \qquad \Psi_{\mathcal{T}}(X) := K(\{T_i\})^* X K(\{T_i\}).$$

Notice that $\Psi_{\mathcal{T}}$ is unital, completely contractive, w^*-continuous, and

$$\Psi_T(S_\alpha S_\beta^* \otimes A C^*) = A T_\alpha T_\beta^* C^*$$

for any $\alpha, \beta \in \mathbb{F}_n^+$, and $A, C \in \{T_1, \dots, T_n, \Delta\}'$.

Proposition 3.1. *Let $\mathcal{T} := [T_1, \dots, T_n]$ be a C_0-row contraction and let $K(\{T_i\})$ be its Poisson kernel. If $\mathcal{N}$ is a subspace of $F^2(H_n)$ invariant under $S_1^*, \dots, S_n^*$ and $K(\{T_i\})$ takes values in $\mathcal{N} \otimes \mathcal{H}$, then the map $\Phi : B(\mathcal{N} \otimes \mathcal{H}) \to B(\mathcal{H})$ defined by*

$$\Phi(X) = K(\{T_i\})^* X K(\{T_i\})$$

is unital, completely contractive, w^*-continuous, and

$$\Phi(B_\alpha B_\beta^* \otimes A C^*) = A T_\alpha T_\beta^* C^*,$$

for every $\alpha, \beta \in \mathbb{F}_n^+$, $A, C \in \{T_1, \dots, T_n, \Delta\}'$, where $B_i := P_{\mathcal{N}} S_i|_{\mathcal{N}}$, $i = 1, \dots, n$.

Proof. Since $\mathcal{N} \subseteq F^2(H_n)$ is an invariant subspace of $S_1^*, \dots, S_n^*$, it is clear that $P_{\mathcal{N}} S_\alpha S_\beta^*|_{\mathcal{N}} = B_\alpha B_\beta^*$ for every $\alpha, \beta \in \mathbb{F}_n^+$. Since $K(\{T_i\}) = (P_{\mathcal{N}} \otimes I_{\mathcal{H}})K(\{T_i\})$, Relation (3.1) implies that

$$\begin{aligned} AT_\alpha T_\beta^* C^* &= K(\{T_i\})^*(S_\alpha S_\beta^* \otimes AC^*)K(\{T_i\}) \\ &= K(\{T_i\})^*(P_{\mathcal{N}} \otimes I_{\mathcal{H}})(S_\alpha S_\beta^* \otimes AC^*)(P_{\mathcal{N}} \otimes I_{\mathcal{H}})K(\{T_i\}) \\ &= K(\{T_i\})^*(B_\alpha B_\beta^* \otimes AC^*)K(\{T_i\}) \end{aligned}$$

for any $\alpha, \beta \in \mathbb{F}_n^+$. This completes the proof. □

Let $C^*(S_1, \dots, S_n)$ be the C^*-algebra generated by $S_1, \dots, S_n$, the extension through compacts of the Cuntz algebra $\mathcal{O}_n$ (see [**5**]). The following result is a generalization of Theorems 8.1 and 9.2 from [**22**], and Theorem 6.2 from [**4**].

Theorem 3.2. *Let $\mathcal{T} := [T_1, \dots, T_n]$ be a contractive sequence of operators with $T_i \in B(\mathcal{H})$. Then there exists a completely contractive linear map*

$$\Phi_{\mathcal{T}} : C^*(S_1, \dots, S_n) \otimes C^*(T_1, \dots T_n)' \to B(\mathcal{H}),$$

such that $\Phi_{\mathcal{T}}(S_\alpha S_\beta^* \otimes A) = AT_\alpha T_\beta^*$, for any $\alpha, \beta \in \mathbb{F}_n^+$ and $A \in C^*(T_1, \dots T_n)'$. Moreover, if $T_1, \dots, T_n$ are commuting, then the result remains true if one replaces S_i by $B_i := P_{F_s^2(H_n)} S_i|_{F_s^2(H_n)}$, $i = 1, \dots, n$.

Proof. For each $0 < r < 1$, let $K_r(\{T_i\})$ be the Poisson kernel associated to $[rT_1, \dots, rT_n]$. Define the linear map

$$\Phi_{\mathcal{T}} : C^*(S_1, \dots, S_n) \otimes C^*(T_1, \dots T_n)' \to B(\mathcal{H})$$

by $\Phi_{\mathcal{T}}(f) = \lim_{r \to 1} K_r(\{T_i\})^* f K_r(\{T_i\})$. Using Relation (3.1), one can see that the limit exists in the uniform topology of $B(\mathcal{H})$ and $\Phi_{\mathcal{T}}$ is unital, completely contractive, and $\Phi_{\mathcal{T}}(S_\alpha S_\beta^* \otimes C) = CT_\alpha T_\beta^*$ for every $\alpha, \beta \in \mathbb{F}_n^+$ and $C \in C^*(T_1, \dots, T_n)'$. The map $\Phi_{\mathcal{T}}$ is called the Poisson transform associated with $\mathcal{T}$.

Assume now that $T_i T_j = T_j T_i$ for any $i, j \in \{1, \dots, n\}$. In this case, the Poisson kernel $K_r(\{T_i\})$ takes values in $F_s^2(H_n) \otimes \mathcal{H}$ for every $0 < r < 1$, where $F_s^2(H_n)$ is the symmetric Fock space. As in the proof of Proposition 3.1, we deduce that

$$K_r(\{T_i\})^*(B_\alpha B_\beta \otimes C)K_r(\{T_i\}) = r^{|\alpha|+|\beta|} CT_\alpha T_\beta^*.$$

Since $K_r(\{T_i\})$, $0 < r < 1$, is an isometry, the map $B_\alpha B_\beta^* \otimes C \mapsto CT_\alpha T_\beta^*$, defined on the span of $\{B_\alpha B_\beta^* \otimes C : \alpha, \beta \in \mathbb{F}_n^+,\ C \in C^*(T_1, \dots, T_n)'\}$, is completely contractive. Therefore, it can be extended to a unital, completely contractive map $\Phi : C^*(B_1, \dots, B_n) \otimes C^*(T_1, \dots T_n)' \to B(\mathcal{H})$ satisfying $\Phi(B_\alpha B_\beta^* \otimes C) = CT_\alpha T_\beta^*$ for all $\alpha, \beta \in \mathbb{F}_n^+$ and $C \in C^*(T_1, \dots, T_n)'$. □

Notice that, for any $A, C \in \{T_1, \dots, T_n, \Delta\}'$, $\alpha, \beta \in \mathbb{F}_n^+$, and $0 < r < 1$, we have

$$K_r(\{T_i\})^*(S_\alpha S_\beta^* \otimes AC^*)K_r(\{T_i\}) = r^{|\alpha|+|\beta|} AT_\alpha T_\beta^* C^*. \tag{3.2}$$

This relation can be used to prove the following result.

Proposition 3.3. *If $\mathcal{T} := [T_1, \dots, T_n]$ is a row contraction, then*

$$\Big\| \sum_{\text{finite}} A_{(\alpha)} T_\alpha T_\beta^* C_{(\beta)}^* \Big\| \leqslant \Big\| \sum_{\text{finite}} S_\alpha S_\beta^* \otimes A_{(\alpha)} C_{(\beta)}^* \Big\|$$

for any $A_{(\alpha)}, C_{(\beta)} \in \{T_1, \dots, T_n, \Delta\}'$. Moreover, if $T_1, \dots, T_n$ are commuting, then one can replace S_i by B_i.

Notice that, in the particular case when $n = 1$, we obtain an extension of the Arveson–Parrott version of the von Neumann inequality (see [**12**]). Let $\mathcal{A}_n$ be the non-commutative disc algebra and let $\mathcal{A}_n^s$ be the norm-closed algebra generated by B_i, $i = 1, \dots, n$, and the identity on F_s^2.

Corollary 3.4. *Let $T_i \in B(\mathcal{H})$, $i = 1, \dots, n$, such that $T_1 T_1^* + \cdots + T_n T_n^* = I_{\mathcal{H}}$, then there exists a completely contractive linear map*

$$\Phi : \mathcal{A}_n \otimes \{T_1, \dots, T_n\}' \to B(\mathcal{H})$$

such that $\Phi(S_\alpha \otimes A) = A T_\alpha$, for any $A \in \{T_1, \dots, T_n\}'$ and $\alpha \in \mathbb{F}_n^+$. Moreover, if $T_1, \dots, T_n$ are commuting, then one can replace S_i by B_i, and $\mathcal{A}_n$ by $\mathcal{A}_n^s$.

Let us remark that all the results presented so far in this section can be extended to the class of sequences of operators with property (P) (see [**22**] for terminology). We leave this task to the reader.

According to Lemma 2.2 and Theorem 2.3, every $f \in F_n^\infty \bar{\otimes} \{T_1, \dots, T_n, \Delta\}'$ has a unique Fourier expansion

$$f \sim \sum_{\alpha \in \mathbb{F}_n^+} S_\alpha \otimes W_{(\alpha)}$$

with $W_{(\alpha)} \in \{T_1, \dots, T_n, \Delta\}'$. For any $0 < r < 1$, define

$$f_r := \sum_{\alpha \in \mathbb{F}_n^+} r^{|\alpha|} S_\alpha \otimes W_{(\alpha)}$$

and

$$f_r(T_1, \dots, T_n) := \sum_{\alpha \in \mathbb{F}_n^+} r^{|\alpha|} W_{(\alpha)} T_\alpha \in B(\mathcal{H}).$$

The convergence of this series is uniform. Indeed, using Theorem 3.2, we infer that

$$\begin{aligned} \Big\| \sum_{|\alpha| \geqslant m} r^{|\alpha|} W_{(\alpha)} T_\alpha \Big\| &\leqslant \Big\| \sum_{|\alpha| \geqslant m} r^{|\alpha|} S_\alpha \otimes W_{(\alpha)} \Big\| \leqslant \sum_{k \geqslant m} r^k \Big\| \sum_{|\alpha| = k} S_\alpha \otimes W_{(\alpha)} \Big\| \\ &\leqslant \Big(\sum_{k \geqslant m} r^k \Big) \Big\| \sum_{|\alpha| \geqslant m} W_{(\alpha)}^* W_{(\alpha)} \Big\|^{1/2}. \end{aligned}$$

Therefore, $\| \sum_{|\alpha| \geqslant m} r^{|\alpha|} W_{(\alpha)} T_\alpha \| \to 0$ as $m \to \infty$. The following result is a generalization of the F_n^∞-functional calculus for C_0-row contractions.

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Theorem 3.5. *Let $f \in F_n^\infty \bar{\otimes} \{T_1, \dots, T_n, \Delta\}'$ and let $\mathcal{T} := [T_1, \dots, T_n]$ be a C_0-row contraction. Then SOT-$\lim_{r\to 1} f_r(T_1, \dots, T_n)$ exists and the map*

$$f \mapsto f(T_1, \dots, T_n) := \text{SOT-}\lim_{r\to 1} f_r(T_1, \dots, T_n)$$

is a WOT-continuous completely contractive homomorphism.

Proof. Since $\mathcal{T}$ is a C_0-row contraction, Relation (3.1) implies that

$$f_r(T_1, \dots, T_n) = K(\{T_i\})^* \Bigl(\sum_{\alpha \in \mathbb{F}_n^+} r^{|\alpha|} S_\alpha \otimes W_{(\alpha)} \Bigr) K(\{T_i\}), \tag{3.3}$$

where $K(\{T_i\})$ is the Poisson kernel associated with $\mathcal{T}$. Since $f = \text{SOT-}\lim_{r\to 1} f_r$ (see the proof of Theorem 2.3), we deduce that SOT-$\lim_{r\to 1} f_r(T_1, \dots, T_n)$ exists. On the other hand, Relation (3.3) and the non-commutative von Neumann inequality show that $\|f_r(T_1, \dots, T_n)\| \leqslant \|f_r\| \leqslant \|f\|$ for any $0 < r < 1$. Therefore, $\|f(T_1, \dots, T_n)\| \leqslant \|f\|$. Since $f(T_1, \dots, T_n) = K(\{T_i\})^* f K(\{T_i\})$, it is clear that $f \mapsto f(T_1, \dots, T_n)$ is WOT-continuous. This completes the proof. □

When $n = 1$, Theorem 3.5 is a generalization of the Sz.-Nagy–Foiaş H^∞-functional calculus for $C_{\cdot 0}$-contractions. We can also obtain a commutative version of Theorem 3.5. Indeed, if $T_1, \dots, T_n$ are commuting, then, according to Proposition 3.1 (when $\mathcal{N} := F_s^2$), the map $\Phi : B(F_s^2 \otimes \mathcal{H}) \to B(\mathcal{H})$ defined by $\Phi(X) := K(\{T_i\})^* X K(\{T_i\})$ is completely contractive and WOT-continuous. Since $\Phi(B_\alpha \otimes A) = AT_\alpha$, the map $B_\alpha \otimes A \mapsto AT_\alpha$ can be extended to a WOT-continuous completely contractive homomorphism from $\mathcal{W}_n^\infty \bar{\otimes} \{T_1, \dots, T_n, \Delta\}'$ to $B(\mathcal{H})$.

Let J be a WOT-closed two-sided ideal of F_n^∞. Set $\mathcal{N}_J := F^2(H_n) \ominus J(1)$, and $B_i := P_{\mathcal{N}_J} S_i|_{N_J}$, $i = 1, \dots, n$. The following theorem generalizes Theorem 3.7 from [**3**] to our setting.

Theorem 3.6. *Let $\mathcal{T} := [T_1, \dots, T_n]$ be a C_0-row contraction, and let J be a WOT-closed, two-sided ideal of F_n^∞ such that $\varphi(T_1, \dots, T_n) = 0$ for every $\varphi \in J$, then the linear map $\Phi : B(\mathcal{N}_J \otimes \mathcal{H}) \to B(\mathcal{H})$ defined by $\Phi(X) := K(\{T_i\})^* X K(\{T_i\})$ is unital, completely contractive, w^*-continuous and*

$$\Phi(B_\alpha B_\beta^* \otimes AC^*) = AT_\alpha T_\beta^* C^*,$$

for every $\alpha, \beta \in \mathbb{F}_n^+$, and $A, C \in \{T_1, \dots, T_n, \Delta\}'$.

Proof. Let $K(\{T_i\})$ be the Poisson kernel associated with $\mathcal{T}$. Since F_n^∞ is the WOT-closed algebra generated by the left creation operators $S_1, \dots, S_n$ and the identity, and the F_n^∞-functional calculus is WOT-continuous, we have

$$\langle K(\{T_i\})h, \varphi \otimes k\rangle = \langle h, \varphi(T_1, \dots, T_n)\Delta^{1/2} k\rangle = 0$$

for any $h, k \in \mathcal{H}$ and $\varphi \in J$. Hence, we deduce that $K(\{T_i\})$ takes values in $\mathcal{N}_J \otimes \mathcal{H}$. Now, using Proposition 3.1, the result follows. □

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We mention that a characterization of the WOT-closed two-sided ideals of F_n^∞ was obtained in [**6**].

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