additional piquancy would have been added if he had progressed slightly further; in particular, some explicit examples of the work of Swinnerton-Dyer and Serre on congruences might have been included.

Postscript: The reviewer was flattered to find himself included in the preface as a member of a quartet who kept the flag of modular function theory flying during the lean years before the recent explosion of interest in the subject. The author is incorrect, however, in assuming that H. Petersson was the only one of the four who did not contribute to the 1956 Bombay Colloquium on zeta-functions; all four were present there and contributed papers.

R. A. RANKIN

KALLENBERG, OLAV (editor), *Random Measures* (Akademie-Verlag and Academic Press, 1977), 104 pp., £6.

The mathematical foundations of point process theory have been developed rapidly in recent years; a monograph was urgently needed to collate the profusion of results. One such volume is the inaccessible German work of Kerstan, Matthes and Mecke. Kallenberg has provided a concise formal treatise incorporating many of his own improvements. One regret is that both works date essentially from 1974 and so do not include the continuing improvements and new concepts such as Papangelou's conditional intensity measures.

Kallenberg considers random measures on locally compact second countable Hausdorff spaces. The generalisation to random measures is mathematically natural and has technical applications, but most readers will find the point process case more intuitive. Few concessions are made to the novice who may well wonder why a point process is defined to be an integer-valued random measure, or what a Palm probability "means". The survey by Daley and Vere-Jones (in "Stochastic Point Processes" edited by P. A. W. Lewis, Wiley, 1972) is useful collateral reading.

"Random Measures" will remain for some years an invaluable reference work.

B. D. RIPLEY

KUSSMAUL, A. U., *Stochastic Integration and Generalized Martingales* (Pitman, 1977), xi+163 pp., £7·00.

The author states his aim "to imbed the theory of stochastic integration into a functional analytic framework". For right-continuous stochastic processes X and Z, $Z \to \int_0^\infty Z_t dX_t$ is a "measure" with values in L^p, the space of p-th power integrable random variables. Kussmaul defines this measure by extension theorems for vector-valued measures. Under localisation he finds the necessary and sufficient condition on X for its existence to be that X is a *quasimartingale*, i.e. the difference of two non-negative supermartingales.

I could not decide on the intended audience. A strong background in Banach spaces is needed, and uniform integrability is taken for granted, yet half the volume is devoted to well-known properties of martingales. I imagine very few readers will not be familiar with this material, so I recommend starting at Section 8.

I was irritated by the use of "modification" and "semimartingale" in new senses; at one point even a "finite set" has a new meaning! This is a photographically produced "Research note in mathematics"; commendably it has a (short) index and a list of symbols but the proofreading has been inadequate.

This treatise can only be recommended to experts in the field.

B. D. RIPLEY

CURTAIN, RUTH F. and PRITCHARD, A. J., *Functional Analysis in Modern Applied Mathematics* (Academic Press, London, 1977), ix+339 pp., £10·80.

Functional analysis has become a major tool in applied mathematics. Nevertheless, the authors point out in the Introduction, for an applied mathematician a "working knowledge of functional analysis . . . is not readily obtained by reading a standard text on functional analysis". Also, in spite of excellent books on applications in specific areas the authors felt "that there was a need for a book

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which illustrated the application of functional analysis in a variety of fields". The book under review is centred around control and optimisation problems and matters related to these (stability, systems theory, numerical methods for optimisation). It appears to be largely based on third year courses at the University of Warwick. There is no attempt to cover applications in other fields such as fluid flow, wave propagation or scattering.

About one third of the book is devoted to an introduction to basic functional analysis. The reader is assumed to have had a first course in functional analysis. Accordingly, this part of the book is in the nature of a review, and much of the material is presented in the form of definitions (often with motivation) and theorems without proof. However, and this is a valuable feature which persists throughout the book, definitions and results are frequently accompanied by illustrative examples. The selection of the material is governed by the applications discussed in the later portions of the book. There are brief sections on integration of real-valued functions and probability theory, there are sections on Banach and Hilbert spaces, linear operators and calculus (Fréchet differentials and Bochner integrals) in these spaces, and there is a brief section on topological spaces.

The second part (a little less than one third of the total) is entitled "Analysis of Abstract Equations". It contains a chapter on differential equations (for both numerical and vector valued functions), delay equations, stochastic differential equations, evolution equations in abstract spaces, and semigroup theory; and a chapter on spectral theory (with special emphasis on compact normal operators and closed linear operators, especially those with compact normal resolvents).

The third and longest part is devoted to applications. Liapunov stability for ordinary differential equations, perturbations of operators and stability of semigroups are followed by linear systems theory for finite-dimensional systems and control theory including filtering and time optimal control for such systems. Then follow optimisation problems including optimisation under constraints and optimal control, and numerical methods, especially those used in optimisation. In conclusion there is an introduction to the control theory of infinite dimensional linear systems.

Throughout the second and third parts the more elementary aspects of the subject are presented in considerable detail, but there is also a gratifying amount of more advanced material. The transition from the former to the latter is sometimes abrupt. For instance, in the chapter on differential equations, a detailed exposition of the most elementary parts of the theory of finite-dimensional systems (Picard–Lindelöf theorem, Gronwall's lemma, linear systems) is followed by a section on stochastic differential equations and one on delay differential equations which will not be easy reading for persons without any previous acquaintance with these subjects — especially since there are no illustrative examples in these sections. Thus, the book will be useful to those readers content with restricting themselves to the more elementary parts, and also to those with sufficient background to appreciate the more advanced presentation, but readers having only the stipulated preparation and wanting to study all of the book may in places find the going rough.

The book is somewhat marred by small imperfections which could have been avoided by a careful revision of the typescript and better proofreading. There is a very large number of misprints (some trivial while others apt to puzzle and occasionally perhaps to mislead a beginner); and there are lapses from grammar, and stylistical infelicities. The mathematical presentation is mostly adequate (if not always elegant and sometimes not easy to follow) but here too there are lapses in wording. In the definition of a vector space "for each $x \in W$, there exists a unique element 0 in W such that $x+0=0+x=x$" will probably not make any reader think that the zero element depends on x but similar lapses in later parts of the book might mislead the beginner. On page 19 continuous functions on an open subset of $\boldsymbol{R}^n$ under the sup norm are said to form a Banach space. On page 29 all essentially bounded measurable functions on a measurable set are said to be integrable (without assuming the set to be of finite measure). On page 46, $f(x) = \sum_{n=1}^{\infty} x_n$ with $x = (x_1, x_2, \ldots)$ is said to define a linear functional on l_∞.

In spite of these minor (and sometimes irritating) faults this is a useful book; and most of the faults could easily be rectified in any reprinting.

A. ERDÉLYI

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