A STUDY OF ASTEROID FAMILIES AND STREAMS BY COMPUTER TECHNIQUES

B. A. LINDBLAD
Lund Observatory
and
R. B. SOUTHWORTH
Smithsonian Astrophysical Observatory

A study of asteroid orbits is made to determine if there exist groupings of similar orbits in the asteroid population. A computer program based on Southworth's D *criterion for similarity in meteor orbits is used. The program successfully sorted out the asteroid families listed by Hirayama, Brouwer, and van Houten et al. A number of new families were detected, several of which appear to be more significant than the minor Brouwer families. Asteroidal streams (jet-streams) are studied and a list of such streams is presented.*

It is well known that the distribution of orbital elements among the minor planets is nonrandom. The frequency distribution of the semimajor axis a exhibits gaps corresponding to commensurabilities with Jupiter. Hirayama (1918, 1928) has shown the existence of families; i.e., groups of asteroids with almost equal values of the orbital elements a, e, and i. Brouwer (1951), who restudied this problem using the proper elements, has added several new families. Arnold (1969) introduced computer methods in the classification of families and found additional families among the numbered asteroids. Alfvén (1969, 1970) has drawn attention to the fact that within the Flora family there exist groups of orbits that exhibit similarity also in the orbital elements ω and Ω.

The orbital elements of about 1700 numbered minor planets are published in the standard asteroid *Ephemeris* (1970). Orbital elements of some further 2000 minor planets from the Palomar-Leiden survey (PLS) have recently been reported by van Houten et al. (1970). In the PLS material, van Houten et al. have verified a number of the Hirayama and Brouwer families. Four new families were discovered.

The PLS considerably increased the number of orbits available for study, and a comprehensive search in the data will probably reveal additional families and streams. The purpose of the present paper is to make such a search with the use of computer techniques.

STREAM DETECTION PROGRAM

D Criterion

The problem of classification based on orbital similarity is well known in meteor astronomy, where the study of meteor streams has necessitated the use of sophisticated computer techniques for the detection and classification of streams. The basis for our stream detection program is Southworth and Hawkins' (1963) criteria for orbital similarity, which for low-inclination orbits may be written

$$D(M, N)^2 = (e_N - e_M)^2 + (q_N - q_M)^2 + \left(2 \sin \frac{i_N - i_M}{2}\right)^2 + \sin i_M \sin i_N$$

$$\times \left(2 \sin \frac{\Omega_N - \Omega_M}{2}\right)^2 + \left(\frac{e_M + e_N}{2} \, 2 \sin \frac{\Omega_N + \omega_N - \Omega_M - \omega_M}{2}\right)^2$$

where M and N represent two orbits to be compared and a, e, i, ω, and Ω are the customary notations for the orbital elements. The stream detection program computes $D(M, N)$ for all possible pairs in the sample under study. If $D(M, N)$ is below a certain stipulated value D_s, the program considers these two orbits as forming a stream. In the continued comparison process, more and more orbits are grouped into the stream. The program finally lists the meteor streams and their members, their mean orbit, and the deviation of each stream member from the mean stream orbit. An extensive survey of photographic meteor orbits using this stream detection program has been made by Lindblad (1971).

The Southworth D criterion is an objective method of classification on the basis of the orbital elements; i.e., it selects concentrations in five-dimensional (q, e, i, ω, Ω) space. The reason for using the perihelion distance q instead of the semimajor axis a is that the perihelion distance q for meteor orbits is better defined than a. In adapting the method to asteroid orbits, we did not consider it necessary to modify the original program. The main problem encountered in our study was how to determine the appropriate rejection level D_s.

Data Sample and Data Preparation

Present and proper orbital elements for 1697 numbered asteroids and for 1232 PLS asteroids were available on cards. The 1697 numbered asteroids have well-defined orbits, and the entire data sample was used in our study. Of the PLS orbits, 28 were excluded because they are already included in the 1697 numbered asteroid sample. In the PLS, the investigators assigned each individual orbit a quality class. The 977 orbits of highest quality (type 1) were used by van Houten et al. in their study of asteroid families. The same data sample is used by us in the family searches described below. In our stream searches, a slightly larger data sample was desirable and orbits of types 2 and 3 were also included. The number of PLS orbits studied for streams was 1232.

DISTRIBUTION OF ORBITAL ELEMENTS

A comparison of the 1697 numbered asteroid orbits and the 1232 PLS orbits of types 1 through 3, using the present-day elements, showed that the distribution functions of orbital elements differ in the two data samples.

The distribution of inclination i is markedly different in the two populations, with an almost complete cutoff in the PLS data at about 20°. This is to be expected because the PLS was limited to the immediate neighborhood of the ecliptic. It follows that high-inclination families, as a rule, will not be detected in the PLS.

Figure 1 depicts the distribution of the longitude of the node in the two data samples. The numbered asteroid sample shows an almost random distribution of Ω, whereas asteroids of the PLS exhibit strong maxima at nodal values near 20° and 200°. Hence, asteroid streams with mean nodal values near

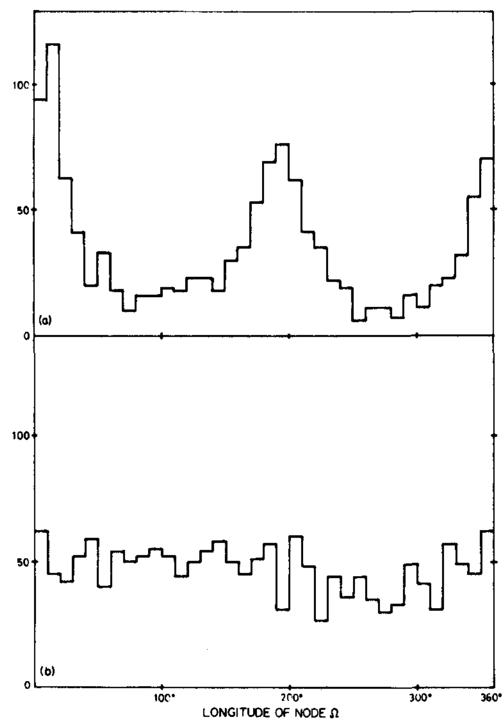

Figure 1.–Distribution of the longitude of the node. (*a*) PLS asteroids; N = 1232. (*b*) Numbered asteroids; N = 1697.

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

110° and 290° may be difficult to detect in the PLS material. In practice this may not be a serious restriction, however, because the asteroidal streams detected by us show a fairly large scatter in Ω.

FAMILIES AMONG NUMBERED ASTEROIDS

Rejection Level D_s

The degree of orbital similarity within the asteroid population is far higher than in the meteor population, and no direct inference from the meteor studies can be made as to the correct numerical value of D_s to use in a family search. In addition, the mathematical definition of D_s is different because only three orbital elements are involved. A trial and error method was therefore adopted. The mean $D(M, N)$ was first computed for the Brouwer families 1, 2, and 3. The mean $D(M, N)$ was found to be 0.045, 0.026, and 0.017, respectively, if only the proper elements a, e', and i' are considered. A D_s value between 0.015 and 0.030 thus appears reasonable. Numerically this D_s value is about one-tenth of that encountered in meteor studies.

For the family searches in the asteroid population, the deck of proper elements was used and constant values of ω' and Ω' were introduced on the cards. The numbered asteroid sample was searched for families at four different rejection levels, $D_s = 0.015, 0.018, 0.020$, and 0.025. The purpose of the study was to investigate at which D_s value the best possible agreement with Brouwer's original family classification was obtained. In the comparison, interest was focused mainly on families 1 through 9.

It was found that the search at $D_s = 0.020$ gave the best agreement with Brouwer, and this D_s value was therefore adopted as the rejection level. The search at $D_s = 0.020$ accepted 48 of the 53 members listed by Brouwer as family 1, considered four as belonging to a satellite family and one as a nonfamily member. In families 2 and 3 the search accepted 57 out of 58 and 33 out of 33 members, respectively.

Previously Known Families

The computer search at $D_s = 0.020$ classified the 1697 numbered asteroids into 25 previously known (Brouwer) families and 198 new families. The total number of asteroids listed as family members was 1026. However, a very large number of the new families have only two or three members and may be considered as chance groupings in the data. If these families are rejected, we find that 652 asteroids belong to families. Thus approximately 40 percent of the numbered asteroid population are in families.

Table I details how the known Brouwer family members were classified by our search at $D_s = 0.020$. Of the 458 asteroids listed by Brouwer as belonging to families, 380 appear as family members in our search. Table I is directly compiled from the computer output without involvement of any "personal

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

TABLE I.—*Classification of Brouwer Family Members*

Brouwer family no.	Number of asteroids in family	Asteroids in satellite families	Total number of asteroids	Remarks	Number of asteroids (Brouwer)
1	48	4	52	...	53
2	57	0	57	...	58
3	33	0	33	...	33
4	7	10	17	...	17
5	3	5	8	High inclination	25
6	22	1	23	...	23
7	61 } 119	1 } 3	62 } 122	...	62
8	8	0	8	6 through 9 combined	9
9	28	1	29	...	31
10	0	0	0	...	0
11	2	0	2	High inclination	6
12	2	3	5	...	6
13	0	0	0	...	7
14	2	0	2	...	8
15	3	3	6	...	6
16	4	3	7	Arnold 66	8
17	2 } 5	2 } 8	4 } 13	17 and 18 combined	10
18	3	6	9	...	12
19	5	0	5	...	7
20	6	2	8	Arnold 70	8
21	3	4	7	Arnold 71	7
22	3	5	8	Arnold 72	8
23	2	0	2	...	5
24	5	2	7	Arnold 74	9
25	6	4	10	Arnold 75	11
26	4	0	4	Arnold 76	8
27	5	0	5	Arnold 77	8
28	0	0	0	High inclination	6
29	0	0	0	High inclination	7
Total	324	56	380	...	458

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

judgment" or "manual handling." The first column gives the family number according to Brouwer. The second column lists the number of Brouwer members in each family according to our search at $D_s = 0.020$. Members of a Brouwer family assigned by our search to a satellite family—or to other groupings—appear in column 3. Column 4 gives the total number of Brouwer family members assigned by our search to families. Any remaining Brouwer family members were rejected. The total number of Brouwer family asteroids involved in the compilation is listed in column 6.

The original Hirayama families 1 through 4 are well represented and have very nearly the same members as in Brouwer's study. The outstanding case is family 3 where all 33 members are classed the same as in Brouwer's study. In contrast, the high-inclination family 5 has suffered losses. This is not unexpected, considering that we used the same rejection level D_s for all orbits irrespective of the inclination value. In a graphical search, it would have been natural to vary the similarity criteria depending on the density of points. If we had searched the sample of high-inclination orbits separately at a less rigorous rejection level, the high-inclination Brouwer family 5 could no doubt have been retained in almost its original form.

As expected, the Flora group is the largest family detected in our search. It contained 160 members, of which 119 are listed by Brouwer and 41 are new identifications. Three of Brouwer's Flora members were assigned to other families and three were rejected. The Flora group is often subdivided, after Brouwer, into four separate families, 6 through 9. There was no clear indication of such a separation in our study. Integrity of the Flora group was investigated by making additional searches at lower D_s values (stricter rejection levels). The use of a stricter rejection level reduced the number of members in the Flora group, but it was in no case possible to disentangle the separate families 6 through 9. Our computer output lists the members within each family in order of inclination. Inspection of the list shows that at all levels of D_s the division lines separating the various Flora families are arbitrarily set at certain inclination values.

Table I shows that 16 out of 19 Brouwer families with numbers 11 through 29 are detected. With the exception of families 17 and 18, all are listed as separate families. In general, the minor families have fewer members in our study than in Brouwer's. Most of the minor families are divided into two or sometimes three subgroups. The frequent existence of subgroups indicates that many of the Brouwer families 11 through 29 are fairly loose associations of orbits. Brouwer appears to have used a more liberal rejection level in classifying these groups than in the study of families 1 through 9.

It is interesting to note that families 16, 20, and 24 were detected by van Houten et al. in the PLS. Family 24, however, was incorporated by them into the Nysa family (van Houten family 32). Arnold, in his study of the numbered asteroids, revised family 24 and introduced a new family 74. This family is essentially the Nysa family of van Houten et al. The Nysa family is thus also discernible in the numbered asteroid population.

Significance of Small Families

In a large data sample, it is to be expected that a number of small-sized families are due to chance. This problem was studied by making searches in samples of random orbits. It was found that more than 50 percent of the two- and three-member families and about 30 percent of the four-member families are spurious groupings in the data. In accordance with these results, only Brouwer families 19, 20, 24, 25, and 27 are considered as statistically significant groupings among the small families.

New Families

A number of previously unknown or poorly studied families were identified by the search at $D_s = 0.020$. The largest new family found had 21 members. There also were 15 new families with from 5 to 11 members. These families are not detailed here. The majority of them appear in a subsequent search in the total asteroidal sample (table III).

The largest new family detected in the numbered asteroid population is the Io family with 21 members. The mean $D(M, N)$ for this group is 0.023. It should be considered as a rather loose association of orbits. This family includes asteroid 85 Io, but it appears to be distinctly different from the Io family detected by van Houten et al. in the PLS.

Arnold's Family Classification

Our discussion has mainly emphasized a comparison with Brouwer's classification. Arnold's computer study will be referred to only briefly. Arnold qualitatively confirmed the major Brouwer families. Several of the minor families, however, were extensively modified in his study.

Our search at $D_s = 0.020$ listed as separate groups nearly all the Brouwer families 1 through 29. However, it is evident from table I (column 3) that some members of the Brouwer families were assigned to other families, or to altogether new groups. By way of illustration, two members of family 24 were incorporated into a new family (Fortuna) and two were rejected altogether. In the present search, the Fortuna group consisted of eight members, five of which were included in Arnold 82. Several similar groupings were studied and were identified with Arnold's new families. At the four-or-more-member level, Arnold families 66, 67, 69, 73, 76, 82, 84, 85, and 90 were discernible, whereas families 72, 86, and 87 were divided into two about-equal-sized groups. Arnold family 70 was divided by our search into several small-sized families, each of low significance.

FAMILIES AMONG THE PLS ASTEROIDS

The PLS data were searched for families by van Houten et al. (1970) using 977 orbits of type 1. The same data sample was used by us, thus allowing a direct comparison between the searches.

The numerical value of D_s is known to be a function of sample size and the precision of the orbital elements. In a family search, D_s should vary inversely as the cube root of the sample size, if the samples are otherwise similar. If $D_s = 0.020$ is accepted as the rejection level in the numbered asteroid population, a rejection level of

$$D_s = 0.020 \left(\frac{1697}{977}\right)^{1/3} = 0.024$$

would have been appropriate for the somewhat smaller PLS sample. However, the stronger concentration to the ecliptic plane suggests a lower D_s value because the density of observed orbits corresponds to a larger sample. Because of this truncation effect, we are inclined to use about the same D_s value as in the numbered asteroid sample. In order not to prejudice the choice, computer searches were made at four rejection levels, $D_s = 0.017$, 0.018, 0.019, and 0.020.

Our study showed that at the stricter rejection levels the recognized families often were split into two or three subgroups. The searches at $D_s = 0.019$ and $D_s = 0.020$ provided reasonable agreement with the classification of van Houten et al. The finally adopted rejection level was $D_s = 0.019$. This level is, perhaps, slightly on the conservative side.

Previously Known Families

Table II compares the classification of van Houten et al. with that of our search at $D_s = 0.019$. Table headings are similar to those of table I. It is seen that 214 out of 386 orbits are classified in the same manner, whereas 110 orbits are assigned to satellite families or to other groups. Inspection of families 1 through 3 suggests that the classification boundaries used by van Houten et al. are slightly wider than those of Brouwer.

The newly introduced families 30, 33, and 34 were readily identified by our search, although about half of their members were assigned to smaller satellite families. Families 4 and 12, with very few members in the PLS sample, were not found in our study at $D_s = 0.019$. Nor were they found in the search at $D_s = 0.020$. Family 33 was split into several groups, two of which were related to Brouwer 19 and 20 and a third which appeared to be related to the Io family introduced by us (table III). These results again indicate that the limits of the van Houten families in general are wider than those of the Brouwer study.

In the classification of the PLS orbits, the newly found Michela-Nysa group (van Houten families 31 and 32) presented some problems. The Michela and Nysa families represent two fairly close groupings, which our search could not disentangle. The division line between the two families appears to be one of inclination much in the same way as in the Flora group. Our investigation

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

TABLE II.—*Classification of Family Members in the PLS Data*

van Houten family no.	Number of asteroids in family	Asteroids in satellite families	Total number of asteroids	Remarks	Number of asteroids (van Houten)
1	40	16	56	....	63
2	5	10	15	....	23
3	28	2	30	....	32
4	0	0	0	....	0
5	0	0	0	....	0
6 to 9	16	19	35	....	36
12	0	0	0	....	7
16	4	2	6	....	10
25	0	4	4	Incorporated in 31 and 32	7
27	2	0	2	....	8
30	14	13	27	....	28
31	14 }86	7 }11	21 }97	Combined with 32	26
32	72 }	4 }	76 }	....	77
33	8	18	26	....	37
34	11	15	26	....	32
Total	214	110	324	....	386

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

produced more members in the Michela-Nysa group than found by van Houten et al. The search at $D_s = 0.019$ listed 181 members in the combined Michela-Nysa group. Of these, 97 were included among the 103 members ascribed to the Michela-Nysa families by van Houten et al. The properties of the Michela-Nysa families deserve further detailed study.

New Families

The search at $D_s = 0.019$ produced a number of previously unknown families or groups. These have not yet been fully investigated and therefore are not detailed here. The more important families were detected in a subsequent search in the total asteroid sample (table III).

FAMILY SEARCHES IN TOTAL SAMPLE

The number of orbits available for study in the combined numbered/PLS asteroid sample was 2674. Of the PLS orbits, 22 of type 1 were excluded because they are already included in the 1697 numbered asteroid sample. The total number of orbits used in the study was 2652. As previously, proper elements were used.

Rejection Level

The appropriate rejection level to use in the family search was estimated as follows. From the relations

$$D_s = 0.020\left(\frac{1697}{2652}\right)^{1/3} = 0.017 \quad \text{and} \quad D_s = 0.019\left(\frac{977}{2652}\right)^{1/3} = 0.014$$

a rejection level slightly smaller than 0.014 or 0.017 may be estimated. Searches were made at four different levels, $D_s = 0.011$, 0.012, 0.013, and 0.014. The investigations at $D_s = 0.011$ and 0.012 separated and identified the low-inclination families but gave a far too severe rejection level for the moderate- and high-inclination groups. Family 2 Eos was split into two groups. The search at $D_s = 0.013$ gave adequate separation of the Brouwer families and this rejection level therefore was adopted. The adopted D_s value again was chosen conservatively.

Previously Known Families

The results of the search at $D_s = 0.013$ were compared with the classifications of Brouwer and van Houten. Families identified were 1 through 9, 16, 18 through 22, 25, 27, and 30 through 34. Family 24 was incorporated into the Nysa group. Families 11 through 15, 17, 23, and 26 were not detected in the total sample; i.e., at the chosen, stricter, rejection level their members were classified as nonfamily objects.

TABLE III.—*New Families*

No.	Family name	Number of members	Mean[a] $D(M, N)$	Members	Remarks[b]
35	Fortuna	6 (4)	0.010	19, 21, 138, 557, 1012, 1190 (1076, 1358, 2552, 4087)	A82
36	Pawlowia	6	.010	1007, 2058, 4641, 6584, 6653, 6700	Related to L39
37	Una	6	.010	160, 847, 1228, 2568, 4030, 4117	A86
38	Arcadia	7 (1)	.005	1020, 1352, 1726, 2088, 2598, 6059, 6554 (272)	A86, very concentrated
39	Nemesis	7 (5)	.011	125, 128, 301, 4058, 4609, 6323, 6646 (58, 380, 1135, 2740, 4863)	Part of A87
40	Merxia	7 (1)	.012	808, 1662, 2615, 4054, 4095, 4633, 4781 (4592)	....
41	Hecuba	6	.011	108, 1107, 1209, 4214, 4529, 6575	Related to vH34
42	Mathilda	5 (3)	.012	253, 869, 1178, 1525, 1555 (505, 1277, 5578)	A84, loose association
43	Valda	7	.010	262, 518, 678, 1391, 4591, 6038, 6744	....
44	Dora	7 (4)	.009	668, 1427, 1734, 2142, 4010, 4013, 6045 (479, 638, 2540, 4040)	....
....	Io	11	.013	85, 141, 390, 888, 1313, 1329, 1431, 1458, 1499, 2802, 6523	A69? Loose association, separate from vH33

[a]Computed from orbital elements a, e', and i'.
[b]A, L, and vH stand for Arnold, Lindblad, and van Houten, respectively.

https://doi.org/10.1017/S0252921100089181 Published online by Cambridge University Press

New Families

Table III lists those new families that had six or more members detected by the search. The first and second columns give the numbers and names suggested for these families, the third and fourth columns give the number of members and the mean value of $D(M, N)$ at the rejection level $D_s = 0.013$. The individual members are given in column 5. Numbers above 2000 refer to the PLS asteroids. Parentheses indicate additional members obtained at the rejection level $D_s = 0.014$. Column 6 lists the family number given by Arnold (1969) to these groupings.

The new families have been named after bright asteroids occurring within their boundaries, and have been numbered 35 through 44 in continuation of the numbering of van Houten et al. It is seen that the new families have members in both data samples studied by us. Further, it should be mentioned that the new families (table III) were also detected in the separate searches in these two samples.

The mean $D(M, N)$ listed in table III is a measure of the concentration within an asteroid family, a low $D(M, N)$ value implying a high degree of concentration. For most families, $D(M, N)$ is of the order of 0.010. This value is lower than that found in our study for the majority of the Brouwer families.

In addition to the families reported in table III, a large number of minor groups were detected by the search. It is believed that a number of the new four- and five-member families are significant, but there is an increasing probability as we proceed to small-sized groups that the associations are due to chance. Subsequent studies may very well give reasons to include some of the smaller groups in an extended listing of families. It is also possible that a future study will revise upward the rejection level D_s, thus allowing more members in the families reported in table III.

ASTEROIDAL STREAMS

An asteroidal stream (jetstream) is defined as an assembly of orbits showing similarity in all five orbital elements a, e, i, ω, and Ω. For the purpose of the study of streams, the present-day elements will be used. From a geometrical point of view, the asteroidal streams are analogous to the meteor streams. Southworth and Hawkins' D criterion can thus be used without modification to search for similar orbits once the rejection level D_s is determined.

Very little information is at present available as to the size and number of streams in the asteroidal population. Alfvén (1969) found three separate streams, denoted A, B, and C, among the members of the Flora family. The statistical significance of these groupings has been discussed by Danielsson (1969). Some additional streams have been listed by Arnold (1969).

Lacking more detailed information, the investigator is faced with the problem of setting the rejection level D_s more or less arbitrarily. Test runs in the numbered asteroid population indicated that D_s values in the range 0.050

to 0.060 gave a reasonable number of streams and provided confirmation of the known streams in the Flora family. Similar results were obtained in test runs in the PLS data.

For the study in the total asteroid population, we rather conservatively have chosen $D_s = 0.044$. The search at this rejection level produced 81 streams with two members, 12 streams with three members, and 36 streams with four or more members. The number of asteroids in streams was 647 out of a total of 2929 orbits. Thus at the rejection level $D_s = 0.044$, approximately 22 percent of the asteroid population was placed in streams.

Table IV lists those streams detected in our search that had seven or more members. In nearly all cases, these streams were detected independently in searches both in the numbered asteroid population and in the PLS data. The possibility that these streams are caused mainly by selection effects peculiar to the PLS therefore appears unlikely.

Alfvén's jetstreams A and C are detected. Alfvén's stream B was discernible, but was split into two groups, one with five and one with four members. At the adopted acceptance level of table IV, these groupings were rejected. The tendency of asteroids to aline their lines of apsides with Jupiter's is well known. An interesting situation occurs in Alfvén's jetstream A, where the search at $D_s = 0.044$ split stream A into two groups of orbits having their lines of apsides oriented roughly symmetrically with respect to Jupiter's perihelion. A similar geometry exists in the Coronis and Denone streams, which also form two sets of orbits symmetrical with respect to the apsidal line of Jupiter's orbit.

The streams listed by Arnold (1969) were compared with our streams (table IV). Only three orbits were common to both searches. The reason for this poor agreement is not known. As can be seen from the definition of $D(M, N)$, the D criterion favors orbits that have their major axes alined. For low-inclination streams, this condition can be met even if there exist rather large differences in ω and Ω. It is possible that the stream search program of Arnold did not emphasize the alinement of the orbital major axis.

The statistical significance of the streams listed in table IV was investigated by making stream searches in random samples. In one search in the combined population, we found, besides numerous two- and three-member streams, four streams with four members, one stream with five members, and one with seven members. It follows that the majority of four-, five-, and six-member streams found in the real sample are significant groupings, and thus could have been included in an extended version of table IV.

The asteroidal streams represent concentrations within the recognized families of Themis, Coronis, Flora, and Nysa. The streams consist of family members that have a similar orientation of the orbital major axis. It is interesting to note that there are preferred directions of alinement. Figure 2 depicts the distribution of the longitude of perihelion π in the 647 stream orbits. Maxima in the distribution are evident at about $\pi = 50°$ and $320°$. The

TABLE IV.—*Asteroidal Streams (Jetstreams)*

Preliminary stream name	Number of members	Mean[a] $D(M, N)$	Family no.	Members	Remarks
Rosa	11	0.055	1	223, 461, 621, 946, 1003, 1674, 4776, 6582, 6634, 6725, 6745	–
Janina	8	.035	1	383, 515, 1074, 1576, 1615, 1687, 2523, 4602	–
Coronis	7	.033	3	158, 243, 993, 1079, 1570, 2549, 4122	–
Denone	10	.034	3	215, 761, 1100, 1128, 1289, 1363, 1497, 1635, 2560, 4521	–
Elvira	8	.036	3	263, 277, 832, 962, 1350, 1442, 4036, 4593	–
Lacrimosa	17	.071	3	208, 321, 452, 658, 720, 975, 1029, 1223, 4545, 4626, 4661, 4893, 6044, 6546, 6555, 6632, 6636	Loose assoc.; doubtful stream
Anahita	19	.058	6, 32	270, 315, 939, 960, 1682, 1699, 2084, 2716, 4065, 4171, 4227, 4663, 6061, 6063, 6074, 6097, 6098, 6189, 7595	–
Nephele	14	.049	1	431, 468, 492, 767, 938, 1073, 1082, 1383, 1445, 2547, 2757, 4590, 4896, 6587	–
Hertha	9	.037	32	135, 1493, 2035, 2164, 4009, 4015, 4078, 6080, 9093	–
Gisela	10	.040	6, 7	244, 296, 352, 703, 1120, 1335, 1422, 1494, 4637, 6199	Alfvén A
–	7	.035	6	810, 1150, 2526, 2659, 2679, 4578, 6619	Alfvén A
Eriphyla	15	.053	3	462, 811, 1010, 1245, 1336, 1423, 1725, 2522, 2567, 2581, 6534, 6542, 6565, 7631, 7633	–
Lucretia	8	.035	7, 8	281, 915, 935, 1016, 2168, 4014, 4537, 6110	Alfvén C

[a]Computed from orbital elements a, e, i, ω, and Ω.

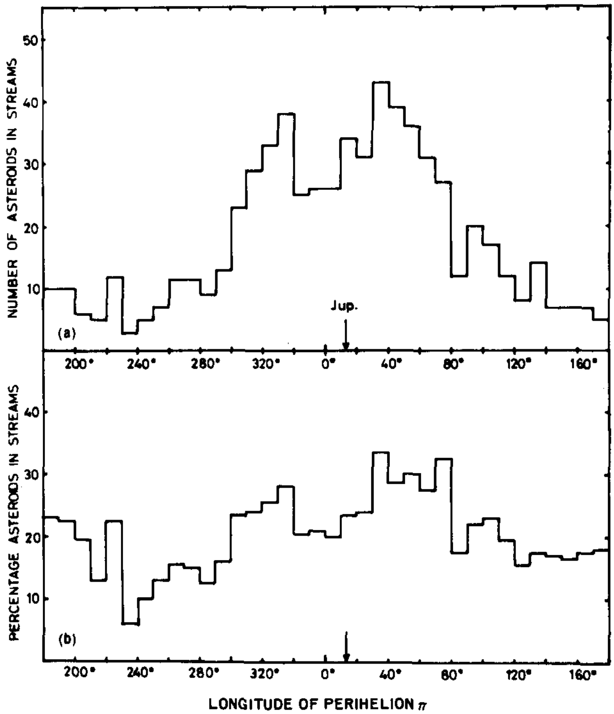

Figure 2.—Distribution of the longitude of the perihelion in 647 stream orbits. (*a*) Number. (*b*) Percentage.

overall distribution (fig. 2(*a*)) is symmetric with respect to a weighted mean longitude of perihelion $\pi = 12\overset{\circ}{.}5$, which agrees closely with the longitude of perihelion in the Jupiter orbit. It is evident that Jupiter plays a predominant role in the orbital history of most of these asteroidal streams.

ACKNOWLEDGMENTS

The authors are indebted to Dr. J. R. Arnold and Drs. van Houten for kindly placing at our disposal card decks of the present and proper elements of the numbered asteroids and PLS asteroids. One of us (Lindblad) is indebted to Professor H. Alfvén for many helpful and stimulating discussions. This work has been partly supported by grants 204-43 and 204-44 from the Swedish Natural Science Research Council and by NASA contract NSR 09-015-033.

REFERENCES

Alfvén, H. 1969, Asteroidal Jet Streams. Astrophys. Space Sci. 4, 84-102.
Alfvén, H. 1970, Jet Streams in Space. Astrophys. Space Sci. 6, 161-174.
Arnold, J. 1969, Asteroid Families and Jet Streams. Astron. J. 74, 1235-1242.

Brouwer, D. 1951, Secular Variations of the Orbital Elements of Minor Planets. Astron. J. **56**, 9-32.

Danielsson, L. 1969, Statistical Arguments for Asteroidal Jet Streams. Astrophys. Space Sci. **5**, 53-58.

Hirayama, K. 1918, Groups of Asteroids Probably of Common Origin. Astron. J. **31**, 185-188.

Hirayama, K. 1928, Families of Asteroids. Jap. J. Astron. Geophys. **5**, 137-162.

Houten, C. van, Houten-Groeneveld, I. van, Herget, P., and Gehrels, T. 1970, Palomar-Leiden Survey of Faint Minor Planets. Astron. Astrophys. Suppl. Ser. **2**, 339-448.

Lindblad, B. 1971, Two Computerized Stream Searches Among Meteor Orbits. Smithson. Contrib. Astrophys. **12**, in press.

Southworth, R., and Hawkins, G. 1963, Statistics of Meteor Streams. Smithson. Contrib. Astrophys. **7**, 261-285.