

# ROTATIONAL VELOCITIES OF EVOLVED A AND F STARS\*

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**Abstract.** Mean rotational velocities are presented for stars on and above the main sequence from spectral type A5 to F9. The velocities are shown to be inconsistent with the hypothesis of stellar rotation in completely uncoupled shells for stars that have expanded in radius by less than a factor of four. They do, however, support the hypothesis of solid-body rotation for these stars. The data for the giant stars are discussed, but no firm conclusions about the mode of rotation in these stars can be drawn.

## 1. Introduction

Rotational velocity is an important parameter in stellar structure and one which, for the interior of stars at least, is largely unknown. Investigations by many workers in the past have produced an extensive collection of  $v \sin i$ 's, and the behavior of surface equatorial velocity along the main sequence is consequently quite well determined. However, our only possibility for understanding internal rotational structure lies in determination of surface  $v \sin i$  along an evolutionary track, together with theoretically predicted stellar radii and moments of inertia. Several investigators, notably Oke and Greenstein (1954), Sandage (1955), Abt (1957, 1958), and Rosendhal (1968), have utilized this technique. Following the approach pioneered by Oke and Greenstein, all these workers have set themselves a modest goal, to determine whether angular momentum is conserved as stars evolve and, if so, which of two hypotheses, solid-body rotation or rotation in completely uncoupled shells, agrees more closely with the observed data. No definite evidence has been found for non-conservation of angular momentum in any of these investigations. However, attempts to discriminate between the two modes of rotation have either been inconclusive, because of too small a sample of available velocities, the possible presence of turbulence, especially in the supergiants, and inaccurate theoretical values for moments of inertia; or have produced conflicting results, apparently because the groups of stars under consideration in these various papers have occupied different regions of the HR diagram. The present investigation has employed the same approach while attempting to circumvent some of these difficulties.

## 2. The Data

A vertical strip in the HR diagram, from spectral type A5 to F9, was selected. All stars in the *Catalog of Bright Stars* (Hoffleit, 1964) known to lie above the main sequence in this strip, together with all catalog members in this region without a listed

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luminosity class, were taken as candidates. In this way we sought to include every possible star in the catalog lying above the main sequence between A5 and F9. The literature was searched for all available rotational velocities for these stars. Velocities for the great majority of the remainder, some 360 stars, were obtained by us, yielding a total population of 579 stars for further analysis.

Since the details of this work will be published elsewhere, it is not appropriate here to discuss at length the procedure used in determining the rotational velocities. In brief, the values of  $v \sin i$  were obtained by comparing observed profiles, traced from spectra with a dispersion of  $13.6 \text{ \AA/mm}$ , with a grid of standard profiles. The standards were lines chosen from observed zero-velocity or near zero-velocity stars of a variety of spectral types and luminosity classes, distributed throughout the region of the HR diagram under consideration. These zero-velocity profiles were then mathematically rotated by varying amounts in  $v \sin i$ . Only one novel aspect of this procedure deserves mention. In addition to three normal profiles of single lines, the 'profiles' referred to above contained two extended sections of spectrum totaling over  $200 \text{ \AA}$  in length. These sections were traced on each star and compared to the grid of model sections, the entire lengths of which had been mathematically rotated. These extended regions proved to be extremely helpful velocity indicators at all ranges of  $v \sin i$ , in effect providing over 150 lines and blends for comparison with the model profiles.

In an effort to obtain more accurate and up-to-date values for stellar moments of inertia, we have utilized unpublished data on stellar models kindly made available by Professor Iben. These new data alter significantly the conclusions to be drawn for the

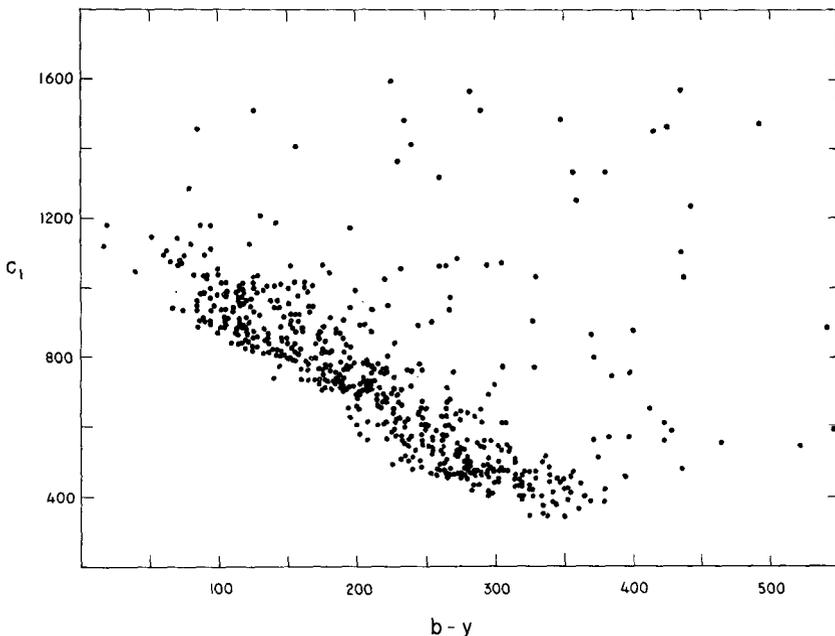


Fig. 1. The  $c_1 - (b - y)$  diagram for all stars.

most massive stars. Iben's models predict that the velocities expected under the hypothesis of solid-body rotation, or case A, are fully 5 times greater for Ia supergiants in this region than the velocities expected under the hypothesis of rotation in shells, hereafter called case B. This value of 5 is to be compared with the factor of 2 generally adopted in previous investigations.

For comparison with theoretical evolutionary tracks, accurate estimates of luminosity and effective temperature for each star are necessary. For this purpose, *uvby* photometry was obtained as an aid to spectral classification for all stars not in the catalog by Strömberg and Perry (1962). The resultant  $c_1 - (b - y)$  diagram for all stars is shown in Figure 1.

### 3. Stars in the Main Sequence Band

The stars were divided into two groups. The first contained all stars lying in the main sequence band. For these stars, the *uvby* photometry was used as an indicator of luminosity and spectral type. Iben's tracks for  $1.5 M_{\odot}$  and  $1.25 M_{\odot}$  were transferred to the  $c_1 - (b - y)$  diagram and the stars broken down into smaller groups for the determination of mean  $v \sin i$ 's as shown in Figure 2. The groups were located so that the stars in regions B and C are evolutionary descendants of region A, and similarly for regions D, E, and F. The initial starting points of the  $1.5 M_{\odot}$  and  $1.25 M_{\odot}$  tracks are also indicated in the figure.

The final data pertaining to these stars are contained in Table I and Figure 3. The errors quoted for the mean values of  $v \sin i$  were determined by considering the distribution of velocities to be a Gaussian and computing the standard error of the mean according to the usual formula

$$\sigma = \sqrt{\frac{\sum_i^N ((v \sin i)_i - \overline{v \sin i})^2}{N(N - 1)}}$$

where  $N$  is the total number of stars in each group. Groups A and D being assumed to represent the main sequence, the ratio

$$K = \frac{\overline{v \sin i}_{ms}}{\overline{v \sin i}}$$

TABLE I  
Stars in the main sequence band

Group	No. of members	$\overline{v \sin i}$	$K_{obs}$	$K_A$	$K_B$
A	102	$132 \pm 6$	1.00	1.00	1.00
B	77	$129 \pm 8$	$1.02 \pm .08$	1.04	1.19
C	61	$120 \pm 7$	$1.10 \pm .07$	1.07	1.50
D	136	$48 \pm 3$	1.00	1.00	1.00
E	45	$51 \pm 5$	$0.94 \pm .12$	0.97	1.13
F	33	$50 \pm 7$	$0.96 \pm .15$	1.00	1.36

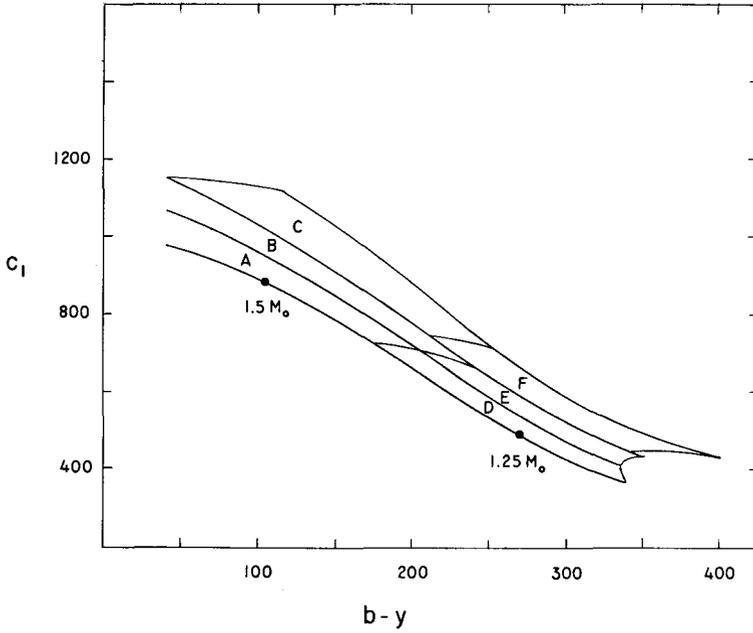


Fig. 2. The groups into which the stars near the main sequence were divided.

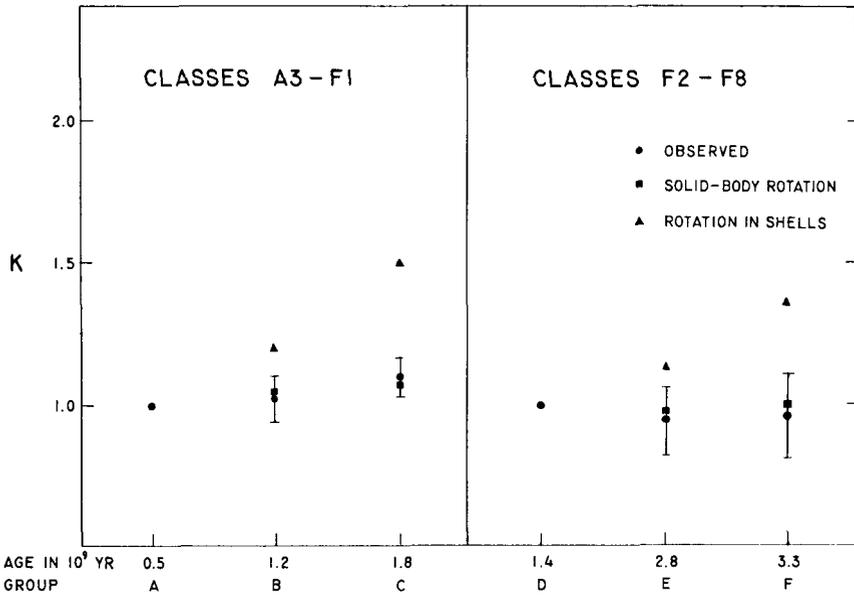


Fig. 3. The observed and theoretical values of  $K$  for stars near the main sequence. Ages on the  $x$ -axis are the mean ages for each group estimated from Iben's models.

where  $\overline{v \sin i}_{ms}$  is the mean value of  $v \sin i$  on the main sequence, was determined from Iben's models according to hypotheses A and B. These predicted values,  $K_A$  and  $K_B$ , are compared with the observed values of  $K$  in the table and in the figure. The data appear to exclude the possibility of rotation in uncoupled shells for these stars, whereas they are extremely compatible with the hypothesis of solid-body rotation.

We note, however, that the identical result might be obtained if the photometry should not in fact accurately indicate luminosity and, hence, age. If this were true, we might expect stars of differing ages to be uniformly spread out over the main sequence band, yielding roughly equal  $\overline{v \sin i}$ 's for all groups in the same sequence. This is exactly what was found. Moreover, Kraft and Wrubel (1965) have calculated the changes in  $c_1$  and  $(b-y)$  to be expected for a star on the main sequence in this region having various values of  $v \sin i$ . They find that stars with high  $v \sin i$ 's are shifted to a higher position in the band, the maximum effect being just about equal to the width of the band. Unfortunately, this movement is in precisely the direction which would tend to obliterate the decline in  $v \sin i$  toward the top of the band predicted by theory B. It is important to note that their results are based on the investigations of Sweet and Roy (1953). Although the later efforts of Roxburgh and Strittmatter (1966) would predict an effect only one-third as large, the uncertainty of the situation makes it desirable to verify that the photometry is indeed a reliable age indicator.

Two procedures were carried out to test the photometry. The first was a simple plot of absolute magnitude above the main sequence predicted by the photometry vs. the magnitude expected from the parallax listed in the *Catalog of Bright Stars* if available. A very strong correlation was found, which had a slope near unity. Secondly, histograms of the velocities in all six groups were compared. If the mechanism of Kraft and Wrubel were operative, we would expect no stars with  $v \sin i$  greater than 150 km/sec in group A and none with  $v \sin i$  over 100 km/sec in group D. We would also expect relatively larger numbers of fast rotators in groups B, C, E, and F. Neither effect is observed. In fact, within statistical uncertainties, the histograms of A, B, and C are identical, the same being true for groups D, E, and F. This is exactly what we would expect if solid-body rotation is true for these stars.

#### 4. Giants and Supergiants

The analysis of the stars above the main sequence band is not as straightforward. The Strömngren indices have not been calibrated for stars as luminous as these, and even if they were, interstellar reddening would still be a problem for most of these stars. We have therefore used spectral types as indicators of luminosity and temperature in this region. Those stars for which accurate types are not available were classified by a combination of criteria: the listed *Catalog of Bright Stars* type, the reddening-free Strömngren indices  $[m_1]$  and  $[c_1]$  (Strömngren, 1966), the mean reddening line in the  $c_1 - (b-y)$  diagram (Strömngren, 1963), and the appearance of the spectra on our plates. Stars that were so classified comprised slightly less than one-half the total number

above the main sequence band. Cases for which the various criteria gave conflicting results were eliminated from further consideration.

The stars were divided into five classes according to luminosity, as follows: Ia supergiants, Ib supergiants, class II, class II-III, and class III; and were further subdivided into an early and a late group within each class.  $\overline{v \sin i}$  and values of the ratio  $K$  were again calculated for a mean point within each group.  $K_A$  and  $K_B$  were obtained by interpolating between Iben's tracks. The results are presented in Table II and Figure 4. In the figure, each triad of data points represents the observed and

TABLE II  
Stars above the main sequence band

Lum. class	Spec. types	No. of members	$\overline{v \sin i}$	$K_{obs}$	$K_A$	$K_B$
Ia	A5-F2	4	$24 \pm 4$	$6.3 \pm 0.8$	4.6-7.2	23.0-35.6
Ib	A5-F2	11	$18 \pm 2$	$8.3 \pm 0.7$	3.4-5.5	7.0-11.6
Ib	F5-F8	6	$15 \pm 2$	$10.0 \pm 0.8$	4.5-7.2	9.4-15.6
II	A5-F3	16	$37 \pm 9$	$4.6 \pm 1.1$	2.5-3.6	4.2- 6.2
II	F5-F9	11	$42 \pm 15$	$4.0 \pm 1.4$	3.1-4.6	5.3- 7.8
II-III	F2-F8	4	$92 \pm 39$	$2.0 \pm 0.8$	2.1-2.7	3.4- 4.3
III	A4-A9	7	$115 \pm 23$	$1.5 \pm 0.3$	1.3-1.4	2.0- 2.3
III	F0-F8	21	$89 \pm 13$	$1.9 \pm 0.3$	1.5-1.9	2.4- 2.8

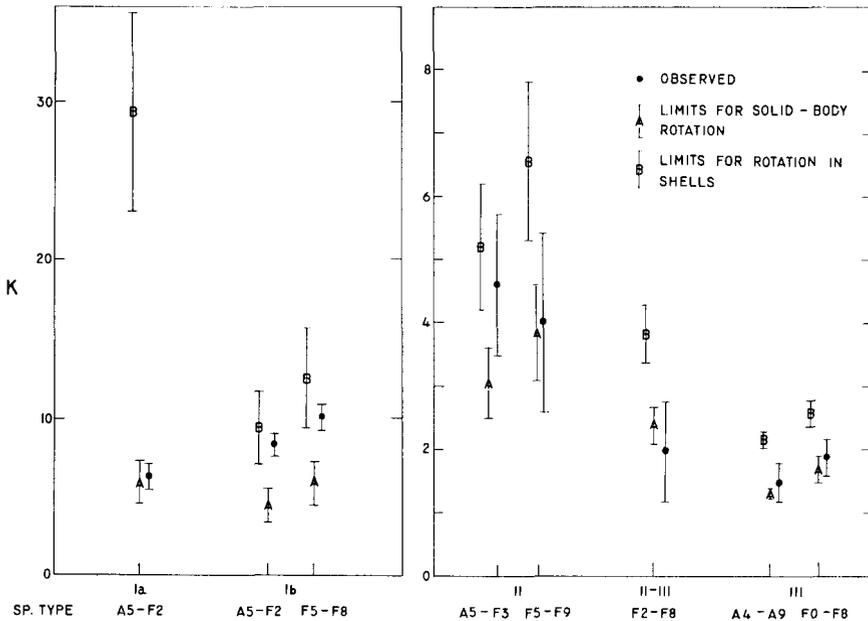


Fig. 4. The observed and theoretical values of  $K$  for giants and supergiants. Each triad of points represents the data for one group of stars as identified below on the  $x$ -axis.

predicted values of  $K$  for one group of stars.  $K_A$  and  $K_B$  have been represented in the table and in the figure as a range of possible values. The principal source of uncertainty in these quantities arises from our ignorance of the exact position of the main sequence in the  $M_{\text{bol}}, \log T_e$  plane and the consequent error in the factor by which a star expands in radius. The upper limits to  $K_A$  and  $K_B$  have been obtained by using the observed values of the bolometric corrections and temperatures given by Johnson (1966) together with the calibration of  $M_v$  for the main sequence by Blaauw (1963). The lower limits are those predicted from observed stellar radii (Harris *et al.*, 1963). These lower limits are also in reasonable agreement with the origin of Iben's tracks in the  $M_{\text{bol}}, \log T_e$  plane.

The data once again support solid-body rotation for the less luminous stars of classes III and II–III, all of which have expanded by less than a factor of 4. We therefore support the conclusion suggested by Abt (1958) that all stars which have expanded by less than a factor of 4 appear to rotate as solid bodies.

The situation for stars of luminosity class II seems to be intermediate between cases A and B. This is to be expected if these stars have undergone a recent transition from solid-body to some other mode of rotation, such as rotation in uncoupled shells. However, it should be emphasized that the mean values in Table II and Figure 4 for class II are very sensitive to the spectral classification assigned to a few stars with higher-than-average velocities. These are all stars classified primarily on the basis of the photometry. If these stars are omitted, the mean for class II, A5–F3, becomes 19 km/sec instead of 37 km/sec, and for class II, F5–F9, it is 32 km/sec instead of 42 km/sec. These revised numbers would be clear evidence for rotation in shells. A star for which an accurate estimate of spectral type is particularly desirable is HR 6531, which has a  $v \sin i$  of 175 km/sec and an estimated type of F5II. If this star is truly a member of class II, it provides clearcut evidence against case B, since the velocity extrapolated back to the main sequence would then considerably exceed break-up velocity.

It is difficult to draw conclusions regarding the supergiants because of the possible effect of turbulence in these stars. The data in Table II and Figure 4 refer to the observed macroscopic broadening uncorrected for turbulence. The true rotational velocities may therefore be considerably lower and the observed data points plotted in the figure higher. Solid-body rotation for the Ib supergiants seems to be fairly definitely excluded, however, except perhaps for the period of time these stars spend near the main sequence. The tantalizing question, of course, is whether the striking agreement between case A and observations for the Ia supergiants is real or merely fortuitous. The observed mean is based on only four stars. However, the value of 24 km/sec obtained adjoins smoothly to Rosendhal's (1968) data (uncorrected for turbulence) for the earlier Ia supergiants. In fact, if we assume that turbulent broadening is unimportant compared to rotational broadening for all Ia supergiants, the mean uncorrected rotational velocities obtained by Rosendhal and by us agree quite well with the velocities predicted under case A at all spectral types earlier than F8.

## 5. Conclusion

In conclusion, it seems clear that the results of all these investigations rule out the possibility that stars which have expanded in radius by less than a factor of 4 rotate in uncoupled shells. The data are in accordance with the hypothesis of solid-body rotation in these stars, but it is possible that some strange exchange of angular momentum is taking place in their interiors such as to speed up the outer layers at the expense of the central parts. The present technique obviously cannot distinguish between these two processes. For class II giants, the situation is still in doubt, while the velocities for the Ia supergiants, naively interpreted, flatly contradict the results for fainter supergiants. One fact is clear, however. It is no longer lack of velocities for a sufficient number of stars which prevents us from eliminating one of our two hypotheses. It is, rather, a need for more accurate absolute magnitudes together with a method for dealing effectively with the phenomenon of turbulence in these giants. Further research in this area must come to grips with these two problems.

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## Discussion

*Abt:* It seems very likely that among the Ia supergiants, and to a lesser extent the Ib's, the primary contributor to the line broadening is turbulence because at each luminosity the minimum broadening is large and the range in broadening is small, whereas for rotation and random orientation one would expect to observe some narrow-lined spectra and a large range in broadening.

*Faber:* Our sample of four supergiants is so small that statistical analysis of them is admittedly dangerous. However, the minimum velocity seen, 16 km/sec, and the maximum seen, 40 km/sec, are about what one would expect if the broadening is due to rotation only and the rotation axes were distributed randomly in space. In any case we do not assert that the broadening in these stars is due principally to rotation; only that one can no longer reject rotation solely on the grounds that the observed velocities are higher than can be imagined under any hypothesis.

*Jaschek:* Did you look into the proportion of spectroscopic binaries in your groups?

*Faber:* That is something we have planned. I did not show separately the means of  $v \sin i$  of the stars we measured and the means of the stars measured in the literature. In almost all cases, the means of this second group of stars are lower than those of the first. We do not think that this could be due to systematic errors in our method because individual  $v \sin i$ 's measured by us agree very well with those already given in the literature. There is, however, a strikingly higher percentage of multiple stars in this second group, as tabulated in the *Catalogue of Bright Stars*, but whether this high number of binaries is related to the low  $v \sin i$ 's also found for these stars we do not know at this time.

*Buscombe:* What is the resolution of the spectrograms, and what interval in the spectrum was studied? Were many of the late A's metallic-line stars?

*Faber:* The spectra have a dispersion of 13.6 Å/mm. The two extended sections of spectrum I referred to bracketed H $\gamma$  and H $\delta$  and included  $\lambda\lambda$  4058–4141 and  $\lambda\lambda$  4263–4382. The other three lines were Fe II, Ti II at  $\lambda$  4549.6, Fe II at  $\lambda$  4508.3 and Fe I at  $\lambda$  4476.0. In general, we avoided metallic-line stars because they are suspected to constitute a peculiar slow-rotating group.

*Jaschek:* Can you distinguish an Am star from a normal star on the basis of your photometric technique?

*Faber:* We see no difference in the  $c_1 - (b - y)$  diagram used for classifying the main-sequence stars. We have not in general made use of the  $m_1$  index for classification where, presumably, metal abundance effects would be visible.

*Roxburgh:* Apart from the two cases you considered it would be worthwhile to do other cases; namely, angular momentum conserved in different chemical regions, angular velocity constant in convective zones, and angular momentum conserved in radiative regions or angular velocity constant in radiative regions.

*Faber:* Definitely.