CORRIGENDUM

EXTREME COVERINGS OF n-SPACE BY SPHERES

Journal of the Australian Mathematical Society 7 (1967), 115–127

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(Received 4 September 1967)

The proof of Theorem 1 is deficient; the error lies in part (ii) of the proof of Lemma 4.1, where the 'sufficient smallness' of ε is not shown to be independent of the matrix T. In order to repair the proof, we need the following refinements of Lemmas 3.1 and 3.3:

LEMMA 3.1′. *Let $f(\mathbf{x}) = \mathbf{x}'A\mathbf{x}$, where A is positive definite. Then there exist positive constants c_1, ε_1 such that for any neighbouring form*

$$g(\mathbf{x}) = \mathbf{x}'(A+\varepsilon T)\mathbf{x} \tag{1}$$

satisfying

$$\operatorname{tr}(A^{-1}T) = 0, \ \max |t_{ij}| = 1 \tag{2}$$

we have

$$d(g) < d(f)(1-c_1\varepsilon^2) \ \textit{whenever} \ 0 < \varepsilon < \varepsilon_1. \tag{3}$$

PROOF. As in the proof of Lemma 3.1, we have

$$d(g) = d(f)(1+k_2\varepsilon^2+k_3\varepsilon^3+\cdots+k_n\varepsilon^n) \tag{4}$$

where

$$A = P'P, \quad T = P'DP, \quad D = \operatorname{diag}(d_1, d_2, \cdots, d_n), \tag{5}$$

$$k_2 = -\tfrac{1}{2}\sum_{i=1}^{n} d_i^2 < 0. \tag{6}$$

Setting $a = \max|a_{ij}|$, $p = \max|p_{ij}|$, $d = \max|d_i|$, we have, from (5) and (6),

$$a \geqq p^2, \quad 1 = \max|t_{ij}| \leqq np^2d, \quad 2|k_2| \geqq d^2.$$

Hence

$$|k_2| \geqq \frac{1}{2n^2a^2},$$

giving a lower bound for $|k_2|$ which is independent of T. Also the coefficients

$k_3, \cdots, k_n$ in (4) are clearly bounded independently of T, since $\max |t_{ij}| = 1$. The result (3) now follows, with any c_1 satisfying

$$0 < c_1 < \frac{1}{2n^2a^2}.$$

We now write the result (3.3) of Lemma 3.2 in full as

$$\mathbf{w} = \mathbf{v} + \sum_{n=1}^{\infty} \varepsilon^n \boldsymbol{\alpha}_n \tag{7}$$

where, as in (3.4), (3.5),

$$\boldsymbol{\alpha}_1 = \boldsymbol{\gamma} - A^{-1}T\mathbf{v}, \tag{8}$$

$$\boldsymbol{\alpha}_n = -A^{-1}T\boldsymbol{\alpha}_{n-1} \qquad \text{for all } n > 1. \tag{9}$$

We now prove, with the above notation,

LEMMA 3.3′.

$$g(\mathbf{w}) = f(\mathbf{v}) + \varepsilon(2\mathbf{v}'A\boldsymbol{\gamma} - \varphi(\mathbf{v})) + \sum_{n=1}^{\infty} \varepsilon^{2n}\psi(\boldsymbol{\alpha}_n) \tag{10}$$

where

$$\varphi(\mathbf{x}) = \mathbf{x}'T\mathbf{x}, \ \psi(\mathbf{x}) = f(\mathbf{x}) - \varepsilon\varphi(\mathbf{x}) = \mathbf{x}'(A - \varepsilon T)\mathbf{x}. \tag{11}$$

PROOF. All series being absolutely convergent for all T satisfying (2) if ε is sufficiently small, we have

$$\begin{aligned} g(\mathbf{w}) &= \mathbf{w}'(A+\varepsilon T)\mathbf{w} \\ &= f(\mathbf{v}) + \sum_{n=1}^{\infty} c_n \varepsilon^n, \end{aligned}$$

where, from (7), (8) and (9), we easily find that

$$c_1 = 2\mathbf{v}'A\boldsymbol{\gamma} - \varphi(\mathbf{v})$$

and, for $n \geqq 2$,

$$c_n = \boldsymbol{\alpha}_{n-1}'A\boldsymbol{\alpha}_1.$$

Using (9) repeatedly, we obtain

$$\boldsymbol{\alpha}_{n-1}'A\boldsymbol{\alpha}_1 = -\boldsymbol{\alpha}_{n-2}'T\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_{n-2}'A\boldsymbol{\alpha}_2 = \cdots$$

whence

$$c_{2n} = \boldsymbol{\alpha}_n'A\boldsymbol{\alpha}_n, \quad c_{2n+1} = -\boldsymbol{\alpha}_n'T\boldsymbol{\alpha}_n \qquad (n \geqq 1).$$

The result (10) now follows at once.

PROOF OF LEMMA 4.1 (ii).

We have to show that an interior form f is extreme if there exists no symmetric T satisfying

https://doi.org/10.1017/S1446788700006297 Published online by Cambridge University Press

(12) $$\operatorname{tr}(A^{-1}T) \geqq 0$$

and, for every maximal vertex $\boldsymbol{v}$ of Π_f,

(13) $$2\boldsymbol{v}'A\boldsymbol{\gamma}-\varphi(\boldsymbol{v}) < 0.$$

As in the original paper (p. 122), we note that any sufficiently close neighbour g of f, which is not a multiple of f, can be written as

(14) $$g(\boldsymbol{x}) = \boldsymbol{x}'(A+\varepsilon T)\boldsymbol{x}$$

where

(15) $$\varepsilon > 0,\ \max |t_{ij}| = 1$$

and

(16) $$\operatorname{tr}(A^{-1}T) = 0.$$

We choose ε_2 so small that the form ψ defined in (11) is positive definite for all T satisfying (15) and all ε satisfying

(17) $$0 < \varepsilon < \varepsilon_2.$$

Since T now satisfies (12), our hypothesis implies that there is a maximal vertex $\boldsymbol{v}$ of Π_f for which

(18) $$2\boldsymbol{v}'A\boldsymbol{\gamma}-\varphi(\boldsymbol{v}) \geqq 0.$$

We denote the corresponding vertex of Π_g by $\boldsymbol{w}$; then, from (10), (17) and (18),

(19) $$\begin{aligned} m(g) &\geqq g(\boldsymbol{w}) \\ &\geqq f(\boldsymbol{v})+\varepsilon(2\boldsymbol{v}'A\boldsymbol{\gamma}-\varphi(\boldsymbol{v})) \\ &\geqq f(\boldsymbol{v}) = m(f). \end{aligned}$$

Choosing also ε_1 as in Lemma 3.1′, we obtain at once from (3) and (19) that

$$\mu(g) > \mu(f)$$

provided only that $0 < \varepsilon < \min(\varepsilon_1, \varepsilon_2)$; and this now shows that f is extreme.

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https://doi.org/10.1017/S1446788700006297 Published online by Cambridge University Press