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CORRECTION TO A RESULT OF JAIN ON A CORRELATED RANDOM WALK

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ABSTRACT. The authors have found that the formula for $U(t)$—the probability generating function (p.g.f) of a return to the starting position, obtained by Jain [1, p. 344] is incorrect and have obtained its correct formulae.

Consider the walk $\{S_n, n=0,2,\ldots\}$ where $S_0=0$, $S_n=X_1+\cdots+X_n$. Let X_i be the random variable associated with the ith step, assuming the value $+1$, -1 or zero according as the ith step is to the right, to the left or a stay, with probabilities as per the t.p.m., for $n\geq 1$,

(1)
$$\begin{array}{cc} \text{From/To} & \begin{array}{ccc}\text{right} & \text{left} & \text{stay}\end{array} \\ \begin{array}{c}\text{right}\\ \text{left}\\ \text{stay}\end{array} & \begin{bmatrix} p_1\alpha & q_1\alpha & 1-\alpha \\ q_2\alpha & p_2\alpha & 1-\alpha \\ \beta & \gamma & 1-\beta-\gamma \end{bmatrix} \end{array} \qquad p_1+q_1=p_2+q_2=1,$$

for $n=0$, $P(X_0=1)=\rho_1(=1-\rho_2)$ is an arbitrary probability.

For $r=0$ or an integer, let

(2)
$$u_r(n)=\begin{bmatrix} u_r^{+1,+1}(n) & u_r^{+1,-1}(n) & u_r^{+1,0}(n) \\ u_r^{-1,+1}(n) & u_r^{-1,-1}(n) & u_r^{-1,0}(n) \\ u_r^{0,+1}(n) & u_r^{0,-1}(n) & u_r^{0,0}(n) \end{bmatrix}$$

where $u_r^{i,j}(n)=P\{S_n=r, X_n=j \mid X_0=i\}$. Also, let

$$u_r^i(n)=\sum_j u_r^{i,j}(n) \quad \text{and} \quad U_r^i(t)=\sum_{n=0}^{\infty} u_r^i(n)t^n, \quad \text{etc.}$$

On using the 'one-step' t.p.ms. (as defined in [2]) for walk (1), we can easily write

(3)
$$P(s)=\begin{bmatrix} p_1\alpha s & q_1\alpha s^{-1} & 1-\alpha \\ q_2\alpha s & p_2\alpha s^{-1} & 1-\alpha \\ \beta s & \gamma s^{-1} & 1-\beta-\gamma \end{bmatrix}$$

and $U_r(t)$ is the coefficient of s^r in $\sum_{n=0}^{\infty}[tP(s)]^n=[I-tP(s)]^{-1}$.

With $\alpha=\beta+\gamma=1$ and $\beta=\rho_1$, the walk (1) reduces to the one considered by

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Jain [1] for which we, then, get

$$(4)\quad U_0(t)=\frac{(wp_1/p_2)^{1/2}}{p_1t(1-w)}\begin{bmatrix} 1-t(wp_1p_2)^{1/2} & q_1t(wp_1/p_2)^{1/2} & 0 \\ q_2t(wp_2/p_1)^{1/2} & 1-t(wp_1p_2)^{1/2} & 0 \\ \rho_1t(wp_2/p_1)^{1/2} & \rho_2t(wp_1/p_2)^{1/2} & 1+\delta t^2 \\ +(q_2-\rho_1)t^2 & +(\rho_1-p_1)t^2 & -2t(wp_1p_2)^{1/2} \end{bmatrix}$$

where

$$\delta=p_1-q_2,\qquad w=\frac{1+\delta t^2-\{(1+\delta t^2)^2-4p_1p_2t^2\}^{1/2}}{1+\delta t^2+\{(1+\delta t^2)^2-4p_1p_2t^2\}^{1/2}}.$$

In terms of Jain's notations ($w=A_1(t)B_{-1}(t)$), on summing each row in (4), we get the following p.g.fs. of conditional return to origin

$$(5)\qquad \begin{gathered} U_0^{+1}(t)=C[1-\delta tA_1(t)],\qquad U_0^{-1}(t)=C[1-\delta tB_{-1}(t)] \\ U_0^0(t)=C[1-tB_1(t)\{\rho_1(p_1-p_2)+p_1(2p_2-1)\}/p_2] \end{gathered}$$

where $C=A_1(t)/[1-A_1(t)B_{-1}(t)]\ p_1t$. The p.g.f. of unconditional return to zero, then, is

$$(6)\qquad U_0(t)=\sum_{i=-1}^{1} U_0^i(t)P(X_0=i)=\rho_1U_0^{+1}(t)+(1-\rho_1)U_0^{-1}(t)$$

For $p_1=p_2=p$, (6) verifies (13) in Seth [3].

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