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GEORGE THOM, Esq., President, in the Chair.

Proof of a Geometrical Theorem.

BY W. J. MACDONALD, M.A.

The middle points of the diagonals of a complete quadrilateral are collinear.

Let ABCDEF be a complete quadrilateral (see fig. 70), and X, Y, Z, the middle points of its diagonals. To prove X, Y, Z, collinear.

Let PQR be the diagonal triangle.

Then BPDQ is a harmonic range.

$$\therefore \quad BX^2 = XP XQ$$

$$\frac{BP}{BQ} = \frac{BX+XP}{BX+XQ} = \frac{\sqrt{XP.XQ}+XP}{\sqrt{XP.XQ}+XQ} = \frac{\sqrt{XP}(\sqrt{XQ}+\sqrt{XP})}{\sqrt{XQ}(\sqrt{XP}+\sqrt{XQ})} = \frac{\sqrt{XP}}{\sqrt{XQ}}$$

Similarly, $\dfrac{AP}{AR} = \dfrac{\sqrt{YP}}{\sqrt{YR}}$ and $\dfrac{QF}{FR} = \dfrac{\sqrt{ZQ}}{\sqrt{ZR}}$

Now $\dfrac{PB}{BQ}\cdot\dfrac{QF}{FR}\cdot\dfrac{RA}{AP} = -1$ ($\because$ triangle PQR is cut by BF)

i.e., $\dfrac{BP}{BQ}\cdot\dfrac{FQ}{FR}\cdot\dfrac{AR}{AP} = 1$

i.e., $\sqrt{\dfrac{XP}{XQ}\cdot\dfrac{ZQ}{ZR}\cdot\dfrac{YR}{YP}} = 1$

$\therefore$ $\dfrac{XP}{XQ}\cdot\dfrac{ZQ}{ZR}\cdot\dfrac{YR}{YP} = 1$

$\therefore$ $\dfrac{PX}{XQ}\cdot\dfrac{QZ}{ZR}\cdot\dfrac{RY}{YP} = -1$

$\therefore$ X, Y, Z, are collinear.

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