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Abstract

This note proves Cellini's conjecture that, in a Coxeter system (W, S) with reflections T, the T-increasing paths in W are self-avoiding. Here, a T-increasing path is a sequence $v, t_1v, \dots, t_n \cdots t_1v$ in W with $t_i \in T$ and $t_1 \prec \cdots \prec t_n$ in a reflection order $\preceq$ of T.

1. Introduction

A reflection order is a special total order of the set T of reflections of a Coxeter system (W, S). Despite significant applications to the study of Bruhat order, Hecke algebras and Kazhdan–Lusztig polynomials (see e.g. § 3.4), they remain poorly understood, and basic conjectures about them from [Dye92, Dye94] remain open.

Denote the standard length function of (W, S) as l. Let Φ be the standard root system, with positive roots Φ_+, and denote the reflection in $\alpha \in \Phi$ as $s_\alpha \in T$. Fix a reflection order $\preceq$ for (W, S) i.e. a total order $\preceq$ of T such that for all $\alpha, \beta, \gamma \in \Phi_+$ with $s_\alpha \preceq s_\gamma$ and $\beta = a\alpha + c\gamma$ for some $a, c \geqslant 0$ one has $s_\alpha \preceq s_\beta \preceq s_\gamma$. Such reflection orders are known to exist. They generalize reduced expressions of longest elements of finite Coxeter groups, in the following sense: if W is finite, the reflection orders are precisely the orders $t_1 \preceq \cdots \preceq t_N$ such that there is a reduced expression $s_1 \cdots s_N$ for the longest element of W such that for each $i = 1, \dots, N$ one has $t_i = s_1 \cdots s_{i-1} s_i s_{i-1} \cdots s_1$. In this note, the following is proved.

Theorem. *Let $t_1 \prec t_2 \prec \cdots \prec t_n$ in T. Then $l(t_1 \cdots t_n) \geqslant n$.*

Define the (undirected) Bruhat graph Ω of (W, T) to be the Cayley graph of W with respect to its generating set T. It is an undirected simple graph Ω with vertex set W and edge set $E := \{\{x, tx\} \mid x \in W, t \in T\}$. Equip it with an edge-labelling $L : E \to T$ defined by $L(\{x, tx\}) = t$. A path (in Ω) is defined to be a sequence $p = (v_0, \dots, v_n)$ in W with $\{v_{i-1}, v_i\} \in E$ for $i = 1, \dots, n$. The path p is said to be a $\preceq$-path if $t_1 \prec \cdots \prec t_n$, and to be self-avoiding if $v_i \neq v_j$ for $i \neq j$. In general, the opposite order $\preceq^{\mathrm{op}}$ of a reflection order $\preceq$ is a reflection order, so there is also a notion of $\preceq^{\mathrm{op}}$-path. The theorem implies the following corollary.

Corollary. (a) *Any $\preceq$-path in Ω is self-avoiding.*

(b) *Let $p = (v_0, \dots, v_n)$ be a $\preceq$-path and $q = (w_0, \dots, w_m)$ be a $\preceq^{\mathrm{op}}$- path with $w_0 = v_0$ and $w_m = v_n$. Then $\{v_0, \dots, v_n\} \cap \{w_0, \dots, w_m\} = \{v_0, v_n\}$.*

(c) *Unless $n = m \leqslant 1$, one has in (b) that $L(\{v_0, v_1\}) \prec L(\{w_0, w_1\})$ and $L(\{v_{n-1}, v_n\}) \succ L(w_{m-1}, w_m)$.*

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Part (a) was conjectured and proved for finite and affine Weyl groups by Cellini [Cel00], who called $\preceq$-paths 'T-increasing paths'. The results in [BI06] (which give rise to efficient algorithms for computation of Kazhdan–Lusztig polynomials) and [BB07, Proposition 6.3] (related to a non-negativity conjecture on the complete **cd**-index of Bruhat intervals) require at some points a special case of (c), which was only known previously to hold for finite and affine Weyl groups because of a dependence of its proof on [Cel00]. The results in [BB07, BI06] are now known to hold for general Coxeter systems, by virtue of the results in this note.

The arrangement of this note is as follows. In § 2, we recall without proof some necessary background. The (very brief, given the background) proof of the theorem and its corollary are given in § 3, followed by some concluding remarks.

2. Background

It is assumed throughout the paper that the reader is familiar with basic facts about Coxeter groups, Bruhat order, root systems etc.; as general references on these topics, see [BB05, Bou68, Hum90]. This section gives additional background concerning reflection orders and related properties of reflection subgroups needed in the proofs.

2.1 Definition of the reflection cocycle from [Dye87, ch. 1] and [Dye90]

Fix a Coxeter system (W, S). Let T, l, Φ, Φ_+, s_α be as in the introduction. Regard the power set $\mathcal{P}(T)$ of T as an additive abelian group under symmetric difference $A + B := (A \cup B)\backslash(A \cap B)$, for $A, B \subseteq T$. Define the 'reflection cocycle' $N : W \to \mathcal{P}(T)$ of (W, S) by $N(w) = \{t \in T \mid l(tw) < l(w)\}$. Then N is characterized by the conditions

$$N(xy) = N(x) + xN(y)x^{-1} \text{ for } x, y \in W, \quad N(s) = \{s\} \text{ for } s \in S. \tag{1}$$

Furthermore,

$$|N(w^{-1})| = l(w) \quad \text{for all } w \in W, \tag{2}$$

where the cardinality of any set X is denoted as $|X|$.

2.2 Properties of reflection subgroups from [Dye87, ch. 1] and [Dye90]

For any reflection subgroup $W' = \langle W' \cap T \rangle$ of W,

$$S' = \chi(W') = \chi_{(W,S)}(W') := \{t \in T \mid N(t) \cap W' = \{t\}\} \tag{3}$$

is a set of Coxeter generators for W'. The corresponding set of reflections and reflection cocycle for (W', S') are $W' \cap T$ and $N' : w \mapsto N(w) \cap W'$ respectively. If $T' \subseteq T$ is any set of reflections of W with $W' = \langle T' \rangle$, then $\bigcup_{w \in W'} wT'w^{-1} = W' \cap T$. Define the root system of W' to be the set $\Phi_{W'} := \{\alpha \in \Phi \mid s_\alpha \in W'\}$. If W'' is a reflection subgroup of W', then $\chi_{(W',S')}(W'') = \chi_{(W,S)}(W'')$.

2.3 Results on dihedral subgroups from [Dye87, (3.18)] and [Dye91, Remark 3.2]

A dihedral reflection subgroup is a reflection subgroup W' which may be generated by two reflections or, equivalently, such that the real vector space $\mathbb{R}\Phi_{W'}$ spanned by its root system is a plane. Any dihedral reflection subgroup W' of W is contained in a unique maximal (under inclusion) dihedral reflection subgroup W'' of W: one has $\Phi_{W''} = \mathbb{R}\Phi_{W'} \cap \Phi$.

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2.4 Definition of the dot action from [Dye92, § 1]

Twisting the W-action on $\mathcal{P}(T)$ by the reflection cocycle N gives another action

$$(w, A) \mapsto w \cdot A := N(w) + wAw^{-1} \quad \text{for } A \subseteq T \text{ and } w \in W \tag{4}$$

of W on $\mathcal{P}(T)$, to be called the dot action.

2.5 Results from [BB05, 5.2], [Dye87, ch. 6], [Dye93, § 2] and [Dye94, § 1]

A total order $\preccurlyeq$ on T is called a reflection order (for (W, S) or of T) if for any $r \prec s$ in T with $\{r, s\} = \chi(\langle r, s\rangle)$ the order induced by $\preccurlyeq$ on the set of reflections of $\langle r, s\rangle$ is

$$r \prec rsr \prec rsrsr \prec \cdots \prec srs \prec s. \tag{5}$$

This is equivalent to the definition in terms of Φ in the introduction.

A subset A of T is called an initial section (of T, *with respect to* S) if there is a reflection order $\preccurlyeq$ on T such that $r \prec s$ for all $r \in A$, $s \in T \backslash A$.

By [Dye93, Lemma (2.7)], the set $\mathcal{A} = \mathcal{A}_{(W,S)} \subseteq \mathcal{P}(T)$ of initial sections of T is stable under the dot W-action on $\mathcal{P}(T)$ and under complementation in T. Order $\mathcal{A}$ by inclusion.

It may be remarked that, by [Dye93, Lemma (2.11)], the map $w \mapsto N(w) : W \to \mathcal{A}$ gives an order isomorphism between W in weak right order (see [BB05, ch. 3] for the definition) and the order ideal of $\mathcal{A}$ consisting of finite initial sections of T. It is well known that W in weak right order is a complete semilattice (see [BB05, 3.2]) and it is conjectured that $\mathcal{A}$ is a complete ortholattice (see [Dye94, Remark 2.14]).

2.6 Facts from [Dye93, Remark (2.4)(ii)]

Let W' be a reflection subgroup of W, and $S' = \chi(W')$. The restriction to $W' \cap T$ of a reflection order $\preccurlyeq$ on T is a reflection order on $W' \cap T$ with respect to S'. Hence if A is an initial section of T with respect to S, then $W' \cap A$ is an initial section of $W' \cap T$ with respect to S'.

2.7 Properties of length functions from [Dye92] and [Dye94]

Fix an initial section A of T with respect to S. Define a length function $l_{(W,S,A)} = l_A = l : W \to \mathbb{Z}$ by

$$l_A(w) := |N(w^{-1})| - 2|N(w^{-1}) \cap A|. \tag{6}$$

This may be interpreted as an A-weighted version

$$l_A(w) = \sum_{t \in N(w^{-1})} \mathrm{wt}_A(t) \tag{7}$$

of (2), where the A-weight $\mathrm{wt}_A(t)$ of $t \in T$ is defined to be

$$\mathrm{wt}_A(t) = \begin{cases} -1 & \text{if } t \in A, \\ 1 & \text{if } t \in T \backslash A. \end{cases} \tag{8}$$

One has from the proof of [Dye92, Proposition (1.1)] that

$$l_A(xy) = l_A(y) + l_{y \cdot A}(x), \quad x \in W, y \in W. \tag{9}$$

If $t \in T$ and $w \in W$, one has by [Dye92, Proposition (1.2)] that

$$\begin{cases} l_A(tw) < l_A(w) & \text{if } t \in w \cdot A, \\ l_A(tw) > l_A(w) & \text{if } t \notin w \cdot A. \end{cases} \tag{10}$$

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By (9), equation (10) is equivalent to its special case with $w=1$, which asserts that $l_A(t)$ is positive or negative according as whether $t \notin A$ or $t \in A$. From (6),

$$\text{if } A, B \in \mathcal{A} \text{ with } A \subseteq B \text{ and } w \in W, \text{ then } l_B(w) \leqslant l_A(w) \text{ in } \mathbb{Z}. \tag{11}$$

The standard length function of (W, S) is $l = l_\emptyset$ and $l_{T+A}(w) = -l_A(w)$. By (6) and (2),

$$-l(w) \leqslant l_A(w) \leqslant l(w) \quad \text{for } A \in \mathcal{A} \text{ and } w \in W. \tag{12}$$

3. Proofs

Fix $t \in T$. Let $\mathcal{M} = \mathcal{M}_t$ be the family of all maximal dihedral reflection subgroups of W which contain t. Let $A \in \mathcal{A}$ with $t \in A$. One has by [Dye92, (1.2.1)] that

$$l_{(W,S,A)}(t) = -1 + \sum_{W' \in \mathcal{M}} (1 + l_{(W', \chi(W'), A \cap W')}(t)). \tag{13}$$

This sum involves only finitely many non-zero terms. The equation (13) can be proved using the interpretation of $l_A(w)$ as a weighted sum in (7), noting that every reflection $t' \in T$ with $t' \neq t$ is contained in a unique element of $\mathcal{M}_t$, by § 2.6.

For the proofs of the theorem and corollary, we need only the equivalence of conditions (a)–(c) from the following lemma; the final assertion is included for completeness.

Lemma 3.1. *Let $A \in \mathcal{A}$ and $t \in T \backslash A$. Then the following conditions are equivalent:*

(a) *$A + \{t\} \in \mathcal{A}$;*

(b) *$t \cdot A = A + \{t\}$;*

(c) *there is a reflection order $\preceq$ of T such that $A = \{s \in T \mid s \prec t\}$.*

If these conditions hold, then $l_A(t) = 1 = -l_{t \cdot A}(t)$.

Proof. The equivalence of conditions (a)–(c) comes from [Dye93, Lemma 2.9] and its proof. The final claim is proved as follows. Note first that, by (9), $l_A(t) + l_{t \cdot A}(t) = l_A(1) = 0$. Hence it is sufficient to show that $l_{t \cdot A}(t) = -1$ where $t \cdot A = \{s \in T \mid s \preceq t\}$. First, one checks that this holds if (W, S) is dihedral; we omit the details of this routine verification. In general, note that in (13) with A replaced by $t \cdot A$, each term $1 + l_{(W', \chi(W'), t \cdot A \cap W')}(t) \leqslant 0$ by (10) since $t \in t \cdot A \cap W'$. Hence for $t \in A$, one has $l_{(W,S,t \cdot A)}(t) = -1$ if (and only if) $l_{(W', \chi(W'), t \cdot A \cap W')}(t) = -1$ for every $W' \in \mathcal{M}$. However, the latter holds by the dihedral case, since $t \cdot A \cap W' = \{s \in T \cap W' \mid s \preceq' t\}$ where $\preceq'$ is the reflection order of $W' \cap T$ obtained by restricting $\preceq$. □

Note that the above lemma describes certain coverings $A \subsetneq A \cup \{t\}$ in the partially ordered set $\mathcal{A}$. A conjecture (cf. [Dye94, Remark 2.14]), that any totally ordered subset of $\mathcal{A}$ is a subset of the set of all initial sections of some reflection order $\preceq$, implies that all coverings in $\mathcal{A}$ arise as in the lemma.

The following result is the key lemma in this paper.

Lemma 3.2. *Let $A \in \mathcal{A}$ be an initial section of the reflection order $\preceq$, and let $t_1 \prec \cdots \prec t_n$ be in A. Let $x = t_1 \cdots t_n$. Then $l_A(x) \leqslant -n$.*

Proof. For $t \in T$, let $A_t := \{s \in T \mid s \preceq t\}$. We prove that $l_A(x) \leqslant -n$ by induction on n. If $n = 0$, then $l_A(x) = l_A(1) = 0$. If $n > 0$, then $t_1 \prec \cdots \prec t_{n-1}$ are all in $A_{t_n} \backslash \{t_n\} = t_n \cdot A_{t_n}$, by Lemma 3.1. Also, $A_{t_n} \backslash \{t_n\} = \{s \in T \mid s \prec t_n\}$ is an initial section of $\preceq$. Assume inductively

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that $l_{t_n \cdot A_{t_n}}(t_1 \cdots t_{n-1}) \leqslant -(n-1)$. Then by (11), (10) (or Lemma 3.1) and (9),

$$l_A(x) \leqslant l_{A_{t_n}}(t_1 \cdots t_n) = l_{A_{t_n}}(t_n) + l_{t_n \cdot A_{t_n}}(t_1 \cdots t_{n-1}) \leqslant -1 - (n-1) = -n. \qquad \square$$

3.3 Proof of the main results

The proof of the main results in the introduction is straightforward from Lemma 3.2, as follows. Let $\preceq$ be a reflection order and $t_1 \prec \cdots \prec t_n$ in T. Define the initial section $A := \{s \in T \mid s \preceq t_n\}$ of $\preceq$. Then (12) and Lemma 3.2 imply that $-l(x) \leqslant l_A(x) \leqslant -n$, proving the theorem. Part (a) of the corollary follows immediately from the theorem.

Now make assumptions as in part (b) of the corollary. Set $t_i = L(\{v_{i-1}, v_i\})$ for $i = 1, \ldots, n$ and $s_i = L(\{w_{i-1}, w_i\})$ for $i = 1, \ldots, m$. We prove part (c) before part (b). Suppose it is not true that $m = n \leqslant 1$. Choose $k \in \mathbb{N}$ maximal such that $k \leqslant \min(m, n)$ and $v_i = w_i$ for all $0 \leqslant i \leqslant k$. Necessarily, $k \leqslant 1$ because $(v_0, \ldots, v_k)$ is both a $\preceq$-path and a $\preceq^{\mathrm{op}}$-path. If $x_k = v_n$, then part (a) implies that $k = m = n \leqslant 1$. Hence $x_k \neq v_n$. Then $k < n$, $k < m$ and $t_{k+1} \neq s_{k+1}$ (or else $v_{k+1} = w_{k+1}$). If $s_{k+1} \prec t_{k+1}$, then $s_m \prec \cdots \prec s_{k+1} \prec t_{k+1} \prec \cdots \prec t_n$, and so

$$(w_m, \ldots, w_{k+1}, w_k = v_k, v_{k+1}, \ldots, v_n = w_m)$$

is a non-self-avoiding $\preceq$-path, contrary to part (a). Hence $t_{k+1} \prec s_{k+1}$. If $k = 1$, this gives that $t_1 \prec t_2 \prec s_2 \prec s_1 = t_1$, which is a contradiction. Therefore, $k = 0$ and $L(\{v_0, v_1\}) = t_1 \prec s_1 = L(\{w_0, w_1\})$. Since $(w_m, \ldots, w_0)$ is a $\preceq$-path and $(v_n, \ldots, v_0)$ is a $\preceq^{\mathrm{op}}$-path with $w_m = v_n$ and $w_0 = v_0$, it follows by symmetry that $L(w_{m-1}, w_m) \prec L(\{v_{n-1}, v_n\})$. This completes the proof of part (c). For use in the proof of part (b), note that part (c) holds weakly (with $\prec$ and $\succ$ replaced by $\preceq$ and $\succeq$) even if $n = m = 1$.

Finally, we prove part (b). Suppose that part (b) is false. By part (a), there must exist i and j with $0 < i < n$, $0 < j < m$ and $v_i = w_j$. There are $\preceq$-paths $p' = (v_0, \ldots, v_i)$, $p'' = (v_i, \ldots, v_n)$ and $\preceq^{\mathrm{op}}$-paths $q' = (w_0, \ldots, w_j)$ and $q'' = (w_j, \ldots, w_m)$. Using, in turn, first the fact that p is a $\preceq$-path, then the weak version of part (c) with (p, q) replaced by (p', q'), then the fact that q is a $\preceq^{\mathrm{op}}$-path and finally the weak version of part (c) with (p, q) replaced by (p'', q''), it follows that

$$t_{i+1} \succ t_i \succeq s_j \succ s_{j+1} \succeq t_{i+1}.$$

This contradiction completes the proof of part (b) and of the corollary.

3.4 Concluding remarks

For $A \in \mathcal{A}$, say that a path $(v_0, \ldots, v_n)$ is a l_A-increasing path if $l_A(v_0) < \cdots < l_A(v_n)$. Define a partial order $\leqslant_A$ on W by setting $v \leqslant_A w$ if there is such a l_A-increasing path with $v_0 = v$ and $v_n = w$. This is called the twisted Bruhat order associated to A. When $A = \emptyset$, $\leqslant_A$ reduces to ordinary Bruhat order. For arbitrary A, the 'spherical' intervals of $\leqslant_A$ (which may or may not include all intervals of $\leqslant_A$, depending on A) have similar properties to Bruhat intervals, but in general there are additional subtleties [Dye93, Dye94].

Fix $A \in \mathcal{A}$, a reflection order $\preceq$ of T and an initial section B of $\preceq$. Also fix $v, w \in W$. One may consider various combinations of conditions such as the following on a Ω-path $p = (v_0, \ldots, v_n)$ from v to w i.e. with $v_0 = v$ and $v_n = w$:

(a) p is l_A-increasing;

(b) p is a $\preceq$-path;

(c) all labels of the edges of p are in B.

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The usual applications of reflection orders to Bruhat order require considering paths of increasing (standard) length in the Bruhat graph i.e. paths satisfying condition (a) with $A=\emptyset$. To illustrate some of the applications, take $A=\emptyset$ and assume that $v \leqslant_\emptyset w$. Then the natural labelling of paths p satisfying condition (a) and with $n=l(w)-l(v)$ determines a dual EL-labelling of the Bruhat interval $[v, w]$ (see [Dye93]), and the pattern of the ascents and descents of such paths determines the **cd**-index of the interval as Eulerian poset (see e.g. [BB07]). Dropping the condition $n=l(w)-l(v)$, the pattern of ascents and descents in such paths p determines the 'complete **cd**-index' (see [BB07]), the topological and combinatorial interpretation of which is less well understood. The Kazhdan–Lusztig R-polynomial $R_{v,w}$, which is crucial in the definition of Kazhdan–Lusztig polynomials but is poorly understood combinatorially, can be interpreted as a (renormalized) generating function for the set of paths p satisfying conditions (a) and (b) [BB05, Dye87, Dye93]. More generally, suitable generating functions of paths satisfying conditions (a), (b) and (c) can be interpreted as 'generalized' structure constants for the Iwahori–Hecke algebra of W (see [Dye93]). Similar results to those above apply with any $A \in \mathcal{A}$, using spherical intervals $[x, y]$ in $\leqslant_A$ and modules (depending on A) for the Iwahori–Hecke algebra [Dye92, Dye94].

The above-mentioned results apply to, and are proved using, fixed A. In contrast, the main idea in this note is to study the effect of varying A on the length functions l_A. A subsequent paper will examine more systematically the effect of varying A on the twisted orders $\leqslant_A$, obtaining new results on and relationships between the twisted Bruhat orders, ordinary Bruhat order and (partly conjecturally) the inclusion-ordered set $\mathcal{A}$.

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