BULL. AUSTRAL. MATH. SOC. 46A99, 47H99, 58C20, 58A99
VOL. 9 (1973), 305-306.

Two topics in the differential calculus on topological linear spaces

John Lloyd

The thesis is mainly concerned with an investigation of the smoothness properties of topological linear spaces. First we extend the requisite general theory of S-categories (as defined by Bonic and Frampton [3]) from Banach spaces to topological linear spaces. Then using the kernel theorem, which states roughly that smoothness properties are preserved under the operations of forming products and subspaces, we prove smoothness results for various important classes of locally convex spaces. For example, we show that every Schwartz space (in particular, every nuclear space) has a collection of C^{∞}-seminorms, which generate its topology. We apply these results to proving the existence of smooth partitions of unity on manifolds modelled on topological linear spaces.

Of course, there are many definitions of the derivative in topological linear spaces. We adopt the definitions of the Fréchet, Hadamard and Gâteaux derivatives investigated in detail by Averbukh and Smolyanov [1], [2]. Each of these three derivatives is a special case of what Averbukh and Smolyanov call a σ-derivative, corresponding to different choices of the class σ of bounded sets.

The other topic referred to in the title is the connection between differentiability, strong continuity and precompactness for non-linear mappings. For example, we show that under certain conditions, if f is a Fréchet differentiable mapping, then f is strongly continuous if and only if f' is strongly continuous and $f'(x)$ is strongly continuous for each x .

Received 12 February 1973. Thesis submitted to the Australian National University, February 1973. Degree approved, June 1973. Supervisor: Dr Sadayuki Yamamuro.

https://doi.org/10.1017/S0004972700043227 Published online by Cambridge University Press

References

[1] V.I. Averbukh and O.G. Smolyanov, "The theory of differentiation in linear topological spaces", *Russian Math. Surveys* **22:6** (1967), 201-258.

[2] V.I. Averbukh and O.G. Smolyanov, "The various definitions of the derivative in linear topological spaces", *Russian Math. Surveys* **23:4** (1968), 67-113.

[3] Robert Bonic and John Frampton, "Smooth functions on Banach manifolds", *J. Math. Mech.* **15** (1966), 877-898.

[4] John W. Lloyd, "Inductive and projective limits of smooth topological vector spaces", *Bull. Austral. Math. Soc.* **6** (1972), 227-240.

[5] John Lloyd, "Differentiable mappings on topological vector spaces", *Studia Math.* **45** (1973), 147-160.

[6] John Lloyd, "Smooth partitionsoof unity on manifolds", *Trans. Amer. Math. Soc.* (to appear).

https://doi.org/10.1017/S0004972700043227 Published online by Cambridge University Press