THE OBSERVATIONAL PROPERTIES OF THE ZZ CETI STARS

Edward L. Robinson

Department of Astronomy
University of Texas at Austin

Introduction

The ZZ Ceti stars are the pulsating white dwarfs lying within a narrow instability strip, extending in temperature from 13,500 to 10,500 K, on the white dwarf cooling sequence. That white dwarfs should be pulsationally unstable cannot be considered surprising, since theoretical investigations of white dwarf pulsations began at least as early as 1949 (Sauvenier-Goffin 1949). As the predicted pulsation periods demonstrate, however, the nature of the pulsations was not correctly foreseen. With very few exceptions (e.g. Harper and Rose 1970) the early investigations assumed that the most likely pulsations to be excited were radial pulsations. Thus, the calculated periods were quite short, typically 2-10 seconds. The first of the pulsating white

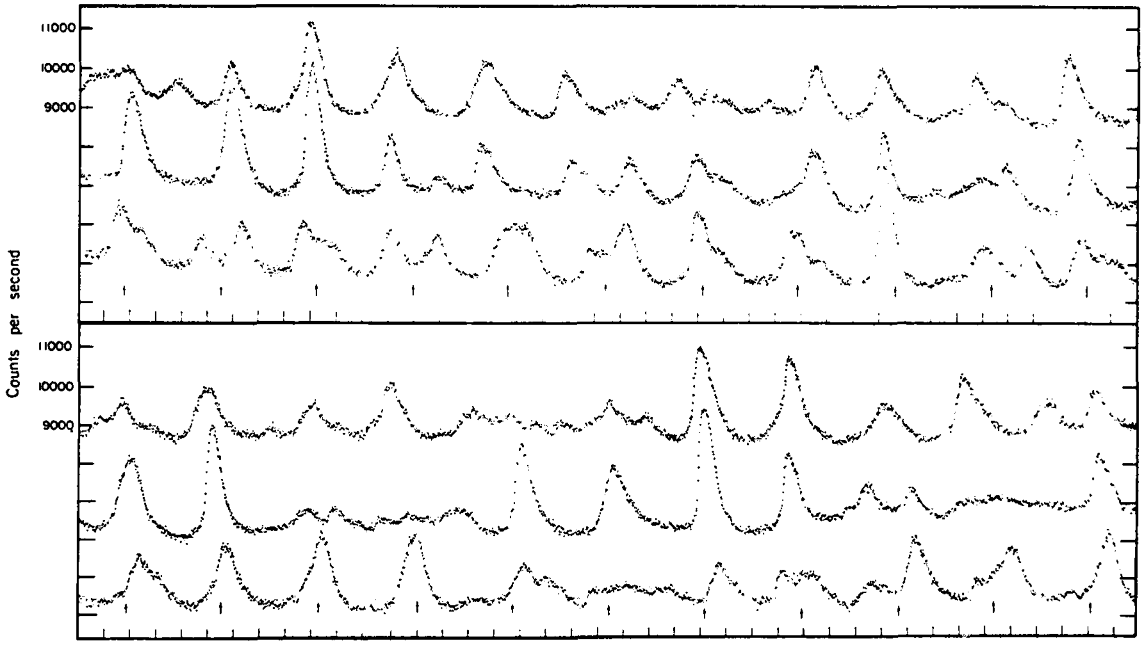

Fig. 1. The light curve of HL Tau-76 on the nights of 7 February, 29 October, and 30 October 1970. The abscissa marks are every 200 seconds. The interval between the arrows is the principal period of 746 seconds. The curves in the upper panel are continued in the lower panel.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

dwarfs actually to be observed was HL Tau-76 (Landolt 1968). A portion of its light curve is shown in Figure 1. The typical interval between successive pulses is about 750 seconds, not 2-10 seconds. This is a serious discrepancy, and one that exists for all of the ZZ Ceti stars discovered since HL Tau-76. Clearly, then, the observational data requires us to modify our understanding of pulsating white dwarfs. A considerable body of this observational data now exists. The purpose of the present paper is to present the data, and to show that it provides a reasonably coherent picture of the pulsating white dwarfs.

The Census of the ZZ Ceti Stars

A total of 13 ZZ Ceti stars has now been found. They are listed in Table 1 along with their spectral types, UBV colors and magnitudes, and references to the paper in which their variability was first reported. The colors and magnitudes have been extracted from the series of papers by Eggen and Greenstein (Eggen 1968, 1969; Eggen and Greenstein 1965; Greenstein 1969). Table 1 is a critical list of ZZ Ceti stars, and includes only the undisputed members of the class. Other, less selective lists of ZZ Ceti stars could be, and have been made. Therefore, we give here our reasons for excluding some specific stars from Table 1.

TABLE 1

THE ZZ CETI STARS

Star	α	(1950)	δ	Sp.	V	B-V	U-B	Ref.
BPM 30551	1^h	$4^m.7$	-46°26'	DA	15.26	+0.29	-0.58	1,2
R548	1	33.7	-11 36	DA	14.10	+0.20	-0.54	3
BPM 31594	3	41.8	-45 58	DA	15.03	+0.21	-0.66	4
HL Tau-76	4	16.8	27 13	DA	14.97	+0.20	-0.50	5
G38-29	4	17.0	36 9	DAs	15.63	+0.16	-0.53	6
GD 99	8	58.7	36 19	DA	14.55	+0.19	-0.59	7
G117-B15A	9	21.2	35 30	DA	15.52	+0.20	-0.56	7,8
GD 154	13	7.6	35 26	DA	15.33	+0.18	-0.59	9
L19-2	14	25.4	-81 7	DA	13.75	+0.25	-0.53	2,10
R808	15	59.5	36 57	DA	14.36	+0.17	-0.56	7
G207-9	18	55.7	33 53	DAn	14.64	+0.17	-0.60	11
GD 385	19	50.4	25 1	DA	15.5	+0.19	-	12
G29-38	23	26.3	4 58	DA	13.10	+0.20	-0.65	13

References:

1) Hesser et al. (1976a)
2) McGraw (1977)
3) Lasker and Hesser (1971)
4) McGraw (1976)
5) Landolt (1968)
6) McGraw and Robinson (1975)
7) McGraw and Robinson (1976)
8) Richer and Ulrych (1974)
9) Robinson et al. (1978)
10) Hesser et al. (1977)
11) Robinson and McGraw (1976)
12) Fontaine and McGraw (1979)
13) Shulov and Kopatskaya (1973)

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

AM CVn and G61-29: These two stars are both close binaries. Their variability is caused by mass transfer, not by stellar pulsations (Faulkner et al. 1972, Smak 1975).

G169-34: According to Richer and Ulrych (1974) G169-34 varies with a period of 465 seconds. McGraw and Robinson (1976) observed this star on several occasions, but it was always constant. It is possible that the star has changed its properties, but it seems more likely that the variations detected by Richer and Ulrych were spurious.

LFT 1679: Hesser et al. (1976b) have given a preliminary report of variations in LFT 1679. Since Hesser et al. suggest that the variations might have been caused by eclipses, we have temporarily excluded LFT 1679 from Table 1 until additional data becomes available.

G44-32: G44-32 is a DC white dwarf (Greenstein 1970) that has been reported to be variable three times (Giclas et al. 1959; Lasker and Hesser 1969; Warner et al. 1970). However, Robinson (1979) argued that all three reports were unreliable, and presented new observations indicating that G44-32 is a constant star. The evidence that G44-32 is a constant star is very strong, but since G44-32 is the only DC white dwarf that is a candidate for inclusion in Table 1, it is important to establish conclusively whether or not the star is variable. Additional observations will be required to do this.

PG 1159-035: McGraw (private communication) has detected pulsations with periods of 535 and 458 seconds in this star, and observations by Nather at McDonald Observatory have confirmed the variability. There is no doubt that PG 1159-035 is variable, and little doubt that it is pulsating. Nevertheless, PG 1159-035 is so different from the ZZ Ceti stars that it cannot, with good judgment, be placed in that class. In particular, PG 1159-035 is too hot ($T_e \gtrsim 30{,}000$ K) to be a ZZ Ceti star, and it is possibly a sub-dwarf rather than a white dwarf.

Other Non-variables: Lists of non-variable white dwarfs which were observed during surveys for variable white dwarfs can be found in Hesser and Lasker (1971, 1972), Hesser et al. (1969), Lawrence et al. (1967), McGraw (1976, 1977), Robinson and McGraw (1976), Robinson et al. (1978), and Richer and Ulrych (1974). Care must be exercised when using these lists because a star may appear - occasionally disguised by an alias - in more than one of them.

Collective Properties of ZZ Ceti Stars

We shall delay until the next section a detailed discussion of the pulsations of the ZZ Ceti stars in order to establish, in this section, the fundamental properties of the class as a whole. Two of the properties of the ZZ Ceti stars are immediately evident from Table 1: all are DA white dwarfs, and all have colors near B-V = +0.20. The restriction of the spectral type to DA cannot be attributed to selection effects. A total of 136 constant white dwarfs are listed in the references given in the previous section. Of the non-variables, 25 are known to have spectral types other than DA, and another 34 are white

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

dwarfs with unknown spectral types. Indeed, two of the variables, GD 154 and BPM 31594, had unknown spectral types when their variability was first detected and only afterwards were found to have DA spectra. Figure 2 is the color-color diagram for all DA white dwarfs which have been examined for variability and have published colors. The filled circles represent the ZZ Ceti stars and the crosses represent the non-variables. The ZZ Ceti stars occupy a narrow instability strip lying between B-V colors of +0.16 and +0.29. The position of the blue edge of the strip is well defined, but because of the general

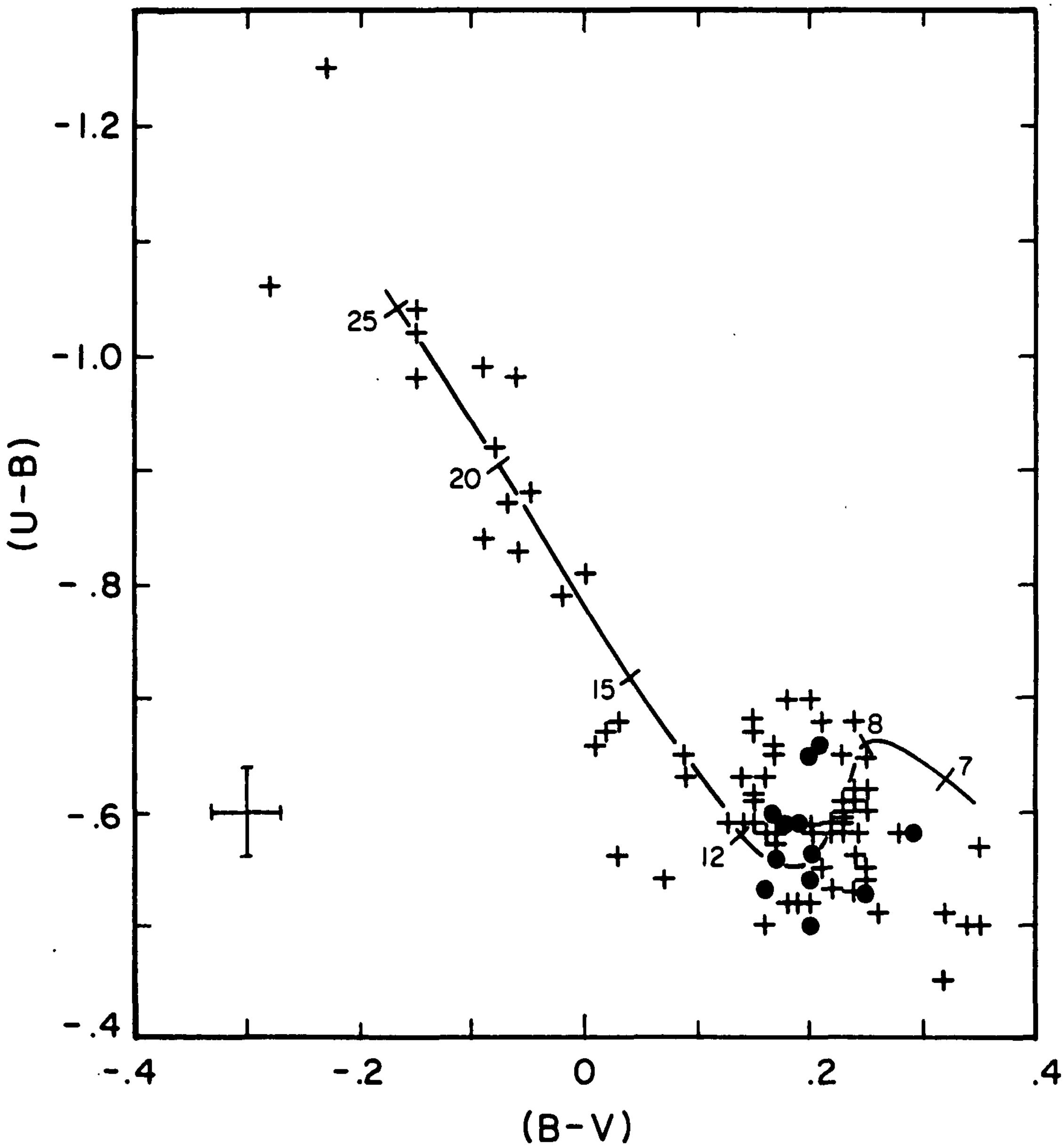

Fig. 2. The Johnson two color diagram for all DA white dwarfs which have been examined for variability and have published colors. The filled circles represent the ZZ Ceti stars and the crosses represent the non-variables.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

paucity of cool white dwarfs, the position of the red edge depends on just one star, BPM 30551, and is less certain. That the instability strip has a finite width of at least 0.10 in B-V is well established by McGraw's (1979) independent measurement of the Strömgren (b-y)colors of the ZZ Ceti stars.

The DA spectral types of the ZZ Ceti stars means that a white dwarf must have hydrogen in its atmosphere in order to be observed as variable. The exact placement of the instability strip emphasizes the importance of hydrogen. The solid line in Figure 2 is the theoretical locus of DA white dwarfs with log g = 8 (Terashita and Matsushima 1969). The dip in the line near B-V = +0.20 is caused by a maximum in the Balmer line and continuum absorption. The variable white dwarfs cluster near this region where the Balmer absorption is strongest. It has been stated several times (e.g. McGraw 1979, Nather 1979) that the ZZ Ceti instability strip lies on an extension of the Cepheid instability strip in the H-R diagram. Although this does appear to be true, it must be noted that the statement requires an extrapolation of the Cepheid instability strip through 5 orders of magnitude in luminosity. That the Cepheid instability strip can be extrapolated at all to the ZZ Ceti instability strip suggests strongly that the Cepheid instability mechanism is also driving pulsations in the ZZ Ceti stars, but the extrapolation is so large that the possibility that other instability mechanisms are acting cannot be excluded.

A third property of the variables is that, although only 13 are known, they must be considered to be very common. Among the DA white dwarfs that have been examined for variability, there are 13 variables and 37 non-variables within the instability strip. Thus, about 25 percent of the DA White dwarfs in the instability strip are variables. This must be a lower limit because variations with low amplitudes are difficult to detect in stars as faint as white dwarfs, so that some of the non-variables could actually be low amplitude variables. On the other hand, it is not possible to argue that all of the DA white dwarfs in the instability strip are variable. Many of the non-variables, especially the brighter ones, have been observed to be constant on two and sometimes three different nights. The solid line in Figure 2 is also the cooling sequence for DA white dwarfs. The cooling sequence passes through the instability strip. Since every white dwarf travels down the cooling sequence and must eventually traverse the instability strip, our statistics indicate that at least one quarter of all DA white dwarfs have been or will become variables.

A fourth property of the ZZ Ceti stars is that, apart from the obvious fact that they vary, they are quite normal and completely indistinguishable from the non-variable white dwarfs. Figure 2 demonstrates that their colors are normal for DA white dwarfs. Their spectra are normal DA spectra. The Hγ line profile of GD 154 is virtually identical to that of the non-variable white dwarf L47 (Robinson 1979). Trigonometric parallaxes have been measured for 6 ZZ Ceti stars and are listed in Table 2 along with the derived absolute visual magnitudes. The mean absolute magnitude of the group is 11.7±.2. The non-variable DA white dwarfs with similar colors listed

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

TABLE 2

ABSOLUTE MAGNITUDES OF ZZ CETI STARS

STAR	TRIGONOMETRIC PARALLAX	MEAN ERROR	M_V	REF.
R548	.014	.002	9.8 ± .3	1
G38-29	.013	.004	11.2 ± .8	2
G117-B15A	.012	.005	10.9 ± 1.1	3
R808	.034	.005	12.1 ± .3	2
G207-9	.030	.004	12.0 ± .3	4
G29-38	.071	.004	12.4 ± .3	5
	WEIGHTED AVERAGE		11.7 ± .2	

REFERENCES:

1) Dahn et al. (1976)
2) Routly (1972)
3) Harrington et al. (1978)
4) Harrington et al. (1975)
5) Riddle (1970)

in the McCook and Sion (1977) catalogue have a mean absolute visual magnitude of 12.5±.2. The variables seem to be slightly brighter than the non-variables, but since their mean magnitude is heavily biased by the unusual parallax of just one variable, R548, the difference is of only marginal significance. The most accurate way to estimate the effective temperatures and gravities of DA white dwarfs is by measuring their colors in the Strömgren uvby system. The Strömgren colors of 10 of the variable white dwarfs have been measured by McGraw and Wegner (McGraw 1979) and are listed in Table 3. The colors are time averages since the colors vary as the luminosity varies. The corresponding effective temperatures and gravities are also given in Table 3 and have been derived by comparing the colors to the theoretical colors of DA white dwarfs calculated by Wickramasinghe and Strittmatter (1972). The effective temperatures range from about

TABLE 3

THE EFFECTIVE GRAVITIES AND TEMPERATURES OF THE VARIABLE WHITE DWARFS

STAR	b-y	u-b	T_e	Log g
R548	.036	.686	12550±150	7.77±.02
HL.Tau-76	.033	.634	13010±350	7.94±.02
G38-29	.063	.678	11900±1000	7.9 ±.4
GD99	.035	.587	13350±1000	8.1 ±.4
G117-B15A	.032	.556	13640±350	8.14±.05
R808	.078	.655	11730±250	7.98±.05
G29-38	.060	.614	12630±150	8.14±.02
BPM 30551	.122	.628	10315±400	7.79±.15
BPM 31594	.028	.665	12870±400	7.80±.25
L19-2	.071	.598	12520±400	8.24±.25

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

10,500 K to 13,500 K and log g ranges from about 7.8 to 8.1. None of these values is unusual for DA white dwarfs. The finite spread in the temperatures and gravities is real and not due to observational error. As a by-product of this study, McGraw was able to show that changes in the effective temperature are sufficient to account completely for the luminosity changes of the variables. This agrees with the earlier study by Warner and Nather (1972). The space motions of the ZZ Ceti stars have been investigated by Sion et al. (1978). They found that the ZZ Ceti stars are kinematically indistinguishable from non-variable DA white dwarfs with the same colors. Thus, the ZZ Ceti stars have normal colors, spectra, temperatures, gravities, and space motions. In addition, Angel (1978) was unable to detect any magnetic field in two of the variables, R548 and HL Tau-76.

From their normality, we conclude that most of the variables are isolated, unperturbed white dwarfs. In particular, they are not all members of close binary systems. Any companion of the variables must be too faint to alter their absolute magnitudes or spectra. This is not possible unless the companion is either a similar DA white dwarf or is at least 2 magnitudes fainter than a white dwarf. In short, the companion must be invisible when next to a DA white dwarf. It is not impossible and perhaps even likely that a few of the variables have close, invisible companions (e.g. Fitch 1973). However, we have shown that the variables are very common. The probability that more than one quarter of all white dwarfs have invisible companions would appear vanishingly small. The normality of the ZZ Ceti stars raises an important question: why do some of the DA white dwarfs in the instability strip vary while others do not vary or vary with undetectably low amplitudes? Clearly there must be some difference in the physical properties of the ZZ Ceti stars that causes them to pulsate. Since the difference has not yet been found, it almost certainly involves some physical property that is difficult to measure with accuracy. Prime candidates are the mass of the white dwarf, and the run of chemical composition through its envelope.

The Properties of the Pulsations

The light curve of each ZZ Ceti star has a characteristic peak-to-peak amplitude which ranges from about 0.02 mag to 0.34 mag among the known variables. The properties of the pulsations depend strongly on the amplitude of the light curve. In order to illustrate the dependence we shall discuss the light curves of two variables, the low amplitude variable R548, and the high amplitude variable G29-38.

The light curve of R548 (=ZZ Ceti itself) in unfiltered light is shown in Figure 3. The amplitude of the variations is about 0.02 mag, the lowest of the known variables, and the time scale of the variations is about 220 seconds. Figure 4 shows power spectra of the light curve of R548 on three successive nights in 1975. At first sight two, but only two, periods are present in the light curve, one near 213 seconds and one near 274 seconds. Each period varies considerably in amplitude

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

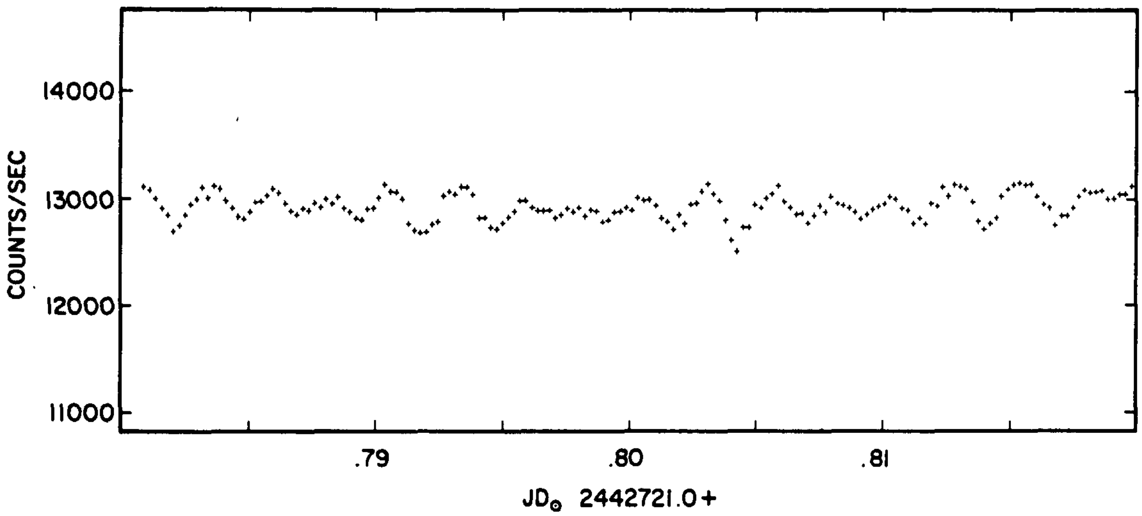

Fig. 3. The light curve of R548. Each point is a 20 second average of the photon counting rate.

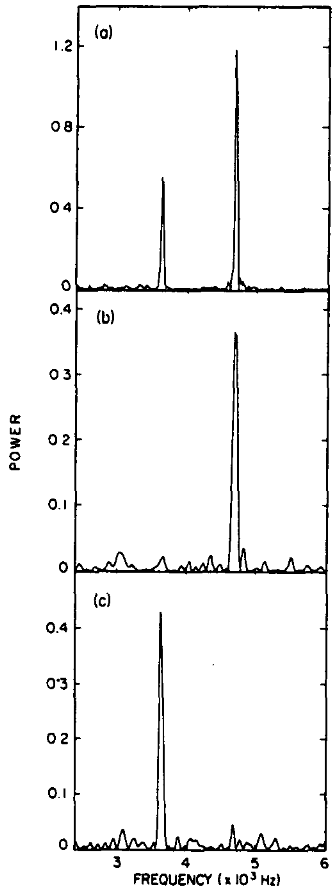

Fig. 4. Power spectra of the light curve of R548 on UT dates (a) 1975 October 6, (b) 1975 October 7, and (c) 1975 October 8.

and slightly in period. A detailed analysis of the light curve demonstrates that there are four, not two, periods present (Robinson et al. 1976). Each of the two periods shown in Figure 4 is actually a pair of independent sinusoidal variations with periods separated by about 0.5 seconds. The amplitudes and periods of the four pulsations are given in Table 4 (Stover et al. 1979). The separation of the two components of the pairs is sufficiently small that each pair appears as a single sinusoidal pulsation that is the linear sum of its two components. The two components beat together slowly, causing the sum pulsation to vary in both amplitude and phase with beat periods of 1.44122 days for the 213 second pair and 1.66528 days for the 274 second pair. Figures 5 and 6 demonstrate that this decomposition into 4 pulsations completely accounts for the behavior of the 213 second and 274 second sum pulsations. In the upper half of each figure the measured amplitudes of the sum pulsations are plotted against phase in the beat cycle, with each dot being the mean amplitude on a separate night. The solid line is the amplitude predicted by letting the pulsation pairs listed in Table 4 beat against each other. The root-mean-square deviation of the points from the lines is 0.0014 mag in both figures, and is consistent with random measurement error. In the lower half of each figure the difference between the observed and predicted (O-C) times of arrival of the pulsations is plotted. The RMS residuals in the

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

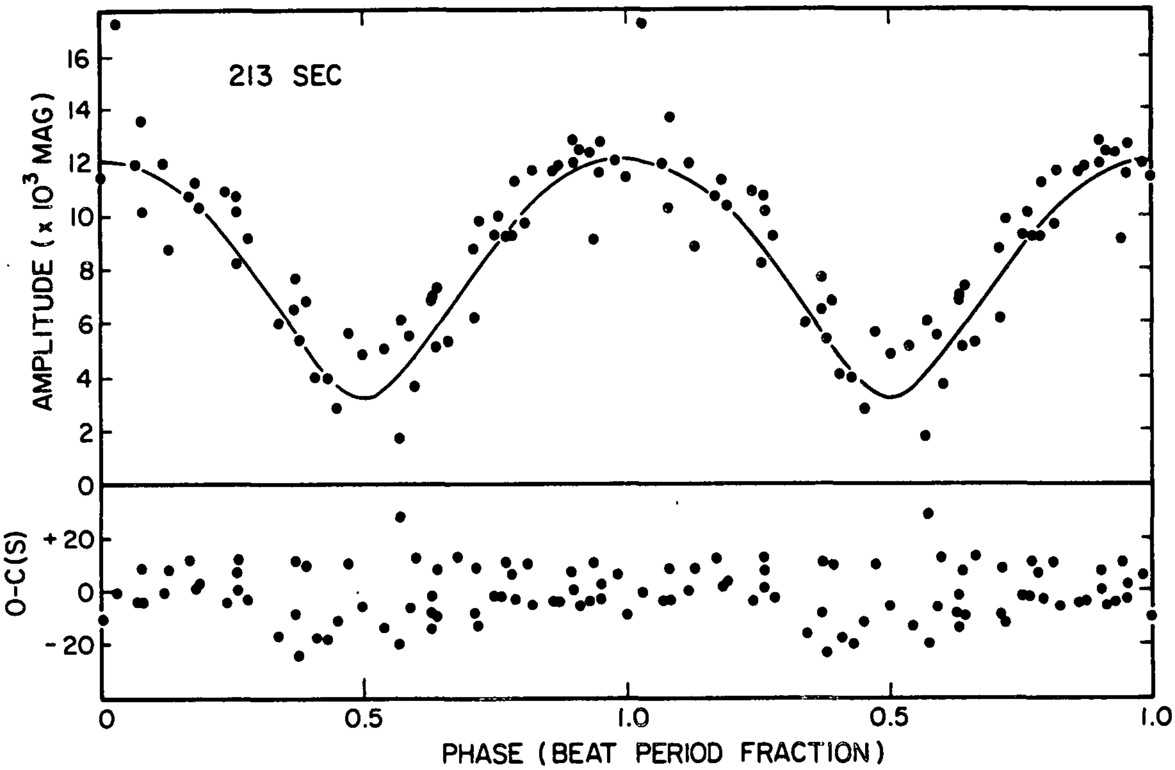

Fig. 5. The points in the upper half of the figure are the measured amplitudes of the 213 second pulsation of R548 folded on its 1.44122 day beat period. The solid line is the amplitude that results from adding the first two pulsations in Table 4. The lower half of the figure displays the difference between the observed pulse arrival times and those predicted by letting the first two pulsations of Table 4 beat against each other.

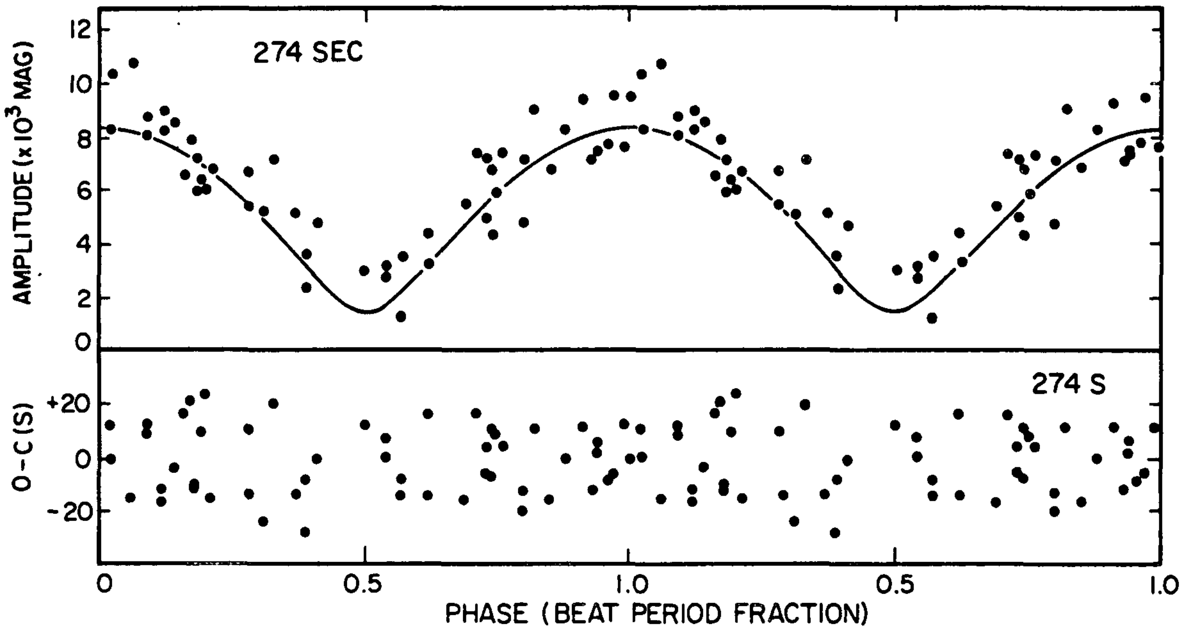

Fig. 6. This figure displays the same information for the 274 second pulsation of R548 as Figure 5 does for the 213 second pulsation.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

TABLE 4

THE FOUR PULSATIONS OF R548

PERIOD (sec)	AMPLITUDE (mag)	PERIOD CHANGE (sec/sec)
212.768427 ±3	0.0044 ±2	$< 7x10^{-13}$
213.132605 ±2	0.0077 ±2	$< 2x10^{-13}$
274.250814 ±4	0.0049 ±2	$< 3x10^{-13}$
274.774562 ±6	0.0034 ±2	$< 9x10^{-13}$

times of arrival are 11 seconds for the 213 second pair and 13 seconds for the 274 second pair, both of which are also consistent with random measurement errors.

The most notable characteristic of the pulsations of R548 is their remarkable constancy. The data on R548 now covers an interval of 9 years, and during this interval the light curve of R548 has not changed its properties. All four pulsations have remained constant in both period and amplitude. The upper limits to the rate of change of the pulsation periods are given in Table 4. The upper limit of $2x10^{-13}$ for the 213.13 second pulsation gives that pulsation the most stable period ever measured for a variable star at visual wavelengths (Stover et al. 1979).

The light curves of G29-38 and G38-29 in unfiltered light are shown in Figure 7 and are typical of the light curves of the large amplitude variables. The peak-to-peak amplitude of the variations is about 0.21 mag in G38-29 and about 0.28 mag in G29-38. The individual pulses are highly variable in shape, but typically are asymmetric with a more rapid rise than decay. The means interval between pulses is about 850 seconds but the light curve is very irregular. The times of arrival of the pulses are not predictable with any simple ephemeris. Figure 8 shows power spectra of the light curve of G29-38 on two successive nights. The power spectra are extremely complex. Literally scores of periods are simultaneously present, and the power spectra change from night to night so that neither the periods nor

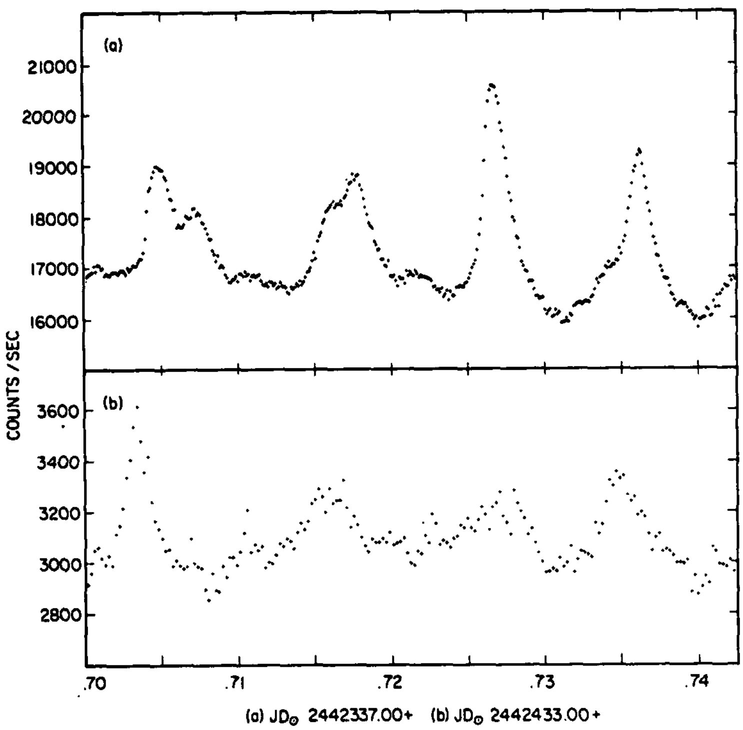

Fig. 7. (a) A portion of the light curve of G29-38. (b) A portion of the light curve of G38-29.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

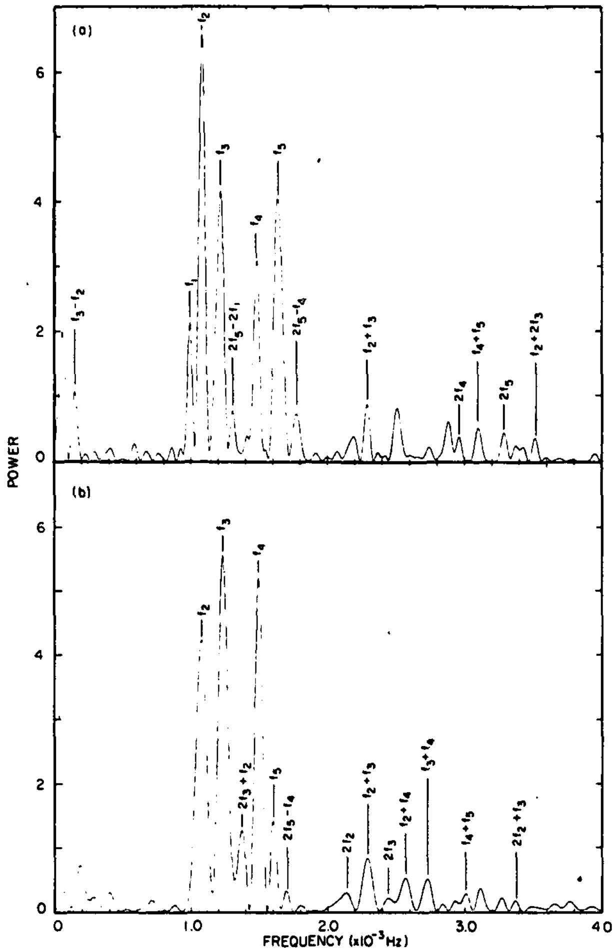

Fig. 8. Two power spectra of the light curve of G29-38. Major frequencies and a few of the frequencies identified as harmonics and linear combinations of the major frequencies are indicated. (a) From 1974 October 16. (b) From 1974 October 17.

their amplitudes are constant. Although this complexity has prevented any detailed understanding of the light curve of G29-38, a few regularities can be found in the power spectra. Periods near 694 seconds, 820 seconds, and 930 seconds usually are present in the spectra. Harmonics of the strongest periods usually are present, and are reflecting the non-sinusoidal appearance of the pulses. Cross frequencies between the strongest periods usually are present; if two strong periods with frequencies f_1 and f_2 are present in the power spectrum, $f_3 = nf_1 \pm mf_2$ is likely to be present also, where n and m are small integers.

The properties of the pulsations of all of the variables are summarized in Table 5. Several valid generalizations can be made about the pulsations. The periods are all very long. Excluding harmonics and cross frequencies, the shortest period is 114 seconds, in L19-2, and the longest period is 1186 seconds, in GD 154. With the possible exception of GD 385, the most recently found variable, every ZZ Ceti star is multi-periodic. L19-2 has at least two periods, G117-B15A has at least three periods, and R548 has four periods. The remaining variables all have large numbers of periods. In the case of HL Tau-76, Desikachary and Tomaszewski (1975) were able to identify 25 periods, but since they included only the periods with fairly large amplitudes, this is a lower limit to the true number. The stability of the periods varies from extremely high in R548 to very low in G29-38. The low stability of the periods may be only apparent and due to incomplete data for some of the stars, but in GD 154, GD 385, and BPM 31594, the instability is real. The light curves of these stars were dominated by a single pulsation period for long intervals of time, after which the pulsation abruptly changed its period or, in the case of GD 385, disappeared altogether. In GD 154, the dominant pulsation period remained at 1186 seconds for more than 30 days, but then changed to 780 seconds in less than 24 hours. This behavior cannot be explained by simple beating phenomena of the kind

TABLE 5

PROPERTIES OF THE PULSATIONS

Star	Peak to Peak Amp. (Mag)	Typical Pulse Interval (Sec)	Dominant Periods (Sec)	Period Stability	Ref.
R548	0.02	220	213 + 274	$\lvert\dot{P}\rvert < 2\times10^{-13}$	1,2,3,4
L19-2	0.04	190	114 + 192	High	5,6
GD 385	0.05	256	256	$\lvert\dot{P}\rvert < 10^{-7}$	7
G117-B15A	0.06	215	216 + 272 + 308	High	8
G207-9	0.06	300	292 + 318 + 557 + 739	High?	9
GD 154	0.10	1200	780 + 1186 + Others	$\lvert\dot{P}\rvert < 10^{-8}$	10
GD 99	0.13	350	260 + 480 + 590 + Others	Moderate	8
R808	0.15	850	513 + 830 + Others	Low	8
BPM 30551	0.18	750	607 + 745 + 823 + Others	Moderate	5,11
BPM 31594	0.21	600	311 + 404 + 617 + Others	Moderate	12
G38-29	0.21	850	925 + 1020 + Others	Low	13
G29-38	0.28	850	694 + 820 + 930 + Others	Low	13
HL Tau-76	0.34	750	494 + 626 + 661 + 746 + Others	Low	14,15,16,17

REFERENCES:
1) Lasker and Hesser (1971)
2) Robinson et al. (1976)
3) Stover et al. (1978)
4) Stover et al. (1979)
5) McGraw (1977)
6) Hesser et al. (1977)
7) Fontaine and McGraw (1979)
8) McGraw and Robinson (1976)
9) Robinson and McGraw (1976)
10) Robinson et al. (1978)
11) Hesser et al. (1976a)
12) McGraw (1976)
13) McGraw and Robinson (1975)
14) Warner and Robinson (1972)
15) Page (1972)
16) Fitch (1973)
17) Desikachary and Tomaszewski (1975)

seen in R548. Finally, there is a strong correlation between the amplitude of the pulsations and their remaining properties. The low amplitude variables in Table 5 have fewer periods in their light curves, the periods are shorter, and the periods are relatively or highly constant. The large amplitude variables have more and longer periods, and the periods are more unstable.

Pulsation Mode Identification

There can be no doubt that the pulsations of the ZZ Ceti stars are non-radial pulsations. The observed pulsation periods are two orders of magnitude too long to be radial pulsations of normal white dwarfs. The extremely close spacing of the pulsation periods in R548 could only be produced by radial pulsations if the pulsations were extremely high overtones, which would further exacerbate the period discrepancy. Non-radial pulsations avoid these problems. There is at least one group of non-radial modes - the g^+ modes - that can have periods long enough to match the observed periods (c.f. the reviews by Cox 1976 and Van Horn 1979). Furthermore, the period spectrum of the non-radial pulsations is very rich and provides a natural framework within which to understand the large number of closely spaced periods.

Even if we consider only the low order non-radial modes, a very large number of modes, and thus periods, could conceivably be excited.

The argument in favor of identifying the pulsations specifically with the g^+ pulsations is only slightly less convincing. Qualitative agreement can be found between the observed pulsation periods and the theoretical pulsation periods of g^+ modes. For example, using Brickhill's (1975) theoretical pulsation periods, one may identify the 213 second pair of pulsations in R548 as the $\ell=2$, $k=1$ (=fundamental) g^+ mode; and the 274 second pair as the $\ell=2$, $k=2$ (=first overtone) g^+ mode. The splitting of the modes into close pairs could be caused by slow rotation. A similar qualitative agreement between theoretical and observational periods has been found for HL Tau-76 (Desikachary and Tomaszewski 1975), BPM 30551 (McGraw 1977), G29-38, and G38-29 (McGraw and Robinson 1975). The most impressive argument in favor of the g^+ modes is that theoretical models of white dwarfs are now, finally, unstable to g^+ mode pulsations. Dziembowski, in his review of the theory of white dwarf pulsations presented at this Colloquium, shows that g^+ modes with $k \sim 6$ and $\ell \sim 5$ to 10 have growth rates as short as 10^{-6}.

The arguments in favor of g^+ mode pulsations are so strong that there is a danger that caveats against a too ready acceptance of this mode identification could hinder rather than help our understanding of the ZZ Ceti stars. Nevertheless, the caveats must be stated. The agreement between the theoretical and observed pulsation periods is qualitative rather than quantitative. The mode assignments are not unique, with McGraw (1977) and Hesser et al. (1976b), for example, giving completely different mode assignments to the periods of BPM 30551. Because of the non-uniqueness, and because the g^+ mode pulsations have such a rich spectrum of periods, one suspects that even a set of random numbers would agree at least qualitatively with some subset of the theoretical periods. The theoretical investigations which Dziembowski will report in his review are not yet conclusive because the g^+ modes he finds to be unstable have periods too short to match those of most of the ZZ Ceti stars. In addition, his theory would seem to require

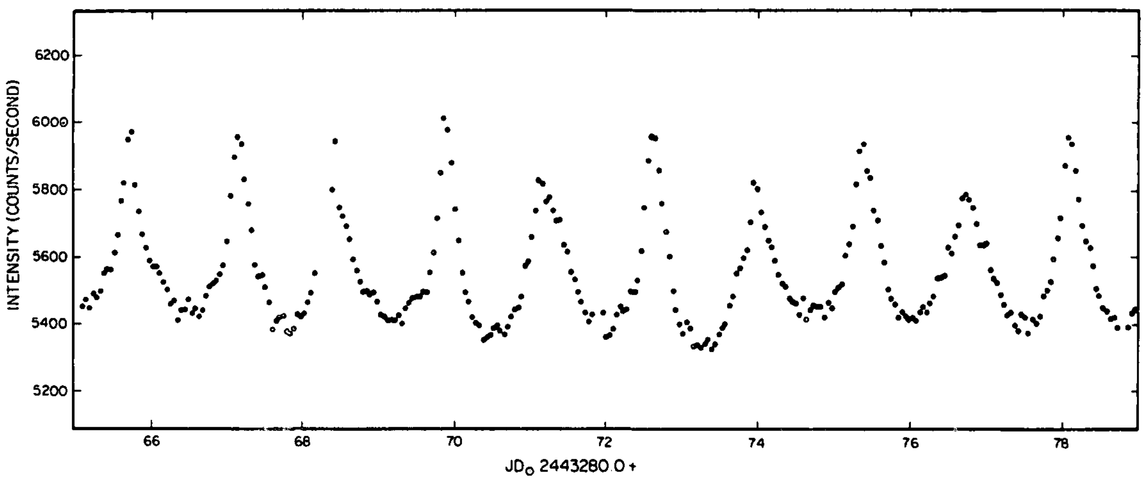

Fig. 9. The light curve of GD 154 in unfiltered light. Each point is the mean counting rate averaged over 40 seconds.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

that the DA spectral types of the ZZ Ceti stars are immaterial to the pulsations.

Perhaps the gravest difficulty with an unmodified g^+ mode interpretation is illustrated by the light curve of GD 154, a portion of which is shown in Figure 9. The amplitude of the pulsation is about 0.10 mag and the period of the pulsation is 1186 seconds (Robinson et al. 1978). GD 154 was observed for 10 nights over a one month interval in 1977. The properties of the light curve were constant on the first 9 nights, and on these nights the light curve was periodic, the rate of change of the 1186 second period being less than 10^{-8}. The power spectrum of the light curve of GD 154 is shown in Figure 10. Most of the power comes from just one dominant period, the 1186 second period, plus its harmonics. The next strongest period, at 780 seconds, is down in power by a factor of 25 from the 1186 second period. If the 1186 second period is a g^+ mode, its period is so long that it must be a very high overtone, $k \approx 20$-30. For talking purposes, let us assign it to $k = 25$. Since the 1186 second period is a single, isolated pulsation period, neither the $k = 24$ nor the $k = 26$ modes are being excited, and, in fact, no other overtone is being significantly excited.

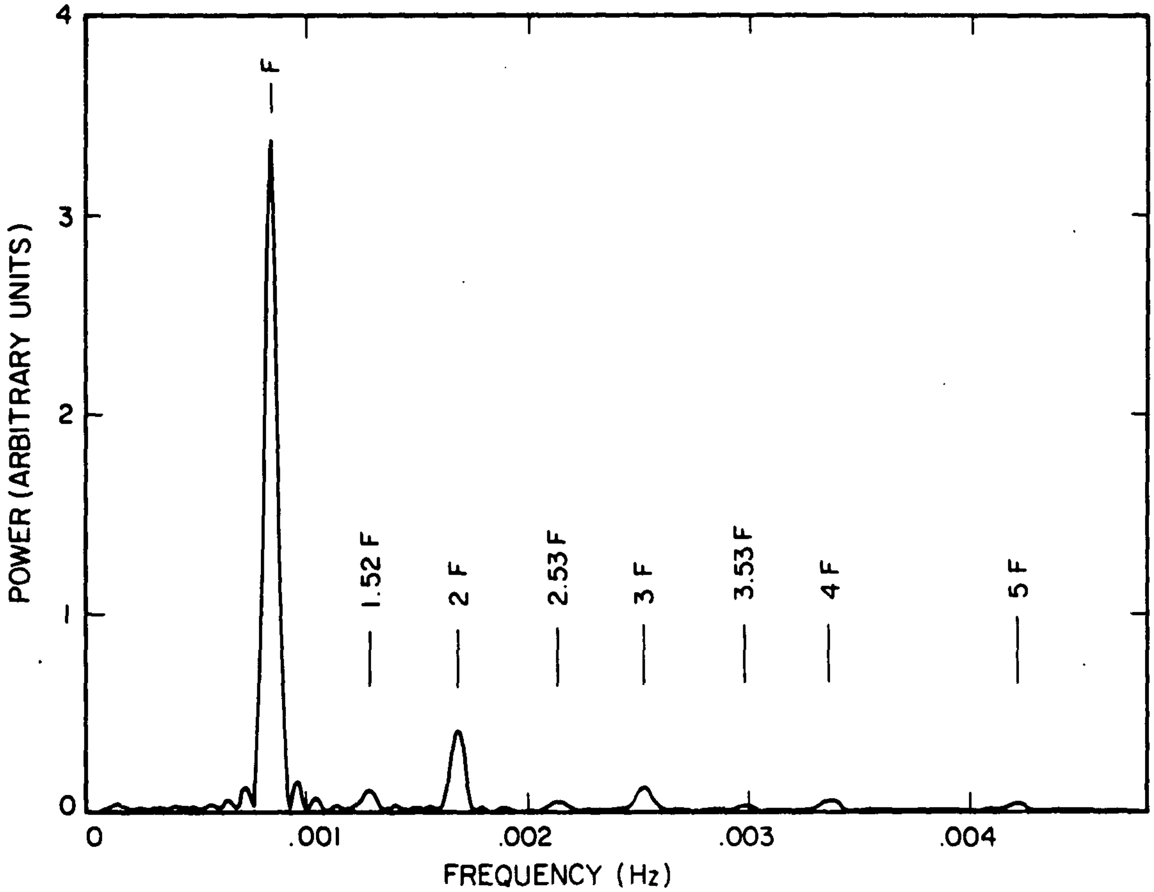

Fig. 10. The power spectrum of the light curve of GD 154. The peak labeled F is the fundamental frequency of the light curve and corresponds to the 1186 second period of the pulsations in Figure 9. The peaks labeled 2F, 3F, 4F, and 5F are harmonics of the fundamental, and the peak labeled 1.52 F is at 780 seconds.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press

One may legitimately doubt that this is possible. It is equivalent to exciting the 25th overtone of an organ pipe without exciting any other overtones. It can be done, but it is very difficult.

Conclusion

The ultimate hope in studying the ZZ Ceti stars is that their pulsations will be uniquely useful tools for probing not only the structure and evolution of white dwarfs, but also the general properties of non-radial and non-linear stellar pulsations. Since the ZZ Ceti stars are multi-periodic, measurement of their pulsation periods should eventually yield enough parameters to give accurate masses, temperatures, and even internal chemical compositions of white dwarfs. The rate of change of the periods should place limits on the cooling rates of white dwarfs. With the exception of the sun, the ZZ Ceti stars are the best examples of non-radially pulsating stars. The large amplitude ZZ Ceti stars provide excellent examples of the effects of non-linearities on stellar pulsations. Now that theoretical studies of white dwarf pulsations are finding the long period g^+ mode pulsations to be unstable, it should soon be possible to make unambiguous mode identifications. Once the mode identifications can be made, there will be good grounds for optimism that the ZZ Ceti stars will fulfill their unique potential.

The research reported here was supported in part by NSF Grants AST 76-23882 and AST-7906340. I gratefully acknowledge my continued indebtedness to R.E. Nather and J.T. McGraw for their collaboration in all aspects of this research.

References

Angel, J.R.P. 1978, private communication.
Brickhill, A.J. 1975, M.N.R.A.S. 170, 405.
Cox, J.P. 1976, Ann.Rev.Astr. and Ap. 14, 247.
Dahn, C.C., Harrington, R.S., Riepe, B.Y., Christy, J.W., Guetter, H.H., Behall, A.L., Walker, R.L., Hewitt, A.V., and Ables, H.D. 1976, Pub.U.S.Naval Obs., Second Series, 24, Part 3.
Desikachary, K., and Tomaszewski, L. 1975, IAU Colloquium No. 29, p.283.
Eggen, O.J. 1968, Ap.J. Suppl. 16, 97.
Eggen, O.J. 1969, Ap.J. 157, 287.
Eggen, O.J., and Greenstein, J.L. 1965, Ap.J. 141, 83.
Faulkner, J., Flannery, B., and Warner, B. 1972, Ap.J. (Letters) 175,L79.
Fitch, W.S. 1973, Ap.J. 181, L95.
Fontaine, G. and McGraw, J.T. 1979, preprint.
Giclas, H.L., Slaughter, C.D., and Burnham, R. 1959, Lowell Obs. Bull. 4, 136.
Greenstein, J.L. 1969, Ap.J. 158, 281.
Greenstein, J.L. 1970, Ap.J.(Letters) 162, L55.
Harper, R.V.R. and Rose, W.K. 1970, Ap.J. 162, 963.
Harrington, R.S., Dahn, C.C., Behall, A.L., Priser, J.B., Christy, J.W., Riepe, B.Y., Ables, H.D., Guetter, H.H., Hewitt, A.V., and Walker, R.L. 1975, Pub.U.S.Naval Obs., Second Series, 24, Part 1.

Hesser, J.E., and Lasker, B.M. 1971, IAU Symposium No. 42, p.41.
Hesser, J.E., and Lasker, B.M. 1972, IAU Colloquium No. 15, p.160.
Hesser, J.E., Lasker, B.M., and Neupert, H.E. 1976a, Ap.J. 209, 853.
Hesser, J.E., Lasker, B.M., and Neupert, H.E. 1976b, IAU Circular No. 2990.
Hesser, J.E., Lasker, B.M., and Neupert, H.E. 1977, Ap.J.(Letters) 215, L75.
Hesser, J.E., Ostriker, J.P., and Lawrence, G.M. 1969, Ap.J. 155, 919.
Landolt, A.U. 1968, Ap.J. 153, 151.
Lasker, B.M., and Hesser, J.E. 1970, Ap.J.(Letters) 158, L171.
Lasker, B.M., and Hesser, J.E. 1971, Ap.J.(Letters) 163, L89.
Lawrence, G.M., Ostriker, J.P., and Hesser, J.E. 1967, Ap.J.(Letters) 148, L161.
McCook, G.P., and Sion, E.M. 1977, Villanova Univ. Obs. Contributions, No. 2.
McGraw, J.T. 1976, Ap.J.(Letters) 210, L35.
McGraw, J.T. 1977, Ap.J.(Letters) 214, L123.
McGraw, J.T. 1979, Ap.J. 229, 203.
McGraw, J.T., and Robinson, E.L. 1975, Ap.J.(Letters), 200, L89.
McGraw, J.T., and Robinson, E.L. 1976, Ap.J.(Letters), 205, L155.
Nather, R.E. 1979, P.A.S.P. 90, 477.
Page, C.G. 1972, M.N.R.A.S. 159, 25P.
Richer, H.B., and Ulrych, T.J. 1974, Ap.J. 192, 719.
Riddle, R.K. 1970, Pub.U.S.Naval Obs., Second Series, 20, Part 3.
Robinson, E.L. 1973, Ap.J. 180, 121.
Robinson, E.L. 1979, Proc. Goddard Conf. on Current Problems in Stellar Pulsational Instabilities, in press.
Robinson, E.L., and McGraw, J.T. 1976, Ap.J.(Letters) 207, L37.
Robinson, E.L., Nather, R.E., and McGraw, J.T. 1976, Ap.J. 210, 211.
Robinson, E.L., Stover, R.J., Nather, R.E., and McGraw, J.T. 1978, Ap.J. 220, 614.
Routly, P.M. 1972, Pub.U.S.Naval Obs., Second Series, 20, Part 6.
Sauvenier-Goffin, E. 1949, Ann.d'Ap. 12, 39.
Shulov, O.S., and Kopatskaya, E.N. 1973, Astrofizika (SSR) 10, 117.
Sion, E.N., Lupie, O.L., and Young, K.N. 1978, P.A.S.P. 90, 154.
Smak, J. 1975, Acta Astr. 25, 227.
Stover, R.J., Hesser, J.E., Lasker, B.M., Nather, R.E., and Robinson, E.L. 1979, preprint.
Stover, R.J., Robinson, E.L., and Nather, R.E. 1978, P.A.S.P. 89, 912.
Terashita, Y., and Matsushima, S. 1969, Ap.J. 156, 203.
Van Horn, H.M. 1979, Proc. Goddard Conf. on Current Problems in Stellar Pulsational Instabilities, in press.
Warner, B., and Nather, R.E. 1970, M.N.R.A.S. 147, 21.
Warner, B., and Nather, R.E. 1972, M.N.R.A.S. 156, 1.
Warner, B., and Robinson, E.L. 1972, Nature Phys. Sci. 239, 2.
Warner, B., Van Citters, G.W., and Nather, R.E. 1970, Nature 226, 67.
Wickramasinghe, D.T., and Strittmatter, P.A. 1972, M.N.R.A.S. 160, 421.

https://doi.org/10.1017/S0252921100076181 Published online by Cambridge University Press